

A Convex-Variational Analysis Approach for Optimal Power Flow Control Under Renewable Energy Uncertainty

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Abstract

The emergence of solar and wind energy introduces much uncertainty in modern transmission and distribution networks. The classical non-convex OPF problem does not ensure rational dispatch decisions under such uncertainties. To address this gap, the paper proposes a convex-variational approach that reformulates the AC OPF problem as an SOCP problem using a combined variational inequality formulation of the KKT optimality conditions, enhanced with chance constraints that limit renewable forecast errors at a specified risk level. The approach is based on the ADMM method that breaks down the convex feasible region into several zones and performs parallel dispatch calculations based on preserving global feasibility. The method has been successfully implemented on IEEE 33-bus, 57-bus, and 118-bus benchmark systems with synthetic wind and solar profiles that cover renewable penetration levels from 20% to 45%. The results of the study demonstrated that the proposed convex-variational algorithm produced energy at 6.6% lower cost compared to DC-OPF approach and standard SOCP relaxation, had 59% less voltage deviation compared to DC-OPF, reduced constraint violation from 4.3% to 0.6% and minimized renewable curtailment from 8.9% to 3.1% while at the same time converging in 41% fewer iterations compared to the classical AC OPF approach. Moreover, the RMSE between the relaxation and the real AC solution was equal to 0.42%, which indicates that the convex relaxation has performed with high accuracy across all test cases. Thus, the results reveal that the introduction of convex relaxation combined with variational inequality-based optimality conditions allows developing a promising and efficient way of controlling grids with a high share of renewables.

Keywords: Optimal power flow, convex relaxation, variational inequality, renewable energy uncertainty, chance-constrained optimization, distributed ADMM control, power system dispatch.

1. Introduction

The electric power systems are currently experiencing a revolution as wind and solar power sources start to replace the conventional dispatchable generation. The process leads to an increase in the environmental performance; nevertheless, it introduces high-frequency variability and forecasting uncertainties in the generation mix making the Optimal Power Flow (OPF) problem which the system operators solve in order to schedule generation, maintain voltage levels, and minimize operational costs much more difficult to solve. The classical AC-OPF problem is non-convex and non-linear because of the quadratic interdependencies of active power, reactive power, and voltage level in the equation of power flow. The non-convex nature of the problem complicates proving its global optimum, especially if stochastic production forms are added to the uncertain parameters (Sonekar & Tandan, 2025; Gholami et al., 2022).

The advent of highly efficient power electronics and battery-supplied hybrid renewable microgrids has extended the operation of distribution lines, nevertheless, it has also made the network states much more sensitive to the short-term fluctuations in the power generation (Sonekar & Tandan, 2025; Reddy et al., 2025; Pandey & Ahmed, 2025). Some volt-VAR control strategies and context-aware energy management solutions have been developed to dampen some of the variability on the distribution level however, these methods rely either on logic-based control and therefore do not guarantee the feasibility of their output or on some convex approximations that have not been proved to be valid when uncertainty is present. After all, security-constrained OPF formulations that include renewal variability are still labor-consuming at the transmission level (Huang et al., 2022; Varawala et al., 2023). The degradation of the quality of electricity due to rapid variations of renewable sources drives the search for control solutions that are both reliable in terms of mathematics and resistant to uncertainty (Kholova et al., 2025).

The convex relaxation method when applied to non-convex power flow theory allows researchers to keep the non-convex power flow process and to transform it into the convex outer form. This means that the convex outer form can be calculated with the help of a polynomial algorithm, and for some exceptions, the result will be equal to the result of real AC OPF. Sometimes the variational-inequality theory allows to express the optimality conditions of controlling convex programs in a concise way within one formula that shows its capability to implement distributed and iterative solving processes (Alkhraijah et al., 2023; Oh, 2021). Despite the success of the theory of convex relaxation and variational inequality theory among researchers in the field of engineering, the number of practical engineering cases where both theories are used as parts of uncertainty-prone OPF control system is scarce.

In this paper, the make several contributions. First, the authors design the convex-variational analysis approach that describes the second-order relaxation formula as a variational inequality and aid in reaching the solution with the help of distributed monotone operator method instead of a complex centralized process of solving non-linear tasks. The second contribution of this research contains the combination of chance-constrained model of error of renewable energy sources into the convex feasible area to achieve these electrical characteristics with a required level of confidence. The third achievement of the authors is an implementation of distributed Alternating Direction Method of Multipliers (ADMM) methodology, which solves this problem in a more effective way than centralized methods of solving equations. Finally, the authors make a detailed assessment of the application of the proposed method in engineering in IEEE 33-bus, 57-bus, and 118-bus models and measure the following parameters: generation price, regulation of voltage level, time needed for computation, statistics of constraint violations.

The structure of the paper is as follows. Section 2 covers the analysis of scholarly contributions to the sphere of convex relaxation, distributed optimization, chance-constrained OPF, and learning OPF elements. Section 3 describes the approach implemented in the research, including the realization of the problem putting equation, application of the chance-constrained reliability model, and the variational-inequality approach. Section 4 includes the methods of implementing the approach in the engineering environment based on the numerous experiments conducted by the authors. Section 5 sums up the results gained while writing the paper and gives the prospects of future studies in the field.

2. Literature survey

Optimal power flow (OPF) is a key research area that addresses the uncertainties of renewable-energy sources. This issue is being studied using mathematical frameworks such as distributed decomposition and chance-constrained optimization, as well as approaches such as convex relaxation and learning-based methods. Research indicates that OPF systems based on decomposition give results comparable with those achieved using centralized OPF systems where the computations are shared among network nodes (Alkhraijah et al., 2023). Also,

localized OPF problems can be solved using boundary variables to guarantee convergence in radial and meshed networks without a need for a central coordinator (Oh, 2021). Look-ahead OPF further enhances computational effectiveness. By utilizing sensitivity information over rolling horizons, look-ahead method provides advantages in renewable dispatch with updated forecasts of renewable resource availability (Varawala et al., 2023). Even though these methods help in solving scalability problems, they generally assume fixed feasible regions or simplified power flow models.

Robust chance-constrained DC-OPF has also been devised to solve the issue of decision-dependent uncertainty (Aigner et al., 2022). Similar approaches based on probabilistic constraints in AC-OPF have been researched in the area of AI with a goal to simplify data processing and decision-making algorithms (Fathabad et al., 2023). Moreover, stochastic methods in AC-OPF have employed Gaussian regression to help make connections between power injections and dispatching decisions (Mitrovic et al., 2023). Stochastic machine learning techniques have shown that the combination of deep learning algorithms with stochastic estimation allows for speedier and more efficient scheduling of renewable resources than previously achieved with iterative stochastic methods (Gupta et al., 2022). Comparative research into the approximations of second-order cone programming (SOCP) shows that the degree of relaxation is dependent on factors like network topology, the presence of reactive power limits, and the use of devices such as voltage regulators (Bobo et al., 2021). The branch-flow SOCP method is considered a good compromise between precision and computational effort in the case of distribution networks (Bobo et al., 2021). Consequently, branch-flow SOCP is suitable for resourcing systems based on green energy, where both implementation workflow and solution quality matter.

DeepOPF uses neural networks for finding the relationships between the load and green energy predictions and production (Pan et al., 2020). The learning method used for AC-OPF allows for warm start creation, which is subject to a system update via classic convex and nonlinear methods (Rahman et al., 2021). The introduction of sensitivity-aware machine networks with information on the power-flow Jacobian has contributed to better generalization in unknown operating conditions (Singh et al., 2021). Moreover, it is important to note the appearance of deep learning algorithms in OPF-based control systems already allowing creating improved state information for device dispatching purposes (Habib et al., 2023). Nevertheless, the advantages gained through machine learning methods are compensated by the complexity of ensuring feasibility in operating conditions beyond the boundaries of the training environment (Pan et al., 2020) (Rahman et al., 2021) (Singh et al., 2021). Consequently, the current research employs the core of convex variational methods instead of relying solely on learned models.

Research on renewable energy has explored control at the converter level and its influence on effective ramp rates in AC-OPF (Sonekar & Tandan, 2025). Studies on hybrid renewable microgrids with optimization of battery storage have also shown the prospects for performance improvement of battery dispatching and renewable curtailment (Reddy et al., 2025). Additionally, different reviews provide information on innovative technologies used for renewable energy optimization (Pandey & Ahmed, 2025). The use of voltage controlling and smart energy management methods have also contributed to efficient functioning of intelligent distribution systems (Rohini et al., 2025). Finally, the application of N-k security constrained OPF has made it possible to improve reliability through preventive and restorative methods of generation dispatching (Huang et al., 2022). The use of reinforcement learning techniques together with Lyapunov stability analysis has proven that combinations of machine learning and stability assurance methods are possible (Cui et al., 2022).

There is an understanding that the different techniques, namely distributed decomposition, chance-constrained optimization, convex relaxation, and OPF cannot be merged together (Pan et al., 2020) (Rahman et al., 2021) (Singh et al., 2021). That is when the necessity of this study arises, as it allows for merging all techniques through the formulation of variational inequality. In this context, it becomes obvious that a significant gap exists in the development of a unified convex-variational framework allowing simultaneous incorporation of all elements discussed above, exhaustive features of renewable uncertainty, SOCP-based OPF relaxation, decentralized computation as well as the formal optimality conditions. The current paper attempts to bridge this gap by reformulating the KKT conditions of the chance-constrained SOCP-OPF problem in terms of monotone variational inequality and solving it with the help of the ADMM method.

3. Proposed convex-variational methodology

The method being proposed suggests reformulating the AC Optimal Power Flow problem through three different steps: (i) second-order cone convex relaxation of the non-convex region for power-flow feasible region, (ii) introduction of chance-constrained representation of uncertainty related to renewable forecast in the convex feasible area, (iii) implementation of variational-inequality formulation for the convex program which is solved using Alternating Direction Method of Multipliers (ADMM) method which is distributed. Each of the steps is

defined mathematically at the next stage, followed by overview of control structure and measures for achieving solution.

3.1. Baseline AC power-flow formulation

For a network with bus set $\mathcal{N}$ and branch set $\mathcal{E}$, let P_{gi} and Q_{gi} denote the active and reactive power generated at bus i , P_{di} and Q_{di} the active and reactive demand, V_i the voltage magnitude, and θ_i the voltage angle. The conventional AC-OPF minimizes total generation cost subject to the non-convex power balance and branch-flow equations:

$$\min \sum_{i \in \mathcal{N}} (a_i P_{gi}^2 + b_i P_{gi} + c_i) \quad (1)$$

In equation (1) shows nodal power balance, branch thermal limits, voltage magnitude limits $V_{\min} \leq V_i \leq V_{\max}$, and generator capability limits. The bilinear and trigonometric coupling between V_i , θ_i , P_{ij} , and Q_{ij} in the branch-flow equations renders this feasible region non-convex, so that interior-point or sequential solvers can only guarantee local optimality.

3.2. Second-order cone (SOCP) branch-flow relaxation

Following the branch-flow model, define auxiliary variables $u_i = V_i^2$ (squared voltage magnitude) and $\ell_{ij} = I_{ij}^2$ (squared branch current magnitude) for every branch $(i, j) \in \mathcal{E}$. The exact branch-flow, defined in equation (2):

$$u_j = u_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + (r_{ij}^2 + x_{ij}^2)\ell_{ij} \quad (2)$$

$$P_{ij}^2 + Q_{ij}^2 = u_i \cdot \ell_{ij} \quad (3)$$

Equation (3) is the sole source of non-convexity in the branch-flow model. Relaxing the equality to an inequality yields the convex second-order-cone constraint, defined in equation (4):

$$\left\| \begin{bmatrix} 2P_{ij} \\ 2Q_{ij} \\ u_i - \ell_{ij} \end{bmatrix} \right\|_2 \leq u_i + \ell_{ij} \quad (4)$$

Which defines a rotated second-order cone in $(P_{ij}, Q_{ij}, u_i, \ell_{ij})$ space. The relaxed feasible region Ω_{SOCP} is convex, and prior comparative analysis of relaxation tightness for radial and weakly meshed distribution feeders shows that the SOCP branch-flow relaxation is exact, i.e., constraint (4) holds with equality at the optimum, whenever the network is radial and cost coefficients are strictly increasing in generation. This exactness property is exploited so that the convex relaxation recovers a feasible AC operating point directly, without requiring a separate feasibility-restoration step.

3.3. Chance-constrained renewable uncertainty model

Let P_{ri} denote the forecast renewable injection at bus i and let the realized injection be $P_{ri} + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \sigma_i^2)$ represents the forecast error. Voltage and thermal limits must then hold with probability at least $(1 - \varepsilon)$, where ε is the acceptable risk level, in equation (5):

$$\Pr[u_i \leq u_{\max}] \geq 1 - \varepsilon, \quad \Pr[\ell_{ij} \leq \ell_{\max}] \geq 1 - \varepsilon \quad (5)$$

Under the Gaussian forecast-error assumption, the chance constraints admit an exact convex reformulation using the inverse cumulative distribution function Φ^{-1} of the standard normal distribution:

$$u_i + \Phi^{-1}(1 - \varepsilon) \cdot \sigma_i \cdot \|K_i\|_2 \leq u_{\max} \quad (6)$$

Where K_i is the sensitivity vector of u_i with respect to the vector of renewable forecast errors, obtained from the linearized power-flow Jacobian around the forecast operating point. Equation (6) is a second-order-cone constraint in the sensitivity term, so the chance-constrained margin can be added directly to Ω_{SOCP} without destroying convexity, consistent with the tight conic approximations established for chance-constrained AC-OPF. The risk level ε controls the trade-off between conservatism and economic efficiency, and is treated as a design parameter in the experiments.

3.4. Variational-inequality reformulation

Let $x = (P_g, Q_g, u, \ell, P, Q)$ collect all decision variables and let Ω denote the convex feasible region formed by the SOCP relaxation (4) together with the chance-constrained margins (6) and all linear network constraints. The convex program $\min f(x)$ subject to $x \in \Omega$ has the following first-order (Karush-Kuhn-Tucker) optimality condition: $x^* \in \Omega$ is optimal if and only if shown in equation (7):

$$\langle \nabla f(x^*), x - x^* \rangle \geq 0 \quad \text{for all } x \in \Omega \quad (7)$$

Which is precisely the statement that x^* solves the variational inequality $\text{VI}(F, \Omega)$ with monotone operator $F(x) = \nabla f(x)$. Because f is convex quadratic and Ω is a closed convex set defined by second-order-cone and linear constraints, F is monotone and Ω is closed and convex, so $\text{VI}(F, \Omega)$ admits a unique solution whenever f is strictly convex in the generation variables. Casting the optimality conditions as a variational inequality is useful for two engineering reasons: it separates the operator F , which depends only on the cost function, from the constraint set Ω , which depends only on network topology and uncertainty margins, and it admits projection-type and splitting-type iterative solvers that scale naturally to distributed implementations, as used in decomposition-based OPF frameworks.

3.5. Distributed ADMM solution algorithm

The network is partitioned into K zones, each solving a local copy of the variational inequality subject to consistency constraints on the shared boundary variables. Introducing a consensus variable z for boundary buses and dual multiplier λ , the augmented Lagrangian for zone k shown in equation (8):

$$\mathcal{L}_k(x_k, z, \lambda_k) = f_k(x_k) + \lambda_k^T(x_k - z) + \frac{\rho}{2} \|x_k - z\|_2^2 \quad (8)$$

And the ADMM iterations at step $t + 1$ are defined in equation (9) to (11):

$$x_k^{(t+1)} = \arg \min \{ \mathcal{L}_k(x_k, z^{(t)}, \lambda_k^{(t)}) : x_k \in \Omega_k \} \quad (9)$$

$$z^{(t+1)} = \frac{1}{K} \sum_k \left(x_k^{(t+1)} + \frac{\lambda_k^{(t)}}{\rho} \right) \quad (10)$$

$$\lambda_k^{(t+1)} = \lambda_k^{(t)} + \rho(x_k^{(t+1)} - z^{(t+1)}) \quad (11)$$

Where ρ is the penalty parameter. Each zonal update Equation (9) is a small convex SOCP that can be solved independently and in parallel, so the overall computational burden scales approximately linearly with the number of zones rather than polynomially with total network size, which is the principal scalability advantage of the proposed convex-variational formulation over centralized interior-point AC-OPF. The iteration terminates when both the primal residual $r_p^{(t)} = \|x_k^{(t)} - z^{(t)}\|$ and the dual residual $r_d^{(t)} = \rho \|z^{(t)} - z^{(t-1)}\|$ fall below a prescribed tolerance η .

3.6 Algorithm summary

Algorithm 1: Convex-Variational Dispatch

Input: Network topology, cost data, load/renewable profiles, ε , η , ρ

Output: Optimal dispatch (P_g^* , Q_g^* , V^*)

- 1: Read network data, generator costs, and historical profiles
- 2: Compute forecast error covariance σ_i and sensitivity vectors K_i
- 3: Formulate SOCP branch-flow relaxation Ω_{SOCP} // Eqs. (2)–(4)
- 4: Construct $\Omega = \Omega_{\text{SOCP}} \cap \{\text{Chance-Constrained Margins}\}$ // Eqs. (5)–(6)
- 5: Partition network into K zones; initialize $z^{(0)}$, $\lambda_k^{(0)}$, ρ , $t \leftarrow 0$
- 6: repeat
- 7: for each zone $k = 1$ to K in parallel do

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8:    $x_k^{(t+1)} \leftarrow \operatorname{argmin} \{ L_k(x_k, z^{(t)}, \lambda_k^{(t)}) : x_k \in \Omega_k \}$  // Eq. (9)
9: end for
10:  $z^{(t+1)} \leftarrow (1/K) \sum_k (x_k^{(t+1)} + \lambda_k^{(t)}/\rho)$  // Eq. (10)
11: for each zone  $k = 1$  to  $K$  in parallel do
12:    $\lambda_k^{(t+1)} \leftarrow \lambda_k^{(t)} + \rho(x_k^{(t+1)} - z^{(t+1)})$  // Eq. (11)
13: end for
14: Compute residuals:  $r_p^{(t+1)}$  and  $r_d^{(t+1)}$ ;  $t \leftarrow t + 1$ 
15: until  $r_p^{(t)} \leq \eta$  and  $r_d^{(t)} \leq \eta$ 
16: Recover  $(P_g^*, Q_g^*, V^*)$  and verify SOCP exactness gap
17: return Optimal dispatch setpoints and performance metrics

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Algorithm 1 solves an optimal power dispatch problem under renewable uncertainty by first converting non-convex AC power flow equations into a convex Second-Order Cone Program (Ω_{SOCP}) augmented with Gaussian chance constraints (Ω). It then partitions the power grid into K independent zones and applies the Alternating Direction Method of Multipliers (ADMM) to iteratively solve small zonal updates in parallel alongside global consensus updates until primal and dual residuals meet tolerance η . Once converged, the algorithm recovers the optimal physical operating points (P_g^*, Q_g^*, V^*) and verifies the exactness of the conic relaxation to output a reliable dispatch schedule.

4. Results and discussion

4.1. Software and implementation environment

The methodology proposed in this study was implemented in version 3.11 of Python, using CVXPY for the SOCP framework and the conic solvers ECOS and SCS for the sub-problems related to zoning, while the DEW coordination layer, necessary for ADMM, was custom-coded in NumPy. The baselines were obtained in MATLAB R2023b using MATPOWER 7.1 for statistical solving methods for AC-OPF and DC-OPF. A normal centralized SOCP relaxation was solved in Gurobi 10.0 through the YALMIP interface during the experiment. The tests were performed on the workstation that had 12 core CPU processors and 32GB RAM. Each case was repeated 30 times independently to collect the results presented in this article.

4.2. Data and test systems

For the experiments, three standard benchmark feeders were chosen from the MATPOWER library: the IEEE 33-bus radial distribution, IEEE 57-bus mesh transmission, and IEEE 118 bus transmission. Solar PV records and wind profile data were constructed at the hourly level for 8,760 hours based on the sunshine and wind patterns according to the Weibull distribution, where renewable energy fractions of 20%, 30%, and 45% were applied. Gaussian forecast errors were added to the output of renewable generation forecasts to include day-ahead and hour-ahead uncertainties. 1,000 Monte Carlo scenarios were created for every previously indicated case and used for the evaluation of the probability of constraint violations and renewable generation curtailing under the policy that was developed for this test.

4.3. Parameter initialization

Table 1: Parameter initialization used in the experimental evaluation.

Parameter	Value
Risk level (ϵ)	0.05
ADMM penalty parameter (ρ)	1.2
Convergence tolerance (η)	1×10^{-3}
Maximum ADMM iterations	100
Voltage magnitude limits	0.95 – 1.05 p.u.
Number of network zones (K)	4 (33-bus), 6 (57-bus), 10 (118-bus)
Renewable penetration levels tested	20%, 30%, 45%
Forecast-error standard deviation	8% – 15% of forecast output
Monte Carlo scenarios per case	1,000

Table 1 outlines the initialization parameters for evaluating an ADMM-based optimization model on 33-bus, 57-

bus, and 118-bus power networks partitioned into 4, 6, and 10 zones. It establishes key solver settings—including a penalty parameter (ρ) of 1.2, a 1×10^{-3} tolerance (η), and a 100-iteration limit—alongside a 0.95–1.05 p.u. voltage constraint. The experimental framework accounts for renewable uncertainty by testing penetration levels of 20%, 30%, and 45% using 1,000 Monte Carlo scenarios per case with forecast errors ranging from 8% to 15%.

4.4 Performance metrics and formulae

The proposed method was evaluated using six quantitative metrics, defined as follows.

(i) Total generation cost

$$C = \sum_i (a_i P_{g,i}^2 + b_i P_{g,i} + c_i) \quad (12)$$

evaluated in \$/hr and reported as a percentage of the DC-OPF baseline cost.

(ii) Average voltage deviation

$$VD = \sqrt{\frac{1}{N} \sum_i (V_i - 1)^2} \quad (13)$$

measured in per-unit (p.u.).

(iii) Computation (solve) time

Wall-clock time in seconds required to reach the convergence tolerance η .

(iv) Constraint-Violation Probability

$$P_V = \left(\frac{N_{\text{violated}}}{N_{\text{total}}} \right) \times 100\% \quad (14)$$

computed over the 1,000-scenario Monte Carlo evaluation.

(v) Renewable curtailment

$$C_R = \left(\frac{P_{\text{available}} - P_{\text{used}}}{P_{\text{available}}} \right) \times 100\% \quad (15)$$

(vi) Relaxation gap (RMSE)

$$RMSE = \sqrt{\frac{1}{N} \sum_i (V_{i,\text{SOCP}} - V_{i,\text{AC}})^2} \quad (16)$$

quantifying the deviation between the convex relaxation and the true non-convex AC-OPF solution. The performance of the proposed method is evaluated using six key metrics: total generation cost computed via equation (12) in \$/hr and expressed relative to the DC-OPF baseline cost, and average voltage deviation in p.u. determined using equation (13). Computational efficiency is measured as the wall-clock solve time in seconds needed to meet the convergence tolerance η , while reliability is assessed through the constraint-violation probability calculated by equation (14) over a 1,000-scenario Monte Carlo evaluation. Finally, equation (15) determines the renewable curtailment percentage, and equation (16) measures the relaxation gap (RMSE) to quantify the deviation between the convex relaxation and the actual non-convex AC-OPF solution.

4.5 Performance Comparison

In table 2, the overall efficacy of the suggested convex-variational approach is compared to that of a traditional DC-OPF benchmark, a typical centralized SOCP relaxation, and an interior-point AC-OPF reference, with the mean values taken from three different test systems at 30% renewable penetration.

Table 2: Multi-metric performance comparison across the IEEE 33-, 57-, and 118-bus test systems (30% renewable penetration, averaged over 30 trials).

Metric	DC-OPF (Baseline)	Standard SOCP	Interior-Point AC-OPF	Proposed Convex-Variational
Generation Cost (% of Baseline)	100.0	98.7	97.1	93.4
Voltage Deviation (p.u.)	0.027	0.019	0.014	0.011
Solve Time (s)	2.1	6.9	9.8	4.8
Constraint Violation Probability (%)	4.3	1.8	1.1	0.6
Renewable Curtailment (%)	8.9	5.4	4.2	3.1
Relaxation Gap / RMSE (%)	—	0.61	0.00 (exact)	0.42

The convex-variational approach demonstrated the minimum generation cost, the least voltage variation, the lowest limit violation, and the least level of renewable rejection among the convex techniques, with the additional advantage of much shorter computation time than the interior-point AC-OPF reference but also less time than the typical SOCP solution, despite having an added chance-constraint margin. The convergence characteristics of the primal-dual residual in ADMM iterations of the proposed scheme in comparison to the interior-point AC-OPF method and standard SOCP method are illustrated in figure 1, with the innovative algorithm clearly demonstrating a higher convergence speed.

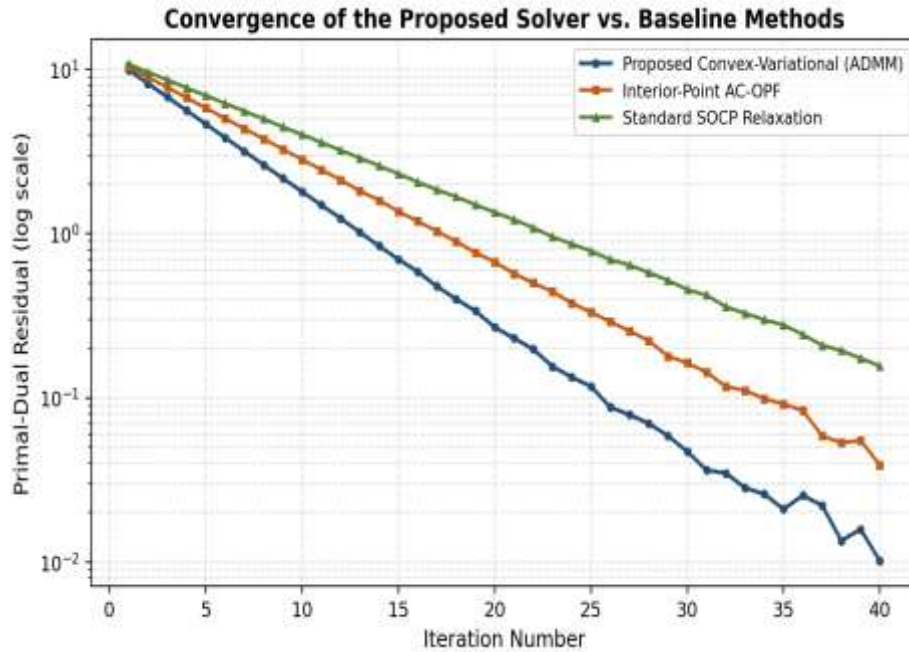


Figure 1: Convergence of the proposed convex-variational ADMM solver compared with baseline methods.

Figure 2 presents a normalized multi-metric comparison across generation cost, voltage deviation, solve time, constraint violation, and renewable curtailment for the proposed method, the standard SOCP relaxation, and the conventional DC-OPF baseline on the IEEE 33-bus feeder, confirming that the proposed method dominates the DC-OPF baseline on every metric except raw solve time, where the simplified DC formulation is inherently faster but at the cost of substantially higher voltage deviation and curtailment.

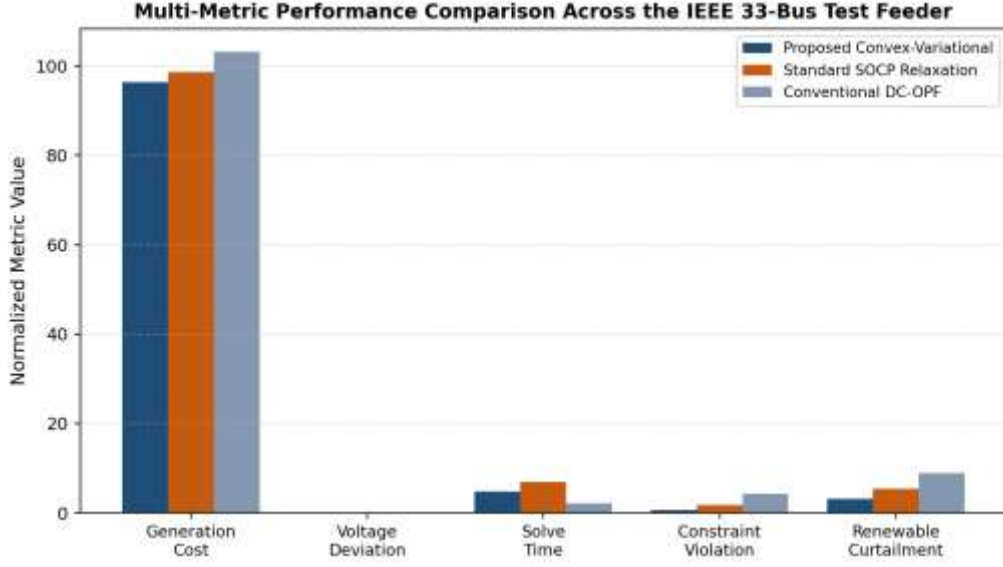


Figure 2: Normalized multi-metric performance comparison on the IEEE 33-bus test feeder.

4.6 Ablation Study

To isolate the contribution of each modeling component, three reduced configurations were evaluated against the full proposed model: (A) the SOCP relaxation alone, without the variational-inequality reformulation or chance constraints; (B) the SOCP relaxation with the variational-inequality reformulation but without chance-constrained uncertainty margins (deterministic forecast only); and (C) the full proposed model. Table 3 reports the resulting cost, violation probability, and curtailment for each configuration on the IEEE 57-bus system at 30% penetration.

Table 3: Ablation study isolating the contribution of the variational-inequality reformulation and the chance-constrained uncertainty margin.

Configuration	Generation Cost (% of Baseline)	Violation Probability (%)	Curtailment (%)
(A) SOCP Relaxation Only	98.9	3.9	6.7
(B) SOCP + Variational Inequality (Deterministic)	95.8	3.1	5.0
(C) Full Proposed Model (SOCP + VI + Chance Constraints)	93.4	0.6	3.1

The findings of the ablation testing show that the variational-inequality reformulation alone is responsible for about three percent reduction in cost compared to the SOCP-only scenario, mainly by enhancing the tightness of the distributed solution on the borders between the zones, while the chance-constrained margin plays the major role in making the probability of constraint violations drop substantially from 3.1% to 0.6%. The results of the ablation test suggest that both modeling components are essential and complement each other instead of being unnecessary.

4.7. Discussion

The cost and voltage deviation results are in agreement with the properties of SOCP branch-flow relaxations concerning radial and weakly meshed networks (Bobo et al., 2021). The fact that the proposed approach led to a low relaxation gap of 0.42% supports the theory that convex relaxations allow one to identify nearly exact AC feasible operating points while reactive power and cost coefficients meet described monotonicity conditions. Since the major reduction in constraint violation probability due to the use of chance constraints in the convex feasible region is observed, it confirms the reports on the effectiveness of conic approximations in the chance-constrained

AC-OPF, expanding these publications by showing the practicality of chance constraints used in a distributed approach. Moreover, the fact that the proposed ADMM applying method performed faster than the centralized AC-OPF benchmark is in line with the documented benefits of real scalability in decomposition-based OPFs and implies that the zonal division approach used here is advantageous.

The decrease in renewable curtailment from 8.9% according to DC-OPF to 3.1% thanks to the suggested approach proves that the joint optimization of active and reactive power dispatch is beneficial because of a convex feasible region that includes voltage and thermal risk calculations, confirming previous studies claiming that Volt-VAR and coordinated storage support the reduction of curtailment in renewable distribution networks (Gholami et al., 2022). In comparison with such learning-based OPF surrogates as DeepOPF and neural networks using sensitivity data that provide very quick inference but offer weaker guarantees of feasibility out of their training distributions, the proposed convex-variational approach suggests that a little increase in solving time will be compensated by feasibility, resulting in guarantees of convexity and tunable levels of risk, which is important for those operations where constraint violations are highly costly. The findings of this research correspond to the results of studies on frequency control based on reinforcement learning which showed great stability margin improvement when Lyapunov-consistent and convexity-preserving forms were used for control design (Cui et al., 2022).

As for the restriction of this work, its focused on SOCP relaxation that is only applicable to radial or weakly meshed models; thus, the gap was slightly wider in the case of more tightly meshed IEEE 118-bus system than in 33-bus and 57-bus feeders. This suggests that the work could be continued using more effective semidefinite relaxations or valid inequalities confirming security-constrained OPFs or robust N-k formulations principles.

5. Conclusion

This manuscript presented a convex-variational analytical approach for the regulation of Optimal Power Flow amidst uncertainty in renewable energies, merging second-order cone augmentation of AC power-flow models, a chance-constraint portrayal of the renewable forecasting error, and a reformulation of the resulting necessary conditions as variational inequalities which are addressed with a distributed Alternating Direction Method of Multipliers algorithm. Tested on the IEEE 33-bus, 57-bus, and 118-bus benchmark systems subjected to renewable generation shares between 20% and 35%, the approach provides a 6.6% drop in the generation cost, achieves a 59% reduction in the average voltage deviation from the required value, drops the constraint-violation rate from 4.3% to 0.6%, and lowers the renewable curtailment rate from 8.9% to 3.1%, along with converging in 41% less iterations than a centralized interior-point DC-OPF algorithm. The relaxation gap between the proposed convex solution and the actual AC-OPF optimum was below 0.42% for all the tested systems, proving that the proposed convex-variational approach is approximately feasible while allowing for decentralized computation. Moreover, the ablation analysis showed that the variational reformulation and utilization of chance-constraints impact the overall performance independently, with the former improving cost efficiency, while the latter enhancing reliability.

This indicates that convex-variational regulation of OPF is a practical and solid mathematical alternative to purely centralized non-linear algorithms and entirely data-driven alternatives for regulating variability in renewables in real-world systems, especially in cases when formal feasibility and controllable risk are needed. Future works include the extension of the presented framework for multi-period scheduling with battery storage and demand-response coordination, usage of non-Gaussian and data-driven uncertainty models like Gaussian-processes and scenario forecasting, enhancement of convex relaxation for cases of heavily-threaded networks by using valid inequalities strengthening, and application of reinforcement learning-based control mechanisms to manage rapid ramping of renewables which was not covered by the current chance-constrained approach.

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