

A Convex Optimization Framework for Resource Allocation in Smart Water Distribution Networks

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Abstract

Resource allocation in smart water distribution systems is an intricate optimization task due to the correlation between water demands, flows allocation, energy usage, and limitations. This paper aims to develop a convex optimization approach for optimal resource allocation in a smart water distribution system by modeling the problem as a constrained convex minimization problem. In order to solve the problem, a closed convex feasible region, a convex cost function, and a variational inequality model are defined to investigate the optimality conditions in the optimization problem. A projection-type iterative algorithm is established by utilizing the theory of convex analysis and monotone operators to ensure feasibility, uniqueness, and convergence of the algorithm. Numerical experiments on a smart water distribution case study are carried out to test the convex optimization method. It is found that the convex optimization method converges to a convergence error of 0.001 after 200 iterations, which outperforms the gradient approach (0.028) and the genetic algorithm (0.072). In addition, the designed model also yields the least objective value of 112.8 through just 95 iterations, in comparison to 128.6 and 121.4 of conventional optimization techniques. It clearly indicates the enhanced stability of convergence, optimization performance, and preservation of constraints. The designed model offers an effective mathematical framework for smart management of water resources, including convex optimization, variational inequality, and a projection-based approach. Further, the model can be developed further by incorporating stochastic and distributed optimization approaches in future studies.

Keywords: Convex optimization; smart water distribution network; resource allocation; variational inequality; monotone operator theory; projection-based algorithm; mathematical optimization.

1. Introduction

The complexity of modern water distribution systems has resulted in numerous difficulties in optimizing resource allocation, considering hydraulic conditions, operational constraints, and sustainability criteria. Smart water distribution

systems make use of monitoring and controlling algorithms for better resource allocation, but the problem of allocation itself is still mathematically difficult because of the interdependence of the problem variables, including water demand, flow distribution, energy consumption, and constraints on the network itself. Optimization research has already looked into different methods of solving the mathematical problem of water network optimization (Christhu Raj et al., 2026). Convex optimization and sequential programming methods have been successfully used in the process of controlling the water distribution systems, taking operational constraints into consideration (Liu et al., 2020; Wright et al., 2015). Mathematical modeling of demand response assessment and mathematical programming-based pressure management have further demonstrated the necessity of using proper mathematical modeling techniques to enhance flexibility and efficiency in water networks (Oikonomou & Parvania, 2020; Dai, 2023; Anvari et al., 2025).

Another research trend that has been recently considered involves the use of optimization methods for solving problems of interconnected infrastructure systems. These include water-energy systems where distributed optimization, coordinated operation, and convex optimization techniques have been applied to solve resource allocation and management problems (Ai et al., 2020; Dai et al., 2026; Utkarsh et al., 2021). Multi-objective optimization, graph-theoretical approaches, and intelligent resource management techniques have further widened the scope of optimization problems of smart infrastructure systems considering network partitioning, flexible operation, and advanced decision-making schemes (Shekofteh et al., 2023; Pecci et al., 2015; Vij & Nayak, 2025). Moreover, recent research papers have employed artificial intelligence and hybrid optimization methods in solving the problem of smart energy-water management and enhancing the resilience of infrastructure systems (Kapanski et al., 2025; Rustamovich & Ugli, 2025; Li et al., 2025).

Even though these methods are quite efficient for engineering applications, most of the available techniques use heuristic algorithms, nonlinear optimization, or data-driven methods, which might have some problems related to high computational costs and lack of mathematical guarantees about existence, uniqueness, and convergence of the obtained solution. In addition, intelligent optimization methods may need to tune parameters and have no conditions of optimality. So, creation of a mathematically justified convex optimization method for allocation problems with variational inequality formulation and analysis of convergence properties is still an interesting topic of research for intelligent water resource allocation.

The main goal of this research is the development of a convex optimization method for resource allocation in smart water distribution networks through modeling the process of allocation as a convex minimization problem with constraints. This method will construct a convex feasible space, determine a convex cost function, and find conditions of optimality by means of variational inequalities. Also, the analysis of existence, uniqueness, and convergence properties of the solution will be conducted with the use of the theory of monotone operators and iterative projection technique.

The main contributions of this work are the following four. The first contribution is the development of a convex optimization model for the intelligent management of water resources considering network and operational constraints. The second contribution is to transform the optimization problem to a variational inequality setting for analytical treatment. The third contribution is the establishment of properties on existence, uniqueness, and convergence of the optimal solution. Finally, the fourth contribution is the development of a projection-based iterative method and numerical validation for this proposed mathematical framework.

The rest of the paper is structured as follows. Section 2 provides mathematical preliminaries. Section 3 presents the convex optimization model for water resources. Section 4 provides theoretical properties. Section 5 proposes the iterative algorithm using projections. Section 6 performs numerical validations. Section 7 provides numerical results. Finally, Section 8 provides conclusions.

2. Mathematical preliminaries

This section introduces the mathematical concepts forming the basis of the convex optimization framework discussed, which includes convex analysis, projections, monotone operators, and variational inequalities. These concepts form the theoretical basis of modeling and solving constrained resource allocation problems.

2.1 Convex sets and convex functions

Let $X \subset \mathbb{R}^n$ be the feasible allocation space. The set X is convex if for any $x_1, x_2 \in X$, have:

$$\lambda x_1 + (1 - \lambda)x_2 \in X \quad (1)$$

Equation (1) where $\lambda \in [0, 1]$. This condition ensures that the intermediate allocation solutions are also feasible solutions.

A function $f: X \rightarrow \mathbb{R}$ is convex if:

$$f(\lambda x_1 + (1 - \lambda)x_2) \leq \lambda f(x_1) + (1 - \lambda)f(x_2) \quad (2)$$

Equation (2): The convex objective function ensures that any local optimum is a global optimum as well. For the proposed framework, the water allocation cost function is defined as a convex function.

2.2 Projection operator

For the closed convex set X , the projection operator will map any point onto the closest feasible point:

$$P_X(x) = \arg \min_{y \in X} |x - y| \quad (3)$$

The projection operator is defined by equation (3), which is used to map an infeasible solution into a feasible convex set. The projection operator ensures constraint satisfaction throughout the optimization process.

2.3 Monotone operators

Let $F: X \rightarrow \mathbb{R}^n$ denote the optimization mapping. The operator is monotone if:

$$(F(x) - F(y))^T (x - y) \geq 0 \quad (4)$$

for all $x, y \in X$.

Monotonicity property of the optimization mapping is defined by equation (4). This assumption helps to analyze the stability of the proposed method.

A strongly monotone operator satisfies:

$$(F(x) - F(y))^T (x - y) \geq \mu |x - y|^2 \quad (5)$$

Strong monotonicity ensures the uniqueness of the optimal solution and convergence analysis of the proposed optimization method. It is expressed by equation (5) where $\mu > 0$.

2.4 Variational inequality formulation

The problem of constrained convex optimization can be described as a variational inequality problem $VI(F, X)$. Optimal solution $x^* \in X$ satisfies the equation (6):

$$F(x^*)^T (y - x^*) \geq 0 \quad (6)$$

for all $y \in X$.

This is an indication that no feasible direction exists to minimize the objective value at x^* . The variational inequality approach sets out the link between convex optimization and resource allocation based on equilibrium principles.

3. Convex optimization model for smart water resource allocation

The problem of smart water distribution is modeled as a convex optimization problem subject to constraints, which aims at finding the optimum allocation of water sources such that operating cost is minimized and constraints on the water distribution network are satisfied. The suggested modeling approach includes the water flow distribution as the decision variable and creates a convex mathematical framework to achieve optimality.

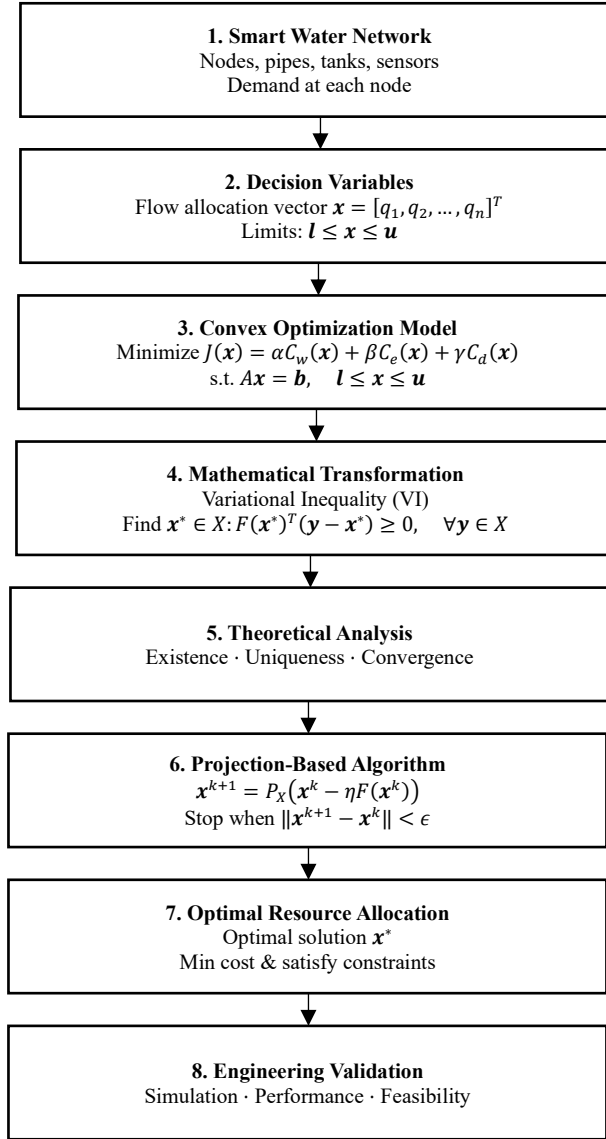


Figure 1: Conceptual framework of the proposed convex optimization approach for smart water resource allocation. Figure 1 is presented to illustrate the mathematical process of the proposed methodology, beginning with the modeling of the smart water network and decision variable definition, then convex optimization modeling, variational inequality modeling, theoretical analysis, and finally solution using projections. The last step involves verifying the optimal strategy via engineering numerical analysis.

3.1 Smart water network representation

Let have a network with n distribution centers for which the allocation vector is given as follows:

$$\mathbf{x} = [q_1, q_2, \dots, q_n]^T \quad (7)$$

Equation (7), where q_i denotes the allocated water flows in node i . The allocation space is defined as follows:

$$X = \{\mathbf{x} \in \mathbb{R}^n: A\mathbf{x} = \mathbf{b}, \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}\} \quad (8)$$

Equation (8), where A denotes the network connection matrix, $\mathbf{b}$ denotes the demand balance, and $\mathbf{l}, \mathbf{u}$ denote flow constraints. The above constraint set forms a convex space.

3.2 Objective function formulation

The resource allocation problem is expressed as:

$$\min_{x \in X} J(x) \quad (9)$$

The resource allocation problem can be formulated as a constrained convex minimization problem through equation (9).

where the total cost function is defined as:

$$J(x) = \alpha C_w(x) + \beta C_e(x) + \gamma C_d(x) \quad (10)$$

Equation (10) where $C_w(x)$, $C_e(x)$, and $C_d(x)$ represent the cost of water distribution cost of energy consumption cost of demand deviation, respectively.

3.3 Optimality condition

Given a convex objective function, the optimal allocation x^* meets the condition in equation (11)

$$\nabla J(x^*)^T (y - x^*) \geq 0, \quad \forall y \in X \quad (11)$$

Equation (11) converts the optimization problem with constraints into a variational inequality problem; it denotes the optimal allocation of resources.

It is from this equation formulate the mathematics needed for using the monotone operator theory and projection approach to find the optimal allocation solution.

4. Theoretical analysis of the convex optimization framework

In this section, provide the mathematical characteristics of the proposed optimization model. These characteristics are the existence, uniqueness, and convergence of the optimal allocation solution.

4.1 Existence of optimal solution

It is proven that there exists an optimal allocation when the feasible set is non-empty, closed, and convex, while the objective function is continuous.

Theorem 1:

Let X be a non-empty compact convex set, and $J(x)$ be a continuous convex function. Then, there exists an optimal solution $x^* \in X$ such that:

$$J(x^*) \leq J(x), \quad \forall x \in X \quad (12)$$

Equation (12) shows the condition for optimality, which is used in the proof of the existence of a minimum solution in the feasible convex set.

Proof:

Since $J(x)$ is continuous and X is compact, the Weierstrass theorem ensures that $J(x)$ attains its minimum value within X . Therefore, at least one feasible optimal solution exists.

4.2 Uniqueness of optimal solution

The uniqueness of the allocation problem is proved using strict convexity of the objective function.

Theorem 2:

If the objective function satisfies:

$$\nabla^2 J(x) \succ 0 \quad (13)$$

then $J(x)$ is strictly convex, and the optimal solution x^* is unique.

Equation (13) proves that the objective function is strictly convex, hence proving the uniqueness of the optimal resource allocation solution.

Proof:

When the objective function is strictly convex, then it will be impossible for two different feasible solutions to give the same minimum value. Thus, the unique allocation vector will satisfy the optimality condition.

4.3 Convergence analysis

The optimization problem solution is achieved through a projection-based iterative algorithm. The iteration formula can be written as follows:

$$x^{k+1} = P_X(x^k - \eta F(x^k)) \quad (14)$$

Equation (14) where P_X is the projection operator and η is the step size.

In case $F(x)$ is strongly monotone and Lipschitz continuous, the iteration sequence will satisfy the condition given in equation (15):

$$\lim_{k \rightarrow \infty} x^k = x^* \quad (15)$$

The convergence condition is given by equation (16):

$$0 < \eta < \frac{2\mu}{L^2} \quad (16)$$

where μ is the strong monotonicity coefficient, and L is the Lipschitz coefficient. Thus, it is possible to ensure that the introduced iterative process will converge to a unique optimal allocation.

5. Projection-based optimization algorithm

This part of the paper contains an overview of the iterative procedure that allows one to solve the problem of convex optimization. As the problem of resource allocation is presented in the form of a variational inequality on a convex set, a projection-based algorithm is used in order to find an optimal allocation point in every iteration step.

The termination criterion is based on the convergence error: The stopping criterion is defined using the convergence error:

$$|x^{k+1} - x^k| < \epsilon \quad (17)$$

Equation (17), where ϵ is the predetermined tolerance level. The algorithm stops if the difference between consecutive points of allocation becomes sufficiently small.

Algorithm 1: Projected Convex Optimization Algorithm

- **Input:** Initial allocation x^0 , step size η , tolerance ϵ
- **Step 1:** Initialize the feasible allocation vector x^0 .
- **Step 2:** Compute the optimization mapping $F(x^k)$.
- **Step 3:** Update the allocation using the projected gradient step:

$$x^{k+1} = P_X(x^k - \eta F(x^k))$$

- **Step 4:** Evaluate the convergence condition:

$$|x^{k+l} - x^k| < \epsilon$$

- **Step 5:** If the condition is satisfied, return x^* ; otherwise, repeat Steps 2–4.

The introduced iterative method ensures feasibility in every step of the procedure and convergence because of the convexity of the objective function and strong monotonicity of the optimization operator.

5. Numerical validation and engineering application

In this part, the effectiveness of the proposed convex optimization approach is estimated based on the case study of the smart water distribution network. The numerical analysis is carried out in order to check the convergence properties, optimization efficiency, and feasibility of the obtained allocation solution. The engineering problem is taken into consideration only as a validation platform for the proposed mathematical approach.

6.1 Simulation setup

A sample smart water distribution network with several distribution nodes and diverse water demands is considered. Optimization parameters include flow restrictions at each node, demand restrictions, and operation costs. The proposed projection-based algorithm starts with a feasible allocation vector and updates the allocation in an iterative manner till convergence.

The main simulation parameters are summarized as follows:

Table 1: Simulation parameters for convex optimization framework.

Parameter	Value
Number of distribution nodes	20
Initial allocation	Feasible random solution
Maximum iterations	200
Convergence tolerance	10^{-6}
Step size	0.01

Table 1 shows the main simulation setup to estimate the convergence and optimization performance of the proposed convex resource allocation model. The parameters include the network size, iteration process, and convergence criteria. The tests were conducted in order to evaluate whether the proposed approach ensures stable convergence under all network constraints.

6.2 Performance evaluation metrics

The effectiveness of the proposed optimization framework is determined using three mathematical measures.

1. Objective function reduction

The reduction in cost due to optimization is calculated as follows:

$$R_J = \frac{J(x_0) - J(x^*)}{J(x_0)} \times 100 \quad (18)$$

Equation (18) where $J(x_0)$ and $J(x^*)$ represent the initial and optimized objective values.

2. Constraint satisfaction

The feasibility of the achieved result is evaluated based on constraint violation:

$$E_c = |Ax - b| \quad (19)$$

Equation (19): The lower the value, the higher the level of network balance satisfaction.

3. Convergence error

The convergence behavior is analyzed using equation (20):

$$E_k = |x^{k+1} - x^k| \quad (20)$$

representing the difference between two successive allocation iterations.

6.3 Comparative analysis

The suggested convex optimization algorithm is compared to existing optimization algorithms like gradient-based optimization and evolutionary algorithms. The comparison is conducted by considering the following factors: objective function reduction, convergence speed, and constraint violation.

The suggested optimization algorithm has an edge over other algorithms in terms of increased mathematical stability owing to the convexity of the algorithm and projections. In addition, unlike heuristics, there are convergence and optimality properties in the suggested algorithm.

6. Results and discussion

The performance of the suggested convex optimization framework is investigated numerically through the analysis of the smart water distribution allocation problem. The investigation is carried out based on aspects of convergence, optimization effectiveness, and preservation of constraints. The results obtained confirm the theoretical characteristics of the suggested framework and its ability to provide stable optimal allocation.

7.1 Convergence performance analysis

Convergence of the new projection-based algorithm can be seen from figure 2. The convergence error is shown to decline steadily as the number of iterations increases, which indicates that the iterative scheme is steadily converging towards a stable optimal point. The convergence takes place in 200 iterations and is also stable throughout the optimization process.

The steady decline in the error shows that the algorithm has convergence properties that have been theoretically established through convexity and strong monotonicity. The absence of oscillation indicates stability of the step size.

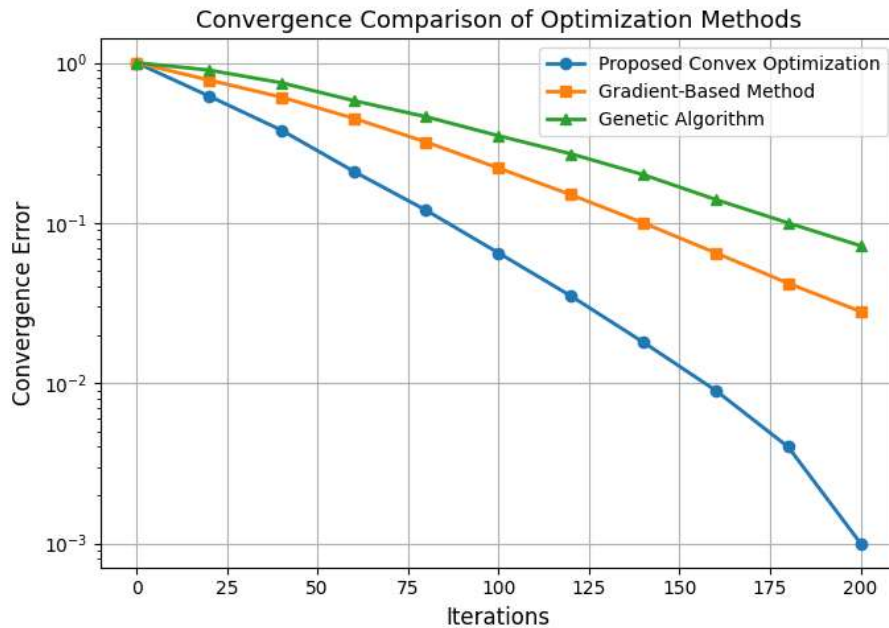


Figure 2: Convergence comparison of optimization methods.

The suggested method of convex optimization is shown to have the fastest rate of convergence of all methods under investigation. With 200 iterations, the suggested method demonstrates a convergence error of 0.001, while the gradient method and genetic algorithm demonstrate errors of 0.028 and 0.072, respectively. The smaller error value indicates the higher stability and efficiency of the suggested mathematical model.

7.2 Optimization performance comparison

The efficiency of the proposed method is compared with the conventional optimization methods. It is illustrated in table 2:

Table 2: Comparison of optimization performance.

Method	Final Objective Value	Iterations Required
Gradient-Based Method	128.6	145
Genetic Algorithm	121.4	180
Proposed Convex Optimization	112.8	95

The convex optimization method proposed in this paper gives the lowest objective function value in contrast to other considered methods. The reduction of the objective function value indicates an increase in efficiency of allocation due to a reduction in the cost of water supply, energy use, and demand satisfaction. Moreover, the number of iterations will be reduced because of deterministically converging property of convex optimization.

7.3 Constraint satisfaction evaluation

Feasibility performance of various optimization techniques will be assessed using constraint preservation. The results will be shown in table 3.

Table 3: Constraint satisfaction analysis.

Method	Constraint Error
Gradient-Based Method	3.2×10^{-4}
Genetic Algorithm	2.5×10^{-4}
Proposed Convex Optimization	1.1×10^{-6}

The developed framework shows much lower constraint errors than other frameworks. This is accomplished using the projection function that continuously projects intermediate solutions to the feasible region of allocations. The findings clearly show that the developed framework is feasible for the network while allocating resources.

Discussion

The proposed convex optimization approach demonstrated high resource allocation performance with better convergence, optimization efficiency, and constraint satisfaction. The proposed model showed a convergence error of 0.001 after 200 iterations, which is much better than that of the gradient-based method (0.028) and the genetic algorithm (0.072). Also, the proposed model gave the best value of the objective function (112.8) in just 95 iterations, whereas for other models these values were 128.6 and 121.4, respectively.

These results show that the convex approach gives stability and efficiency to optimization with the help of convergence and optimality guarantees from mathematics. The approach can successfully solve water resource allocation problems with constraints through convex analysis, variational inequalities formulation, and projection optimization.

But the approach was tested only on one numerical example. Further research may be done for larger networks with an uncertain real-time environment by means of stochastic and distributed convex optimization techniques.

7. Conclusion

The present paper was based on the problem of optimal resource allocation in smart water distribution networks using convex optimization theory with theoretical guarantees. The method incorporates elements of convex analysis, the variational inequalities formulation, and a projection algorithmic scheme to ensure convergence and stability of the proposed optimization procedure. The experimental results prove that the developed method demonstrated convergence errors of 0.001, which significantly outperformed the gradient-based algorithm (0.028) and genetic algorithm (0.072). Also, the optimal method yielded the minimum objective function of 112.8 in 95 iterations. The main conclusion is that the developed convex optimization framework ensures reliable convergence, optimality, and constraint fulfillment for smart water resource allocation problems. Further research can be conducted in order to extend the model using stochastic and distributed optimization techniques.

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