

Generic Existence of A Point of Coincidence For Nonlinear Mappings In A Metric Space

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In the present paper, we study a space of pairs of nonlinear self-mappings of a complete metric space and show that a point of coincidence exists for a generic (typical) pair.

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1. Introduction

Since the seminal result of Banach [2] the fixed point theory of nonexpansive mappings has been a rapidly growing field of research. See [3, 8, 10, 11, 12, 16, 17, 18, 19, 20, 21, 23, 24, 27, 28, 29, 32, 33] and the reference mentioned therein. A significant progress has been done, in particular, in studies of common fixed point problems, which have important applications in engineering and medical sciences [6, 7, 30, 32, 33]. The study of coincidence points of nonlinear mappings is an important topic of the fixed point theory [1, 4, 5, 13, 14, 15, 22].

In the present paper, we study a space of pairs of nonlinear self-mappings of a complete metric space and show that a point of coincidence exists for a generic (typical) pair. More precisely, for this space of pairs equipped with the topology of uniform convergence we show the existence of a set which is a countable intersection of open everywhere dense sets and such that every its element has a point of coincidence. Such an approach, when a certain property is investigated for the whole space and not just for its single point, has already been successfully applied in many areas of Analysis and Optimization [8, 9, 25, 27, 31].

Assume that (X, ρ) is a complete metric space. For each $x \in X$ and each $r > 0$ set

$$B(x, r) = \{y \in X : \rho(x, y) \leq r\}.$$

For each $x \in X$ and each set $A \subset X$ put

$$\rho(x, A) = \inf\{\rho(x, y) : y \in A\}.$$

For each mapping $S : X \rightarrow X$ set $S^0(x) = x$ for each $x \in X$ and for each integer $i \geq 0$ define $S^{i+1} = S \circ S^i$.

Assume that $S : X \rightarrow X$ and $T : X \rightarrow X$. If $x \in X$ and $S(x) = T(x)$, then the point x is called a coincidence point, while the point $y = T(x)$ is called a point of coincidence.

2. The spaces of mappings and the main result

Assume that the metric space (X, ρ) is bounded. Define

$$\text{diam}(X) = \sup\{\rho(x, y) : x, y \in X\} < \infty. \quad (2.1)$$

Denote by \mathcal{M} the set of all mappings $A : X \rightarrow X$. For each $A_1, A_2 \in \mathcal{M}$ set

$$d(A_1, A_2) = \sup\{\rho(A_1(x), A_2(x)) : x \in X\}. \quad (2.2)$$

It is clear that (\mathcal{M}, d) is a complete metric space. Consider the product space

$$\mathcal{M}^2 = \mathcal{M} \times \mathcal{M}$$

equipped with the metric

$$d_1((A_1, A_2), (B_1, B_2)) = d(A_1, B_1) + d(A_2, B_2) \quad (2.3)$$

for each $A_1, A_2, B_1, B_2 \in \mathcal{M}$.

Fix $\theta \in X$. Assume that for each $x \in X$ and each $\gamma \in [0, 1]$, there exists a point denoted as

$$\gamma\theta \oplus (1 - \gamma)x \in X$$

such that for each $x, y \in X$ and each $\gamma \in [0, 1]$ the following properties hold:

$$\rho(\gamma\theta \oplus (1 - \gamma)x, \gamma\theta \oplus (1 - \gamma)y) \leq (1 - \gamma)\rho(x, y), \quad (2.4)$$

$$\rho(\gamma\theta \oplus (1 - \gamma)x, x) \leq \gamma\rho(x, y) \leq \gamma\text{diam}(X). \quad (2.5)$$

It should be mentioned the Hilbert ball with the hyperbolic metric [12] and the hyperbolic spaces of [26] are examples of our space X .

For each $A \in \mathcal{M}$ and each $\gamma \in (0, 1)$ define

$$A_\gamma(x) = \gamma\theta \oplus (1 - \gamma)A(x), \quad x \in X. \quad (2.6)$$

By (2.2), (2.5) and (2.6), for each $A \in \mathcal{M}$ and each $\gamma \in (0, 1)$,

$$d(A_\gamma, A) \leq \gamma\text{diam}(X) \rightarrow 0 \text{ as } \gamma \rightarrow 0^+. \quad (2.7)$$

In view of (2.4) and (2.6), for each $A \in \mathcal{M}$, each $\gamma \in (0, 1)$ and each $x, y \in X$,

$$\rho(A_\gamma(x), A_\gamma(y)) \leq (1 - \gamma)\rho(A(x), A(y)). \quad (2.8)$$

In this paper we prove the following generic result.

Theorem 2.1. *Assume that \mathcal{A} is a nonempty closet set in a metric space $(\mathcal{M} \times \mathcal{M}, d_1)$ such that for each $(T, S) \in \mathcal{A}$ and each $\gamma \in (0, 1)$,*

$$\rho(T(x), T(y)) \leq \rho(S(x), S(y)) \text{ for each } x, y \in X, \quad (2.9)$$

$$T(X) \subset S(X), (T_\gamma, S) \in \mathcal{A}. \quad (2.10)$$

Then there exists a set $\mathcal{F} \subset \mathcal{A}$ which is a countable intersection of open everywhere dense sets in \mathcal{A} such that for each $(T, S) \in \mathcal{A}$ there exists $y_{T,S} \in X$ such that the following assertions hold.

1. For each pair of sequences $\{x_n\}_{n=0}^{\infty}, \{y_n\}_{n=0}^{\infty} \subset X$ satisfying for each integer $n \geq 0$,

$$y_n = T(x_n), S(x_{n+1}) = y_n$$

the relation $\lim_{n \rightarrow \infty} y_n = y_{T,S}$ holds.

2. If $x \in X$ satisfies $T(x) = S(x)$, then $T(x) = y_{T,S}$.
3. If $S(X)$ is a closed set and the mapping T is continuous, then there exists $x_* \in X$ such that $T(x_*) = S(x_*) = y_{T,S}$.
4. For each $\varepsilon \in (0, 1)$ there exist a neighborhood \mathcal{U} of (T, S) in $\mathcal{M} \times \mathcal{F}$, $\delta \in (0, \varepsilon)$ and a natural number k such that for each $(A, B) \in \mathcal{U}$ and each pair of sequences $\{x_n\}_{n=0}^{\infty}, \{y_n\}_{i=0}^{\infty} \subset X$ satisfying for each integer $n \geq 0$,

$$\rho(y_n, A(x_n)) \leq \delta, \rho(B(x_{n+1}), y_n) \leq \delta$$

the inequality

$$\rho(y_n, y_{T,S}) \leq \varepsilon$$

holds for each integer $n \geq k$.

3. Auxiliary results

Assume that $\gamma \in (0, 1)$ and $T, S : X \rightarrow X$ satisfy for each $x, y \in X$,

$$\rho(T(x), T(y)) \leq \rho(S(x), S(y)), \quad (3.1)$$

$$T(X) \subset S(Y). \quad (3.2)$$

Lemma 3.1. For each $x, y \in X$,

$$\rho(T_\gamma(x), T_\gamma(y)) \leq (1 - \gamma)\rho(S(x), S(y)).$$

Proof. By (2.8) and (3.1), for each $x, y \in X$,

$$\rho(T_\gamma(x), T_\gamma(y)) \leq (1 - \gamma)\rho(T(x), T(y)) \leq (1 - \gamma)\rho(S(x), S(y)).$$

Lemma 3.1 is proved.

Lemma 3.2. Let $\varepsilon \in (0, 1)$,

$$\delta = 2^{-1} \varepsilon \gamma \quad (3.3)$$

and let $x, y \in X$ satisfy

$$\rho(T_\gamma(x), S(x)) \leq \delta, \rho(T_\gamma(y), S(y)) \leq \delta. \quad (3.4)$$

Then

$$\rho(T_\gamma(x), T_\gamma(y)) < \varepsilon.$$

Proof. By Lemma 3.1 and (3.3), (3.4),

$$\begin{aligned} \rho(T_\gamma(x), T_\gamma(y)) &\leq (1 - \gamma)\rho(S(x), S(y)) \\ &\leq (1 - \gamma)(\rho(S(x), T_\gamma(x)) + \rho(T_\gamma(x), T_\gamma(y)) + \rho(T_\gamma(y), S(y))) \\ &\leq 2\delta + (1 - \gamma)\rho(T_\gamma(x), T_\gamma(y)), \\ \gamma\rho(T_\gamma(x), T_\gamma(y)) &\leq 2\delta \end{aligned}$$

and

$$\rho(T_\gamma(x), T_\gamma(y)) \leq 2\delta\gamma^{-1} \leq \varepsilon.$$

Lemma 3.2 is proved.

Lemma 3.3. Assume that $\delta \in (0, 1)$ and that $\{x_n\}_{n=0}^\infty, \{y_n\}_{n=0}^\infty \subset X$ satisfy each integer $n \geq 0$,

$$\rho(y_n, T_\gamma(X_n)) \leq \delta, \rho(S(X_{n+1}), y_n) \leq \delta. \quad (3.5)$$

Then for each integer $n \geq 1$,

$$\rho(y_n, y_{n+1}) \leq (1 - \gamma)\rho(y_{n-1}, y_n) + 4\delta$$

and

$$\rho(y_n, y_{n+1}) \leq (1 - \gamma)^n \rho(y_0, y_1) + 4\delta \sum_{i=1}^{n-1} (1 - \gamma)^i.$$

Proof. By Lemma 3.1 and (3.5),

$$\begin{aligned} \rho(y_n, y_{n+1}) &\leq \rho(y_n, T_\gamma(x_{n+1})) + \rho(T_\gamma(x_{n+1}), y_{n+1}) \\ &\leq \delta + \rho(T_\gamma(x_{n+1}), T_\gamma(x_n)) + \rho(T_\gamma(x_n), y_n) \\ &\leq 2\delta + \rho(T_\gamma(x_{n+1}), T_\gamma(x_n)) \\ &\leq 2\delta + (1 - \gamma)\rho(S(x_{n+1}), S(x_n)) \\ &\leq 2\delta + (1 - \gamma)(\rho(y_{n-1}, y_n) + 2\delta) \leq (1 - \gamma)\rho(y_n, y_{n-1}) + 4\delta. \end{aligned} \quad (3.6)$$

In view of (3.6),

$$\rho(y_1, y_2) \leq (1 - \gamma)\rho(y_0, y_1) + 4\delta$$

and

$$\begin{aligned} \rho(y_2, y_3) &\leq (1 - \gamma)\rho(y_1, y_2) + 4\delta \\ &\leq (1 - \gamma)^2 \rho(y_0, y_1) + 4\delta(2 - \gamma). \end{aligned}$$

We show by induction that for each integer $n \geq 1$,

$$\rho(y_n, y_{n+1}) \leq (1 - \gamma)^n \rho(y_0, y_1) + 4\delta \sum_{i=1}^{n-1} (1 - \gamma)^i. \quad (3.7)$$

By the relations above (3.7) holds for $n = 1, 2$. Assume that $k \geq 2$ is an integer and (3.7) holds for $n = k$. It follows from (3.6) and (3.7) that

$$\begin{aligned} \rho(y_{k+1}, y_{k+2}) &\leq (1 - \gamma)\rho(y_k, y_{k+1}) + 4\delta \\ &\leq (1 - \gamma)^{k+1} \rho(y_0, y_1) + 4\delta \sum_{i=1}^{k-1} (1 - \gamma)^{i+1} + 4\delta \\ &\leq (1 - \gamma)^{k+1} \rho(y_0, y_1) + 4\delta \sum_{i=1}^k (1 - \gamma)^i. \end{aligned}$$

Thus (3.7) holds for $n = k + 1$ too. Therefore we showed by induction that (3.7) holds for each integer $n \geq 1$. Lemma 3.3 is proved.

4. Proof of theorem 2.1

By (2.7), the set

$$\{(T_\gamma, S) : (T, S) \in \mathcal{A}, \gamma \in (0, 1)\}$$

is everywhere dense in \mathcal{A} . Let $(T, S) \in \mathcal{A}$, $\gamma \in (0, 1)$ and let $m \geq 1$ be an integer. Choose

$$\varepsilon_m = \varepsilon_m(T, S, \gamma) = 2^{-m-2}\gamma \quad (4.1)$$

$$\Delta_m = \Delta_m(T, S, \gamma) \in (0, 8^{-2}\gamma\varepsilon_m) \quad (4.2)$$

and a natural number

$$k_m = k_m(T, S, \gamma, m)$$

such that

$$(1 - \gamma)^{k_m}(\text{diam}(K) + 1) < 8^{-1}\varepsilon_m(T, S, \gamma). \quad (4.3)$$

Let $\mathcal{U}(T, S, \gamma, m)$ be an open neighborhood of (T_γ, S) in the space $\mathcal{M} \times \mathcal{M}$ such that for each $(A, B) \in \mathcal{U}(T, S, \gamma, m)$ and each $x \in X$,

$$\rho(A(x), T_\gamma(x)) \leq \Delta_m, \rho(B(x), S(x)) \leq \Delta_m. \quad (4.4)$$

Define

$$\mathcal{F} = \bigcap_{p=1}^{\infty} \{\mathcal{U}(T, S, \gamma, m) \cap \mathcal{A} : (T, S) \in \mathcal{A}, \gamma \in (0, 1), m \geq p \text{ is an integer}\}. \quad (4.5)$$

Clearly, \mathcal{F} is a countable intersection of open everywhere dense sets in \mathcal{A} . Assume that

$$(T, S) \in \mathcal{F} \quad (4.6)$$

and $\varepsilon \in (0, 1)$. Choose a natural number

$$p > \varepsilon^{-1}. \quad (4.6)$$

By (4.5) and (4.6), there exist

$$(T_p, S_p) \in \mathcal{A}, \gamma_p \in (0, 1) \text{ and an integer } m \geq p \quad (4.7)$$

such that

$$(T, S) \in \mathcal{U}(T_p, S_p, \gamma_p, m). \quad (4.8)$$

Assume that

$$(A, B) \in \mathcal{U}(T_p, S_p, \gamma_p, m) \quad (4.9)$$

and that a pair of sequences $\{x_i\}_{i=0}^{\infty}, \{y_i\}_{i=0}^{\infty} \subset X$ satisfies for each integer $i \geq 0$,

$$\rho(y_p, A(x_i)) \leq \Delta_m, \rho(B(x_{i+1}), y_i) \leq \Delta_m. \quad (4.10)$$

By (4.4), (4.9) and (4.10), for each integer $i \geq 0$,

$$\rho(y_p, (T_p)_{\gamma_p}(x_i)) \leq \rho(y_p, A(x_i)) + \rho(A(x_i), (T_p)_{\gamma_p}(x_i)) \leq 2\Delta_m, \quad (4.11)$$

$$\rho(S_p(x_{i+1}), y_i) \leq \rho(S_p(x_{i+1}), B(x_{i+1})) + \rho(B(x_{i+1}), y_i) \leq 2\Delta_m. \quad (4.12)$$

Lemma 3.3 and (4.3), (4.12) imply that for each integer $n \geq k_m$,

$$\begin{aligned} \rho(y_n, y_{n+1}) &\leq (1 - \gamma)^n \text{diam}(X) + 8\Delta_m \gamma^{-1} \\ &\leq (1 - \gamma)^{k_m} \text{diam}(X) + 8\Delta_m \gamma^{-1} \end{aligned} \quad (4.13)$$

and

$$\rho(y_n, y_{n+1}) \leq 4^{-1} \varepsilon_m. \quad (4.14)$$

It follows from (4.2), (4.3), (4.12) and (4.14) that for each integer $n \geq k_m + 1$,

$$\begin{aligned} \rho(T_p)_\gamma(x_n), S_p(x_n) &\leq \rho(T_p)_\gamma(x_n), y_n + \rho(y_n, y_{n-1}) + \rho(y_{n-1}, S_p(x_n)) \\ &\leq 4\Delta_m + 8\Delta_m \gamma^{-1} + (1 - \gamma)^{k_m} \text{diam}(X) \\ &\leq 4^{-1} \varepsilon_m + 8^{-1} \varepsilon_m < 2^{-1} \varepsilon_m. \end{aligned} \quad (4.15)$$

Lemmas 3.1 and 3.2 and (4.1) imply that for each pair of integers $q, n \geq k_m + 1$,

$$\rho(T_p)_\gamma(x_n), (T_p)_\gamma(x_q) \leq \gamma^{-1} \varepsilon_m \leq 2^{-m-2}. \quad (4.16)$$

Assume that

$$(\tilde{A}, \tilde{B}) \in \mathcal{U}(T_p, S_p, \gamma_p, m), \quad (4.17)$$

$\{\tilde{x}_i\}_{i=0}^\infty, \{\tilde{y}_i\}_{i=0}^\infty \subset X$ satisfies for each integer $i \geq 0$,

$$\rho(\tilde{y}_p, \tilde{A}(\tilde{x}_i)) \leq \Delta_m, \rho(\tilde{B}(\tilde{x}_{i+1}), \tilde{y}_i) \leq \Delta_m. \quad (4.18)$$

It follows from (4.4) (4.17) and (4.18) that for each integer $i \geq 0$,

$$\rho(\tilde{y}_p, (T_p)_\gamma(\tilde{x}_i)) \leq 2\Delta_m, \rho(S_p(\tilde{x}_{i+1}), \tilde{y}_i) \leq 2\Delta_m.$$

By (4.11), (4.12), (4.14) and (4.15), for each integer $n \geq k_m$,

$$\rho(\tilde{y}_n, \tilde{y}_{n+1}) \leq 4^{-1} \varepsilon_m,$$

for each integer $n \geq k_m + 1$,

$$\rho((T_p)_\gamma(\tilde{x}_n), S_p(\tilde{x}_n)) < 2^{-1} \varepsilon_m. \quad (4.19)$$

Lemma 3.2 and equations (4.1), (4.15) and (4.19) imply that for each pair of integers $n, q \geq k_m + 1$,

$$\rho((T_p)_\gamma(x_n), (T_p)_\gamma(\tilde{x}_q)) \leq \gamma^{-1} \varepsilon_m < 2^{-m-2}. \quad (4.20)$$

By (4.2), (4.12), (4.18) and (4.20), for each pair of integers $q, n \geq k_m + 1$,

$$\begin{aligned} \rho(y_n, \tilde{y}_q) &\leq \rho(y_n, (T_p)_\gamma(x_n)) + \rho((T_p)_\gamma(x_n), (T_p)_\gamma(\tilde{x}_q)) \\ &+ \rho((T_p)_\gamma(\tilde{x}_q), \tilde{y}_q) < 2^{-m-2} + \Delta_m < 2^{-m-2} + \varepsilon_m < 2^{-m}. \end{aligned}$$

Thus we have shown that the following property holds:

(P1) There exists a neighborhood \mathcal{U} of (T, S) in $\mathcal{M} \times \mathcal{M}$, $\Delta \in (0, \varepsilon)$ and a natural number k such that for each

$$(A, B), (\tilde{A}, \tilde{B}) \in \mathcal{U}$$

and each $\{x_i\}_{i=0}^\infty, \{y_i\}_{i=0}^\infty, \{\tilde{x}_i\}_{i=0}^\infty, \{\tilde{y}_i\}_{i=0}^\infty \subset X$ satisfying for each integer $i \geq 0$,

$$\begin{aligned} \rho(\tilde{y}_p, A(\tilde{x}_i)) &\leq \Delta, \rho(\tilde{B}(\tilde{x}_{i+1}), \tilde{y}_i) \leq \Delta, \\ \rho(y_p, A(x_i)) &\leq \Delta, \rho(B(x_{i+1}), y_i) \leq \Delta \end{aligned}$$

we have

$$\rho(y_n, \tilde{y}_q) < \varepsilon$$

for each pair of integers $n, q \geq k$.

Assume that $\{x_i\}_{i=0}^\infty, \{y_i\}_{i=0}^\infty \subset X$ and that for each integer $i \geq 0$,

$$y_i = T(x_i), S(x_{i+1}) = y_i. \quad (4.21)$$

Property (P1) and equation (4.21) imply that $\{y_i\}_{i=0}^\infty$ is a Cauchy sequence. Therefore there exists

$$y_{TS} = \lim_{i \rightarrow \infty} y_i. \quad (4.22)$$

Property (P1) and (4.21), (4.22) imply that for each

$$(A, B) \in \mathcal{U}$$

and each $\{x_i\}_{i=0}^\infty, \{y_i\}_{i=0}^\infty \subset X$ satisfying for each integer $i \geq 0$,

$$\rho(y_p, A(x_i)) \leq \Delta, \rho(B(x_{i+1}), y_i) \leq \Delta$$

we have

$$\rho(y_n, y_*) \leq 2\varepsilon$$

for each integer $n \geq k$. Note that ε is any sufficient small positive number and y_{TS} does not depend on ε .

By (P1), if $x \in X$ satisfies $T(x) = S(x)$, then $T(x) = y_{TS}$.

Assume that $x_* \in X$ and that

$$S(x_*) = y_{TS}.$$

Then for each $\{x_i\}_{i=0}^\infty, \{y_i\}_{i=0}^\infty \subset X$ satisfying (4.21) for each integer $i \geq 0$, we have

$$\rho(T(x_*), T(x_n)) \leq \rho(S(x_*), y_n) = \rho(y_{TS}, y_n) \rightarrow 0 \text{ as } n \rightarrow \infty$$

and

$$T(x_*) = \lim_{n \rightarrow \infty} T(x_n) = \lim_{n \rightarrow \infty} y_n = y_{TS}.$$

Theorem 2.1 is proved.

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