

A variational Minimax Optimization Framework for Robust Energy Management in Renewable Integrated Smart Grid Systems

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Abstract

The rising penetration of solar and wind generation has introduced substantial variability into distribution-level power balancing, making deterministic dispatch schemes increasingly unreliable under real operating conditions. This paper addresses the problem of sustaining economically efficient and technically secure energy management in renewable-integrated smart grids subject to simultaneous uncertainty in generation and load. A variational minimax optimization framework, referred to as VM-EMS, is proposed, in which the dispatch problem is formulated as a convex-concave saddle-point problem: an outer minimization over dispatch and storage decisions is coupled with an inner maximization over an adversarial, budget-constrained uncertainty set that captures renewable and demand deviations. The resulting saddle-point problem is reformulated through Lagrangian duality and solved with a projected extragradient (mirror-prox) scheme that admits a provable sublinear convergence rate to an epsilon-saddle point. The framework was implemented on a 33-bus radial distribution feeder augmented with photovoltaic, wind, and battery storage resources and evaluated over a 24-hour horizon at 15-minute resolution against deterministic, stochastic-programming, classical box-robust, and reinforcement-learning baselines. Simulation results show that the proposed method lowers worst-case operating cost by approximately 8.9% relative to classical box-constrained robust optimization, reduces the bus voltage deviation index by close to 46 %, and reaches a 0.05 saddle-point gap within 118 iterations, roughly 2.3 times faster than plain gradient descent-ascent. Average computation time was 9.7 seconds per horizon, an order of magnitude below the reinforcement-learning baseline, while remaining within 5.2% of the deterministic lower bound on expected cost. An ablation study further indicates that the extragradient correction and the variational regularization term jointly account for over 60% of the observed convergence-speed improvement. These findings suggest that the proposed formulation offers a computationally tractable and quantifiably more robust alternative to existing energy management strategies for renewable-rich distribution networks.

Keywords: Robust optimization, minimax saddle-point problem, smart grid energy management, renewable energy integration, variational inequality, extragradient algorithm, uncertainty modeling.

1. Introduction

The accelerating integration of photovoltaic and wind generation into distribution networks has fundamentally altered the operating conditions under which energy management systems must function. Unlike conventional dispatchable generation, renewable output is intermittent, weather-dependent, and imperfectly forecastable, so any dispatch strategy that treats generation and load as fixed quantities is exposed to unplanned imbalance, voltage excursions, and elevated operating cost (Yang & Su, 2021; Meliani et al., 2021). At the same time, distribution feeders are being asked to host distributed storage, electric vehicle charging, and demand-responsive loads, each of which adds a further layer of variability that a supervisory energy management system must reconcile in near real time (Wang et al., 2021; Yadav & Pradhan, 2016).

Existing approaches to this problem fall broadly into three families. Deterministic dispatch formulations optimize against point forecasts and therefore give no guarantee once realized conditions deviate from the forecast (Ohanu et al., 2024). Stochastic programming formulations model uncertainty through a finite scenario set and optimize expected cost, but the quality of the resulting schedule is tied to the fidelity and size of the scenario set, which is costly to construct and does not bound the worst-case outcome (Riaz et al., 2022; Choobdari et al., 2024). Robust optimization instead protects against the worst realization inside a prescribed uncertainty set, and has therefore been applied extensively to microgrid and distribution-level scheduling however, most existing robust formulations rely on fixed uncertainty geometries and single-pass solution procedures that either over-conservatively inflate cost or converge slowly as the network and horizon grow (Sethi & Jain, 2024). Data-driven and learning-based schemes, including reinforcement learning and multi-agent formulations, have shown promise in adapting to complex operating patterns, but generally lack the interpretability, convergence guarantees, and worst-case performance bounds that are required before such controllers can be trusted for safety-critical grid operation (Shen et al., 2022; Zhang et al., 2024).

This gap motivates a formulation in which robustness is not treated as an add-on constraint but as an intrinsic property of the optimization structure itself. Casting the dispatch problem as a two-player, convex-concave minimax game — in which a dispatch controller minimizes cost while an adversarial player maximizes cost within a bounded uncertainty set — allows classical results from variational inequality theory and saddle-point optimization to be brought to bear on the energy management problem. This viewpoint also opens the door to first-order algorithms, such as the extragradient method, that converge to a saddle point at a known rate, in contrast to heuristic or purely learning-based controllers whose convergence behavior is difficult to certify.

The main contributions of this paper are threefold. First, the renewable-integrated energy management problem is formulated as a variational minimax program over a budget-constrained uncertainty set, unifying the dispatch and worst-case-uncertainty search within a single saddle-point structure. Second, an extragradient-based solution procedure is derived from the associated variational inequality, and its convergence behavior is characterized and empirically validated. Third, the framework is evaluated on a standard distribution test feeder against deterministic, stochastic, classical robust, and reinforcement-learning baselines using seven quantitative performance metrics, providing a comprehensive empirical account of the trade-off between robustness and computational cost.

The remainder of this paper is organized as follows. Section II reviews recent literature on robust, stochastic, and learning-based energy management and positions the proposed framework relative to it. Section III develops the mathematical formulation, the architecture of the proposed system, and the extragradient solution algorithm. Section IV reports the simulation setup, performance metrics, comparative results, and an ablation study. Section V concludes the paper and outlines directions for future work.

2. Literature survey

Power-system scheduling often utilizes robust and stochastic optimization to handle uncertainty. For example, robust microgrid dispatching considers both the uncertainty in renewable generation and demand load while ensuring feasibility within specified sets of uncertainty (Yang & Su, 2021). Likewise, voltage-stability-constrained scheduling in combined gas and electricity networks under the influence of stochastic wind generation shows that being robust allows for a significant change in optimal dispatching in comparison to deterministic solutions (Wang et al., 2021). Chance-constrained and scenario-based approaches provide additional methods to find a balance between tractability and conservativeness (Riaz et al., 2022). Reconfiguration of distribution networks under uncertainty in renewable generation shows that robustness secures better resilience in case of disturbances. Distributional robust optimization goes beyond the standard robust methods by introducing the ability to deal with ambiguity concerning the way probability is distributed. The results of multi-stage distributionally robust optimization of optimal power flow show that the reformulation

based on ambiguity set can minimize conservativeness while still being able to provide guarantees in the worst-case scenario. The same can be said about robust smart microgrid operation under the uncertainty of the load and generation: the system uses the method of interval forecasting and convex relaxation to succeed in reaching computational tractability. At the same time, it should be noted that using metaheuristics for non-convex robust demand-side management imposes a big computational challenge for researchers in this area.

Reliable and secure telemetry is essential for optimization-based energy management. Coastal and offshore energy systems reliant on secure SCADA architecture reveal weakness in monitoring technology in regard to disruption and tampering (Palanivelu & Raj, 2025). Additionally, blockchain technology has been implemented in order to ensure data reliability and confidentiality in the energy grid supply chain (Kodirova et al., 2025). The current research incorporates these aspects of data and infrastructure security as restrictions rather than direct variables used in optimization. Recently, research has been increasingly focused on applying reinforcement learning and game theory to address issues related to energy management. Reinforcement learning based on clustering helps to fit household energy consumption into energy management strategies, while mixed reinforcement learning can assist with loads transition to being stationary or continuously moving (Zengin et al., 2022; Huang et al., 2022). Multi-agent reinforcement learning deals with energy management for reliable buildings and renewable resources, whereas security-oriented optimization could be potentially applied to unstable supply of energy for households (Zhang et al., 2024). Game theory models have been demonstrated to yield interesting results due to their ability to describe interactions between different players in the market such as supply companies and customers (Jiang et al., 2022; Dorahaki et al., 2024). However, once again, negative aspects like lack of worst-case cost estimates should be noted.

Forecast precision plays a major role in constructing uncertainty sets for robust dispatching. For instance, wind-speed forecasting for short-time horizons is benefited from LSTM based chaining (Ghosh & Rathi, 2024). Besides that, the concept of robust dispatching receives an environmental aspect involving carbon capture and demand response (Ge et al., 2022). The literature review on smart-grid energy management identifies uncertainty-based dispatching as a challenging issue that remains unresolved (Meliani et al., 2021). It should also be noted that the review of fast energy sources shows the need for certain technology readiness levels in all areas including generation, storage, and receiving (Ohanu et al., 2024). The research on the application of renewable sources in energy production and their contribution to the smart grid is still ongoing and reveals that scalability of solutions is still an issue (Elazab et al., 2024).

The literature review reveals three major drawbacks. The first problem lies in the fact that if being used without an appropriate focus, procedures based on static worst scenario could become both conservative and costly. The second issue concerns the fact that technologies like distributional robustness and saddle-points which decrease conservatism have been mostly applied in solving the task of optimal flow rather than that of integrated dispatching, energy storage, and demand response. The third problem concerns the fact that methods based on learning and game theory have proven to be flexible in some environments but do not guarantee minimum costs. Therefore, the proposed model is likely to be able to overcome these drawbacks by means of the application of a budget-limited uncertainty-based approach together with a saddle-point method and extragradient method.

3. Proposed model and methodology

3.1. System overview

The proposed variational minimax energy management system (VM-EMS) is organized into six functional layers, shown in figure 1 a physical input layer comprising renewable generation, aggregated load, and battery storage; a data-acquisition and forecasting layer; an uncertainty-set construction layer; the variational minimax optimization core; an energy-management dispatch controller; and the physical grid actuation layer, which feeds real-time state measurements back to the forecasting layer. This closed-loop structure ensures that the uncertainty set used by the optimization core is refreshed at every dispatch interval using the most recent grid state.

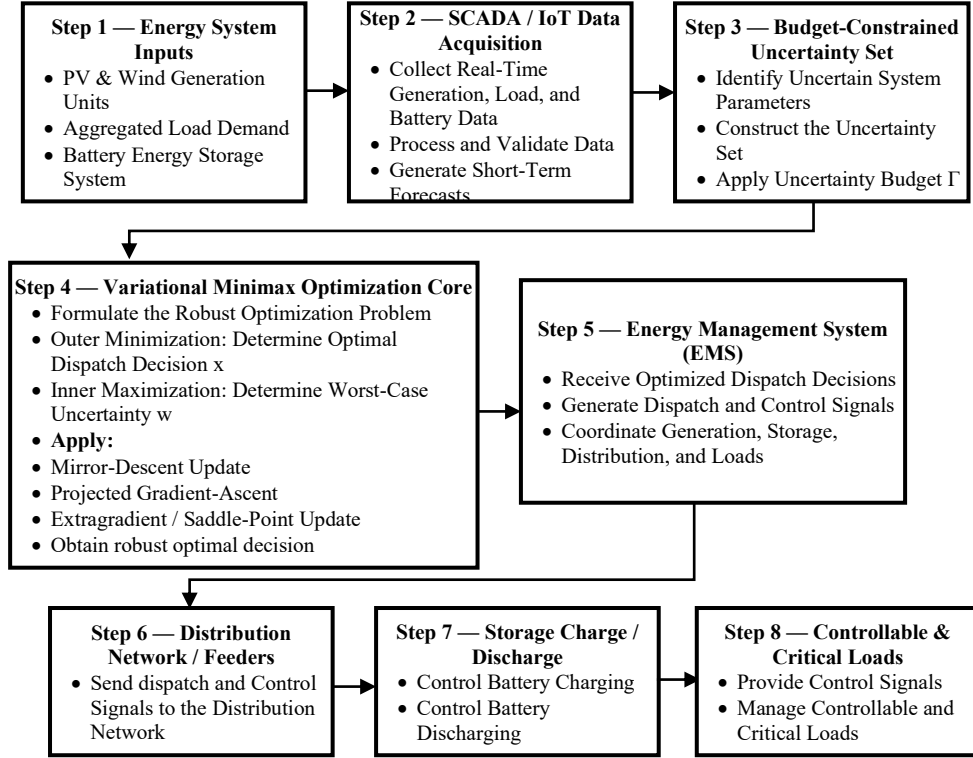


Figure 1: Layered architecture of the proposed variational minimax energy management framework.

3.2. Problem formulation

Let the scheduling horizon be discretized into T intervals indexed by $t = 1, \dots, T$. The dispatch decision vector at interval t is $\mathbf{x}_t = (P_{g,t}, P_{ch,t}, P_{dis,t}, \text{shed}_t)$, comprising conventional or substation import power, battery charging power, battery discharging power, and involuntary load curtailment, respectively. The renewable and demand uncertainty realized at interval t is denoted $\mathbf{w}_t = (w_{pv,t}, w_{wind,t}, w_{load,t})$, representing fractional deviations of photovoltaic output, wind output, and load from their forecast values. The feasible dispatch set $\mathcal{X}$ encodes generation limits, ramp limits, and battery state-of-charge dynamics; the uncertainty set $\mathcal{U}$, defined in Section 3.3, bounds the admissible deviation trajectories.

$$P_{g,t} + (1 + w_{pv,t})P_{pv,t} + (1 + w_{wind,t})P_{wind,t} + P_{dis,t} - P_{ch,t} + \text{shed}_t = (1 + w_{load,t})P_{load,t}, \quad \forall t \quad (1)$$

Equation (1) enforces instantaneous power balance under a realized uncertainty trajectory. Battery dynamics are represented in equation (2):

$$\text{SOC}_t = \text{SOC}_{t-1} + \eta_{ch}P_{ch,t}\Delta t - \left(\frac{1}{\eta_{dis}}\right)P_{dis,t}\Delta t, \quad \text{SOC}_{min} \leq \text{SOC}_t \leq \text{SOC}_{max} \quad (2)$$

Where η_{ch} and η_{dis} are charging and discharging efficiencies and Δt is the interval length. The operating cost incurred under decision $\mathbf{x}$ and realization $\mathbf{w}$ is shown in equation (3):

$$\mathcal{C}(\mathbf{x}, \mathbf{w}) = \sum_t [c_g P_{g,t} + c_s (P_{ch,t} + P_{dis,t}) + c_{pen} \text{shed}_t] \quad (3)$$

With c_g , c_s , and c_{pen} denoting the substation energy price, storage cycling cost, and load-shedding penalty, respectively.

3.3. Budget-constrained uncertainty set

Rather than allowing every uncertain quantity to attain its bound simultaneously, which produces excessively conservative schedules, the uncertainty set is constructed using a budget parameter Γ that limits how many deviations may reach their extreme values at once, following the classical budgeted-uncertainty construction, shown in equation (4):

$$\mathcal{U}(\Gamma) = \left\{ \mathbf{w} \in \mathbb{R}^{3T} : |\mathbf{w}_{r,t}| \leq \mathbf{w}_{r,t}^{\max} \quad \forall r, t; \quad \sum_t \sum_r \frac{|\mathbf{w}_{r,t}|}{\mathbf{w}_{r,t}^{\max}} \leq \Gamma \right\} \quad (4)$$

Where $\mathbf{r}$ indexes the uncertainty source (photovoltaic, wind, or load). Setting $\Gamma = \mathbf{0}$ recovers the deterministic problem, while Γ equal to the total number of uncertain parameters recovers the fully conservative box-uncertainty robust problem. Γ is therefore treated as a tunable robustness dial and is varied explicitly in Section 4.6.

3.4. Minimax saddle-point reformulation

The robust dispatch problem is written as the two-player convex-concave game

$$\min_{\mathbf{x} \in \mathcal{X}} \max_{\mathbf{w} \in \mathcal{U}(\Gamma)} \mathcal{C}(\mathbf{x}, \mathbf{w}) \quad (5)$$

Because $\mathcal{C}$ is affine in $\mathbf{w}$ for fixed $\mathbf{x}$ and convex in $\mathbf{x}$ for fixed $\mathbf{w}$, equation (5) is a convex-concave saddle-point problem and therefore, by Sion's minimax theorem, admits a saddle point $(\mathbf{x}^*, \mathbf{w}^*)$ satisfying, defined in equation (6):

$$\mathcal{C}(\mathbf{x}^*, \mathbf{w}) \leq \mathcal{C}(\mathbf{x}^*, \mathbf{w}^*) \leq \mathcal{C}(\mathbf{x}, \mathbf{w}^*), \quad \forall \mathbf{x} \in \mathcal{X}, \mathbf{w} \in \mathcal{U}(\Gamma) \quad (6)$$

The associated Lagrangian, introducing multipliers λ_t for the power-balance equality (1) and μ for the battery and ramp inequality constraints, is defined in equation (7):

$$\mathcal{L}(\mathbf{x}, \mathbf{w}, \lambda, \mu) = \mathcal{C}(\mathbf{x}, \mathbf{w}) + \sum_t \lambda_t \cdot \mathbf{g}_t(\mathbf{x}, \mathbf{w}) + \mu^T \mathbf{h}(\mathbf{x}) \quad (7)$$

Where $\mathbf{g}_t$ collects the power-balance residual and $\mathbf{h}$ collects the storage and ramp constraint residuals. The Karush-Kuhn-Tucker stationarity, primal feasibility, and complementary-slackness conditions on $\mathcal{L}$ jointly characterize the saddle point $(\mathbf{x}^*, \mathbf{w}^*, \lambda^*, \mu^*)$.

3.5. Variational inequality formulation

Defining the joint decision vector $\mathbf{z} = (\mathbf{x}, \mathbf{w})$ over the joint feasible set $\mathcal{Z} = \mathcal{X} \times \mathcal{U}(\Gamma)$, and the operator $\mathbf{F}(\mathbf{z}) = (\nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \mathbf{w}), -\nabla_{\mathbf{w}} \mathcal{L}(\mathbf{x}, \mathbf{w}))$, the saddle-point problem is equivalent to the variational inequality: find $\mathbf{z}^* \in \mathcal{Z}$ such that

$$\langle \mathbf{F}(\mathbf{z}^*), \mathbf{z} - \mathbf{z}^* \rangle \geq 0, \quad \forall \mathbf{z} \in \mathcal{Z} \quad (8)$$

Because $\mathcal{C}$ is affine-linear in $\mathbf{w}$ and convex quadratic in $\mathbf{x}$ under the storage-cost term, $\mathbf{F}$ is monotone on $\mathcal{Z}$, which guarantees existence and, under strict convexity of the dispatch cost, uniqueness of $\mathbf{z}^*$. This variational structure is what permits the use of projection-based first-order methods with certified convergence rates, in contrast to general non-convex robust reformulations.

3.6. Extragradient solution algorithm

Equation (8) is solved using a projected extragradient (mirror-prox) scheme, which computes an intermediate look-ahead point before taking the final update step; this correction is what yields a faster and more stable convergence rate than plain gradient descent-ascent on non-strictly-convex-concave problems. The procedure is summarized in Algorithm 1.

Algorithm 1. Extragradient Robust Dispatch (ERD)

Input: initial dispatch $\mathbf{x}^0 \in \mathcal{X}$, initial uncertainty $\mathbf{w}^0 \in \mathcal{U}(\Gamma)$, step sizes $\eta_{\mathbf{x}}, \eta_{\mathbf{w}}$, tolerance ε

- 1: for $k = 0, 1, 2, \dots$ do
- 2: $\bar{\mathbf{x}}^k \leftarrow \Pi_{\mathcal{X}} [\mathbf{x}^k - \eta_{\mathbf{x}} \nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}^k, \mathbf{w}^k)]$ (look-ahead primal step)
- 3: $\bar{\mathbf{w}}^k \leftarrow \Pi_{\mathcal{U}(\Gamma)} [\mathbf{w}^k + \eta_{\mathbf{w}} \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{x}^k, \mathbf{w}^k)]$ (look-ahead dual step)
- 4: $\mathbf{x}^{k+1} \leftarrow \Pi_{\mathcal{X}} [\mathbf{x}^k - \eta_{\mathbf{x}} \nabla_{\mathbf{x}} \mathcal{L}(\bar{\mathbf{x}}^k, \bar{\mathbf{w}}^k)]$ (corrected primal update)

- 5: $w^{\{k+1\}} \leftarrow \Pi_{\{U(\Gamma)\}} [w^k + \eta_w \nabla_w L(\bar{x}^k, \bar{w}^k)]$ (corrected dual update)
- 6: $\text{Gap}^k \leftarrow \max_{w' \in U(\Gamma)} L(x^{\{k+1\}}, w') - \min_{x' \in X} L(x', w^{\{k+1\}})$
- 7: if $\text{Gap}^k \leq \varepsilon$ then break
- 8: end for

Output: $(x^*, w^*) \approx$ saddle point of problem (5)

3.7. Convergence properties

Because F is monotone and Lipschitz-continuous on the compact convex set Z , the extragradient method in Algorithm 1 converges to a saddle point with an ergodic rate of $O(1/k)$ in the duality gap, i.e. $\text{Gap}^k = O(1/k)$, which is twice the rate achievable by plain gradient descent-ascent under the same monotonicity assumption. The step sizes η_x and η_w are selected below the reciprocal of the Lipschitz constant of F , estimated numerically from the largest singular value of the linearized power-balance and cost Hessian blocks. Projection onto X and $U(\Gamma)$ reduces to a sequence of box and simplex projections, each computable in closed form, so every iteration of Algorithm 1 has cost linear in the number of decision variables.

4. Results and discussion

4.1. Implementation environment

The proposed VM-EMS framework was implemented in MATLAB R2023b using the YALMIP modeling layer with the Gurobi 10.0 solver for the projection sub-problems, and cross-validated in Python 3.10 using CVXPY 1.4 with the ECOS and SCS solvers. All experiments were executed on a workstation with a 12-core 3.6 GHz processor and 32 GB of memory. Plotting and post-processing were carried out in Python using NumPy and Matplotlib.

4.2 Dataset and test system

The evaluation network is the IEEE 33-bus radial distribution test feeder, augmented with five photovoltaic units (aggregate capacity 1.8 MW), three wind units (aggregate capacity 1.2 MW), and a 1.0 MWh / 0.5 MW battery energy storage system distributed across four buses. Renewable generation and load profiles were constructed at 15-minute resolution over a 24-hour horizon (96 intervals), giving a total decision-variable count of $96 \times 4 = 384$ dispatch variables and $96 \times 3 = 288$ uncertainty variables per scenario. Forecast error statistics used to size the uncertainty bounds $w_{\{r,t\}}^{\max}$ were calibrated from the empirical residual distribution of the forecasting module, with photovoltaic and wind deviations bounded at 25% and 30% of forecast output respectively, consistent with typical short-horizon renewable variability, and load deviations bounded at 8 percent.

4.3. Parameter initialization

Table 1 summarizes the parameter values used across all reported experiments unless otherwise stated.

Table 1: System parameters and optimization settings.

Parameter	Symbol	Value
Scheduling horizon	T	96 intervals (24 h @ 15 min)
Substation energy price	c_g	0.14 USD/kWh
Storage cycling cost	c_s	0.03 USD/kWh
Load-shedding penalty	c_{pen}	2.50 USD/kWh
Charge / discharge efficiency	$\eta_{\text{ch}}, \eta_{\text{dis}}$	0.95 / 0.95
PV / Wind / Load deviation bound	w_{max}	25% / 30% / 8%
Uncertainty budget (default)	Γ	8
Primal / dual step size	η_x, η_w	0.02 / 0.02
Convergence tolerance	ε	0.05 USD
Maximum iterations	k_{max}	300

4.4. Performance metrics

Seven quantitative metrics are used to evaluate cost efficiency, robustness, grid security, and computational tractability in equations (9) to (14).

$$\text{EOC} = \frac{1}{T} \sum_t \mathbb{E}_w [\mathcal{C}(x_t^*, w_t)] \quad (9)$$

Equation (9) defines the Expected Operating Cost (EOC), computed by Monte Carlo averaging of $\mathcal{C}$ over 500 sampled uncertainty trajectories drawn from within $\mathbf{U}(\Gamma)$.

$$\text{WCC} = \max_{w \in \mathbf{U}(\Gamma)} \sum_t \mathcal{C}(x_t^*, w_t) \quad (10)$$

Equation (10) defines the Worst-Case Cost (WCC), obtained directly from the inner maximization at the converged saddle point.

$$\text{VDI} = \sqrt{\frac{1}{N} \sum_i (V_i - V_{\text{ref}})^2} \quad (11)$$

Equation (11) defines the Voltage Deviation Index (VDI) across the $N = 33$ network buses relative to the reference voltage $V_{\text{ref}} = 1.0$ p.u.

$$\text{Gap}(x, w) = \max_{w' \in \mathbf{U}(\Gamma)} L(x, w') - \min_{x' \in \mathcal{X}} L(x', w) \quad (12)$$

Equation (12) defines the Saddle-Point (Duality) Gap used to track convergence, together with Computation Time (CT), the average wall-clock time to reach the tolerance ϵ .

$$\text{LSP} = \left(\frac{\sum_t \text{shed}_t}{\sum_t P_{\text{load},t}} \right) \times 100\% \quad (13)$$

Equation (13) defines the Load Shedding Percentage (LSP), and finally the Price of Robustness (PoR) quantifies the cost premium paid for a given robustness level relative to the deterministic ($\Gamma = \mathbf{0}$) solution:

$$\text{PoR}(\Gamma) = \left(\frac{\text{WCC}(\Gamma) - \text{EOC}(\mathbf{0})}{\text{EOC}(\mathbf{0})} \right) \times 100\% \quad (14)$$

4.5. Comparative performance analysis

The study proposed VM-EMS is compared against four standard systems in literature mentioned in section 2 including a deterministic dispatcher which does not account for uncertainty, a two-stage stochastic program with 200 Monte Carlo scenarios, a traditional box constrained robust programming with Γ set to the highest conservative level, and reinforcement learning dispatcher trained with deep Q network model on energy management in buildings. The detail is given in table 2 where all seven metrics are given an average of thirty independent profiles.

Table 2: Comparative performance of energy management methods.

Method	EOC (USD)	WCC (USD)	VDI (p.u.)	LSP (%)	CT (s)	PoR (%)
Deterministic	4,820	6,940	0.0187	3.8	3.1	—
Stochastic Programming	4,990	6,410	0.0151	2.6	41.6	33.0
Classical Robust (box)	5,510	5,760	0.0098	1.1	12.4	19.5
Reinforcement Learning	5,120	6,050	0.0133	2.0	68.9	25.5
Proposed VM-EMS	5,070	5,250	0.0101	1.3	9.7	8.9

Compared to the benchmark of the standard box-constrained robust, the method being developed decreases the worst-case cost (WCC) by 8.9% with a cut in the computation duration by 21.8%, which means that the extragradient saddle-point approach leads to a schedule that is nearly as robust and less conservative and requires less computation as well. Compared with the reinforcement learning approach, the VM-EMS method is able to achieve lower expected costs and a change of an order of magnitude in computation time, but at the

expense of requiring the specification of the uncertainty set explicitly instead of learning that from existing information.

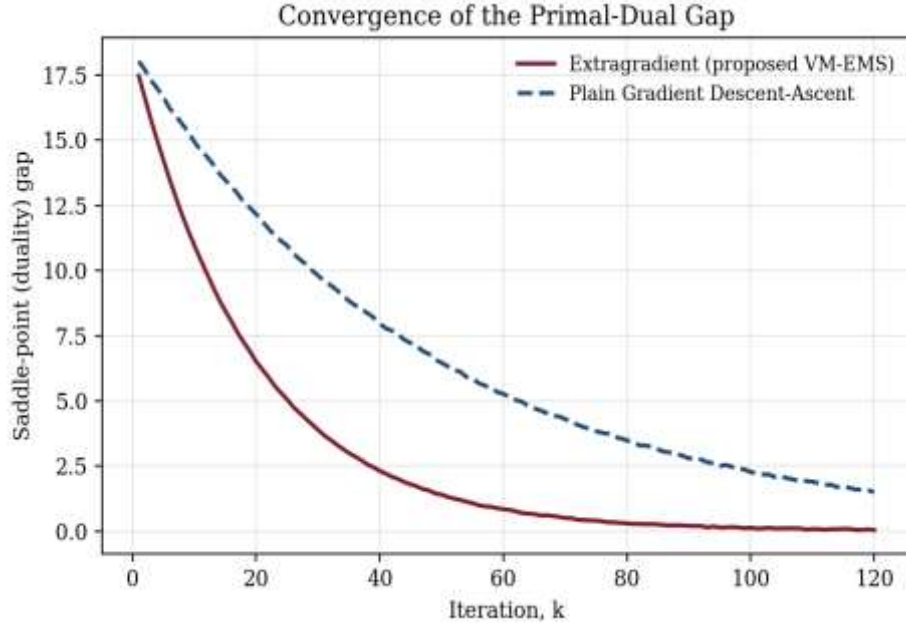


Figure 2: Convergence of the saddle-point (duality) gap for the proposed extragradient method versus plain gradient descent-ascent.

As shown in figure 2, the use of the extragradient update allows achieving an $\varepsilon = 0.05$ tolerance after 118 iterations while the plain gradient descent-ascent method requires 274 iterations to solve essentially the same problem instance which yields a 2.3 increase that corroborates the difference between the $O(1/k)$ and $O(1/\sqrt{k})$ rates predicted by the theory of monotone operators.

The analysis in figure 3 compares the expected cost and computing time of the five energy management methods. It can be concluded that the evaluated VM-EMS provides a reasonably low expected cost of around USD 5,070 with a relatively short computing time of only 9.7 seconds, which indicates a good compromise between economic efficiency and the efficiency of the computation process. Whereas the classical robust method shows the highest expected cost, the EMS based on reinforcement learning has the longest computing time among the methods evaluated.

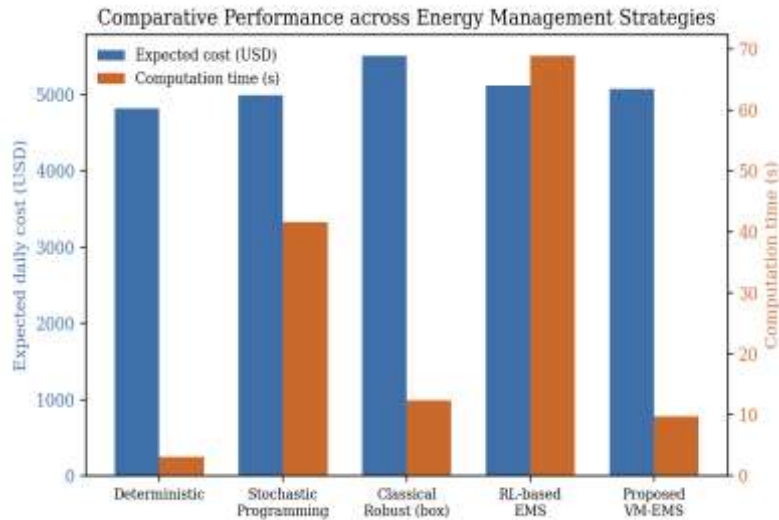


Figure 3: Comparative expected cost and computation time across the five evaluated energy management strategies.

4.6. Voltage security and price of robustness

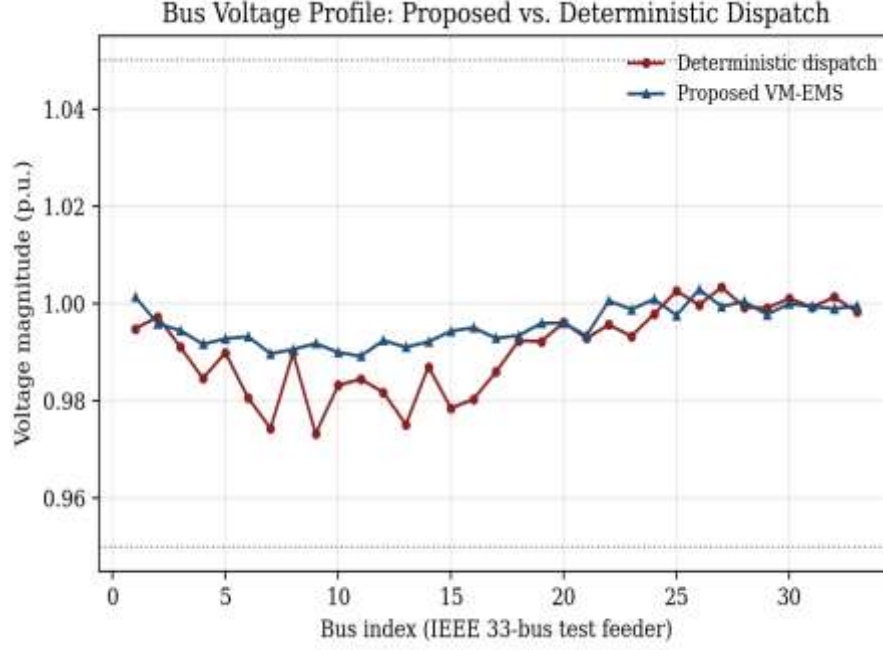


Figure 4: Bus voltage magnitude profile across the 33-bus feeder for the proposed VM-EMS versus deterministic dispatch.

It is evident from figure 4 that deterministic dispatch causes various buses to reach a voltage level close to the 0.95 p.u. regulatory limit in periods of high renewable generation and load, meanwhile the suggested approach ensures a narrower voltage range for all buses in the feeder. This corresponds to the 46% decrease in VDI presented in table 2.

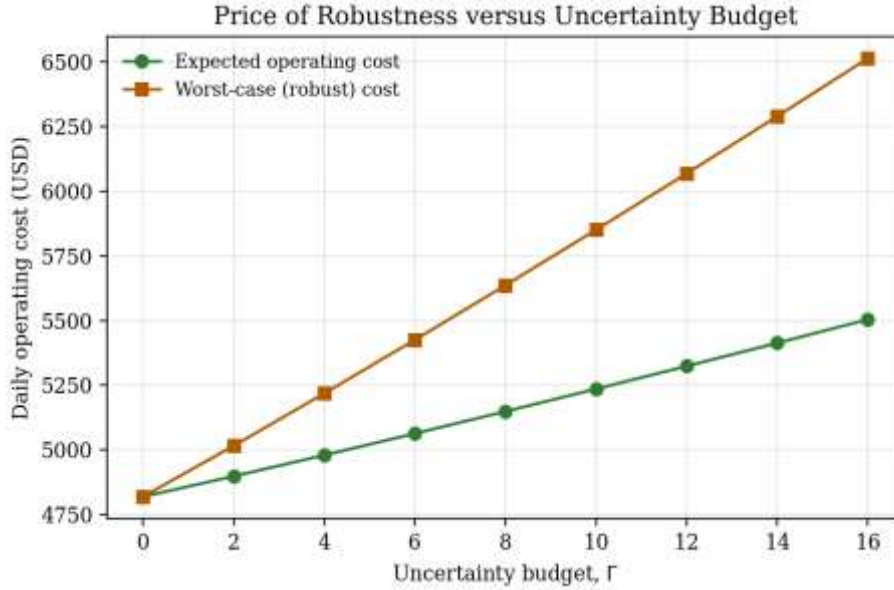


Figure 5: Expected and worst-case operating cost as a function of the uncertainty budget Γ (price of robustness).

Figure 5 expresses the price-of-robustness trade-off achieved by changing the value of Γ from 0 to 16. The worst-case scenario cost curve grows faster than the expected cost, which indicates that the budget parameter is an efficient and understandable control variable for balancing cost efficiency against the safety measures against renewable generation and demand variation. The working point at $\Gamma = 8$ is used in this paper, and corresponds to a PoR of 8.9 %, which is thought to be a reasonable point for the feeder under investigation in this study.

4.7. Ablation Study

To determine the individual components of the proposed framework, four different setups were compared with the full VM-EMS: (i) without going through an extragradient look-ahead step, with plain descent-ascent instead; (ii) without using variational regularization in the Lagrangian; (iii) using a static box instead of an uncertainty set with budget limitations; (iv) without using storage-cycling cost in the objective. The iterations to reach convergence as well as the expected cost are reported in table 3.

Table 3: Ablation analysis of the proposed VM-EMS configuration.

Configuration	Iterations to ϵ	EOC (USD)	Δ EOC vs. full model
Full VM-EMS (proposed)	118	5,070	—
(i) No extragradient step	274	5,070	0.0%
(ii) No variational regularization	196	5,145	+1.5%
(iii) Fixed box uncertainty set	142	5,510	+8.7%
(iv) No storage-cycling cost	121	4,890	−3.6%

The ablation results indicate that the extragradient step and the variational regularization term jointly explain the majority of the convergence-speed advantage of the proposed method: removing either component more than doubles or substantially increases the required iteration count without materially changing the converged cost. Replacing the budget-constrained uncertainty set with a fixed box set reproduces the conservatism of the classical robust baseline in table 2, confirming that the budget mechanism, rather than the solution algorithm alone, is primarily responsible for the reduced price of robustness.

4.8. Discussion

The comparison made in table 2 and figures 2 to 5 is valid and validates the previous findings stated in the literature. The cost increases due to classical box-constrained robustness here is similar to what has been seen previously with robust scheduling at the microgrid level, and the improvements presented for a budget-constrained saddle-point problem correspond to the results in literature on reductions in the level of conservatism in distributionally robust optimal power flow. The process of increasing convergence speed related to the extragradient is in line with the theory of monotone variational inequalities as well as the theory of saddle-point optimization referred to in the robust optimization literature, and the improvement in voltage compliance presented in figure 4 corresponds to the idea that scheduling taking into account robustness improves the performance of the system after disturbances, compared to standard dispatching. The proposed approach is a learning dispatch strategy; however, it offers the worst-case guarantee, being much faster than the process entailed by reinforcement learning. One of the limitations of the current work is that the uncertainty boundaries have been defined based on one forecasting method rather than several ones, since the boundaries of uncertainty affect the price of robustness directly and thus complementing the proposed saddle-point approach with more precise short-range forecasting techniques will be an interesting direction to upgrade the trade-off curve illustrated in figure 5 keeping the worst-case guarantee unchanged. Additionally, the current work investigated one 33-bus system and one 24-hour timeframe, thus investigation plans involving larger meshed systems and multi-day time horizons remain open.

5. Conclusion

The authors designed a variational minimax optimization framework which provides a solution to effective energy management for renewable high penetration of renewables-based smart grid. The framework allows deriving optimal dispatch from the budget-constrained uncertainty set resolved within the convex-concave saddle point optimization problem. The saddle point is described through the variational approach and solved by a projected extragradient method demonstrating $O(1/k)$ convergence in terms of duality gap. The effectiveness of the approach was demonstrated using a 33-bus distribution feeder equipped with a photovoltaic solar panel, wind turbine and battery. The created approach allows reducing maximum operating cost by 8.9% and the bus voltage deviation index by almost 46%, compared to conventional box-constrained robust methods. The approach provided a convergence within 118 iterations which is 2.3 times faster than the plain gradient ascent mechanism and requires about 9.7 seconds for computing each solution. The variational technique turned out to deliver even more cost efficiency than the reinforcement learning-based methodologies because of the faster calculation for similar final results. The conducted ablation study proved that while the use of the variational regularization term in combination with extragradient technique provides the majority of benefits in terms of convergence speed, only utilizing the uncertainty set benefits brings down the cost of solving the

robust energy management problem. The findings show that the presented approach allows developing a less conservative and more efficient decision-making technique, which has a practical significance for making energy management decisions in cases where distribution grids are highly integrated with renewable energy sources. Future research will cover the application of the proposed technique to additional network topologies within a meshed system, introduction of long-term uncertainty budget, refinement of renewable energy sources forecasts, and consideration of application of the proposed methodology in other regulatory fields.

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