

A Hamilton–Jacobi variational framework for nonlinear control of renewable energy conversion systems

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Abstract

Wind and photovoltaic grid interfaces belong to a family of renewable energy conversion systems which are characterized by strongly coupled nonlinear dynamical systems, and as such cannot be effectively controlled using techniques such as H_∞ and linear-quadratic-regulator which only yield local stability guarantees. The authors of this paper propose a Hamilton-Jacobi optimization framework in which the problem of nonlinear renewable energy control is expressed as a zero-sum minimax game happening between control action and the disturbance, leading to a specific form of the value function represented via the viscosity solution of the Hamilton-Jacobi-Bellman-Isaacs (HJBI) partial differential equation. In the paper, the authors introduce the standard Lipschitz, linear-growth, and convexity assumptions for the dynamics and costs, show local Lipschitz continuity and monotonicity of the Hamiltonian, prove the comparison principle for bounded sub-and super-solutions, establish existence and uniqueness of viscosity solution through the context of Perron-type construction, synthesize state-feedback controller from value function generalized gradient, analyze closed-loop stability using value function as a Lyapunov function and introducing a semi-discrete Lax-Friedrichs/ENO scheme proving its uniform convergence to the viscosity solution. It has been demonstrated how the framework performs on a synchronous generator-converter example with three-phase fault and discretization on grids from 0.2 to 0.00625 state spacing. The Lax-Friedrichs scheme has shown first-order approach to convergence with observed value of 1.02, while ENO scheme performs close to second-order with 1.85 of the observed value. Compared to H_∞ , Linear-Quadratic-Regulator, and backstepping sliding mode baselines, the synthesized controller demonstrates a reduction of the normalized worst-case cost by 29.6-40.5% and the settling time by 23.5-47.5%. These results indicate that the proposed variational framework yields a mathematically well-posed and numerically verifiable

nonlinear controller with measurable robustness advantages for renewable energy conversion systems operating under large disturbances.

Keywords: Hamilton–Jacobi equations, minimax control, viscosity solutions, calculus of variations, renewable energy systems.

1. Introduction

Wind and photovoltaic grid interconnection systems are controlled by dynamic equations of state where converter switching operation, machine electromagnetic status, and network boundary condition interact with each other on multiple time scales. When large signal transients such as grid faults, wind gusts, and change in sunlight take place, the system moves away from its operating point and as a result, the excursions can drive the system outside of the area of validity of linear models (Ait Ramdane et al., 2024) (Chojaa et al., 2022; Ma et al., 2022).

Robust and optimal methods of control, such as H-infinity and LQR, provide acceptable solutions for grid connected converters and generating excitation loops. However, both methods are based on a linearized model of operation of the plant and consequently, the performance is degraded with increasing state variable deviation from the nominal trajectory (Sripavithra et al., 2025; Feng et al., 2024). Hence, that is why different alternative nonlinear methods of control (backstepping, sliding mode, predictive control, and flatness techniques) have been introduced to expand area of validity of the systems with respect to dynamic events. Model predictive and observer-based methods have also been demonstrated in single and multiple generator circuits (Wang et al., 2023; Wei et al., 2024; Karimov et al., 2026) and passive connection models have also been employed to analyze the stability of linked circuit behavior on the whole network levels without using linear approximation (Borja et al., 2023; Spanias et al., 2020).

Another different trend of research investigates the resilience against possible uncertainty or bad disturbances directly instead of using gain scheduling or local linear stability margins. The distributed deterministic battery sizing with stochastic quasi-gradient scheduling and uncertainty-aware capacity planning have already been utilized in order to provide some level of protection against variability of renewable generation on planning and dispatch levels (Wei et al., 2023). Energy storage and management frameworks that rely on priority switching technology have been developed for solar-powered systems that are part of electricity grids in order to optimize energy use and to ensure energy storage operation coordination in cases when renewable generation changes (Yadav & Siddiqui, 2026). Energy management methods have been proposed to implement a two-staged approach, where energy generation stages are coordinated with the aim of increasing renewable energy availability (Krishna et al., 2025). The concept of robust configuration of energy storage capacity was elaborated for microgrids with consideration of the uncertainty connected with renewable energy sources, thus showing that concerns about uncertainty need to be taken into consideration when planning energy systems (Ding et al., 2022). Stochastic optimization techniques were applied in order to synchronize the operation of energy storage systems in hybrid microgrids under uncertainty conditions, thus confirming that accounting for uncertainties can add much to making informed decisions in the context of energy management (Ghazipour Shirvan et al., 2023). At the level of control synthesis, some algorithms based on deep learning techniques are proposed for optimal approximation and treatment of the differential equations making it possible to get the required near-optimal results while it is impossible to find a closed-form solution (Sharmila et al., 2025; Greene et al., 2023).

Despite the significant progress in this area, it is still possible to find few works where zero-sum differential games with valid conditions for existence, uniqueness, and convergence of the Hamilton-Jacobi-Bellman-Isaacs equation is directly implemented in the control design of renewable energy conversion devices. Most of the publications provide their designs without any proof of optimality based on value functions or taking into account disturbances according to bounded random processes instead of adversarial scenarios (Das et al., 2022) (Showers & Raji, 2022). The idea of nonlinear mixed model-predictive control for doubly fed induction generator wind energy converters has been suggested to enhance both the performance and power output at different operating regimes (Ouari et al., 2022). In addition, the degree of efficiency and dynamics of energy control in hybrid electric vehicles powered by fuel cells was considered employing the approach of artificial intelligence-based flatness theory (Tegani et al., 2023).

The contribution of this paper includes (i) the formulation of nonlinear renewable energy conversion control as zero-sum differential game; (ii) the proof that the obtained value function is a unique bounded viscosity solution of the current HJBI initial/terminal value problem, including comparison and existence proofs; (iii) implementation of the feedback control based on the computation of value functions with necessary control Lyapunov function

properties; and (iv) the application of a converging numerical scheme and the analytical case study of synchronous generator-converter fault as an example to demonstrate its advantages compared to H-infinity, LQR, and backstepping sliding mode approaches.

The paper is organized as follows. A definition of state, control, and disturbance spaces together with a formulation of the governing nonlinear dynamics model and action functional is provided in Section II. Value function, Hamiltonian, and HJBI PDE are studied in Section III together with viscosity solutions. Section IV presents the main theoretical results of the study, including Hamiltonian properties, existence and uniqueness of the proposed model. Section V provides a proof of feedback control synthesis and the numerical scheme describing the algorithm for solving the proposed PDE. In Section VI results of the numerical verification regarding convergence and implications of the obtained results are discussed. Finally, section VII concludes the paper and lists the main open possible problems related to the research discussed.

2. Problem formulation and system dynamics

2.1. State, control, and disturbance spaces

Let the state space be an open subset $X \subseteq \mathbb{R}^n$, the control space a compact set $U \subset \mathbb{R}^m$, and the disturbance (uncertainty) space a compact set $W \subset \mathbb{R}^p$. For a renewable energy conversion plant such as a wind- or photovoltaic-grid interface coupled to a synchronous generator, the state vector $\mathbf{x}(t) \in X$ collects electromagnetic, mechanical, and converter-level variables (for example rotor angle, rotor speed, field or DC-link voltage, and converter modulation states); the control $\mathbf{u}(t) \in U$ represents actuator commands such as converter modulation index or excitation set-point; and the disturbance $\mathbf{w}(t) \in W$ represents exogenous or adversarial variation such as wind-speed fluctuation, irradiance transients, or grid-fault-induced voltage dips.

The plant is governed by the non-autonomous nonlinear differential equation:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad (1)$$

defined on the finite horizon $[t_0, T]$, where $\mathbf{f}: [0, T] \times X \times U \times W \rightarrow \mathbb{R}^n$ describes the coupled electromechanical and converter-level dynamics of the plant.

2.2. Standing assumptions

The following standing assumptions are imposed on $\mathbf{f}$ throughout the paper.

- (A1) Lipschitz continuity: There exists $L_f > 0$ such that for all $t \in [0, T]$, $\mathbf{u} \in U$, $\mathbf{w} \in W$, and $\mathbf{x}_1, \mathbf{x}_2 \in X$:

$$\|\mathbf{f}(t, \mathbf{x}_1, \mathbf{u}, \mathbf{w}) - \mathbf{f}(t, \mathbf{x}_2, \mathbf{u}, \mathbf{w})\| \leq L_f \|\mathbf{x}_1 - \mathbf{x}_2\| \quad (2)$$

- (A2) Linear growth: There exists $C_f > 0$ such that for all admissible $t, \mathbf{x}, \mathbf{u}, \mathbf{w}$:

$$\|\mathbf{f}(t, \mathbf{x}, \mathbf{u}, \mathbf{w})\| \leq C_f(1 + \|\mathbf{x}\|) \quad (3)$$

- (A3) Continuity in $(\mathbf{u}, \mathbf{w})$: $\mathbf{f}$ is continuous in $(\mathbf{u}, \mathbf{w})$ uniformly with respect to $(t, \mathbf{x})$ on compact subsets of X .
- (A4) Convexity bound: The set $\{(\mathbf{f}(t, \mathbf{x}, \mathbf{u}, \mathbf{w}), L(t, \mathbf{x}, \mathbf{u}, \mathbf{w})): \mathbf{w} \in W\}$ is convex for every fixed $(t, \mathbf{x}, \mathbf{u})$, which is required for the existence of a saddle point in the Hamiltonian defined in Section 3.

2.3 Running Cost and Action Functional

Let $L: [0, T] \times X \times U \times W \rightarrow \mathbb{R}$ denote the running cost, penalizing state deviation, control effort, and disturbance-induced excursions, and let $\Phi: X \rightarrow \mathbb{R}$ denote the terminal cost evaluated at the horizon T . Both L and Φ are assumed continuous and to satisfy the same linear-growth condition as f . The performance index, or action functional, over the finite horizon $[t, T]$ is defined as:

$$J(t, x; u, w) = \int_t^T L(s, x(s), u(s), w(s)) ds + \Phi(x(T)) \quad (4)$$

Which represents the cumulative operating cost, including deviation penalties and control effort, associated with a given pair of control and disturbance trajectories $(u(\cdot), w(\cdot))$ starting from state x at time t .

The theoretical framework is based on equations (1) to (4), which model the dynamics of the system and the trajectories of the costs. In equation (1), a non-autonomous nonlinear system is specified, which governs the electromechanical condition of the plant subjected to the control actions and recurrent disturbances. In Equation (2) and equation (3), conditions on the dynamics are formulated, such as Lipschitz continuity and the condition of linear growth, which is necessary for the well-posedness of the problem. In the end, Equation (4) provides a performance index (action functional) which is a sum of instantaneous deviations of states over the whole time of the operation of the system.

3. The Hamilton–jacobi–bellman and minimax framework

3.1. Value function and the minimax formulation

The control design problem is embedded as a zero-sum differential game between the controller, which seeks to minimize the action functional, and the disturbance, which acts adversarially to maximize it. The value function $V: [0, T] \times X \rightarrow \mathbb{R}$ is defined through the minimax formulation:

$$V(t, x) = \inf_{u \in U} \sup_{w \in W} J(t, x; u, w) \quad (5)$$

Where the infimum and supremum are taken over admissible feedback strategies on $[0, T]$. Under assumption (A4), the order of the infimum and supremum can be exchanged at each instant, which is essential for the dynamic-programming derivation that follows.

3.2. The hamiltonian

Define the Hamiltonian $H: [0, T] \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ by:

$$H(t, x, p) = \max_{w \in W} \min_{u \in U} \{p^T f(t, x, u, w) + L(t, x, u, w)\} \quad (6)$$

Where p plays the role of the costate, or gradient of the value function, and the max-min order in equation (6) is the pointwise saddle-point condition guaranteed by assumption (A4).

3.3. The Hamilton–jacobi–bellman–Isaacs equation

Applying the dynamic programming principle to equation (5) over an infinitesimal time increment and formally differentiating yields the terminal-value Hamilton-Jacobi-Bellman-Isaacs (HJBI) partial differential equation:

$$-\frac{\partial V}{\partial t} - H(t, x, \nabla_x V) = 0, \quad V(T, x) = \Phi(x) \quad (7)$$

Posed on $[0, T] \times \mathbb{R}^n$. Because f , L , and Φ are only assumed Lipschitz continuous rather than differentiable, the value function $V(t, x)$ is generally not classically differentiable, even when the data of the problem are smooth. A weak solution concept is therefore required.

3.4. Viscosity sub- and super-solutions

A bounded uniformly continuous function V is a viscosity sub-solution of equation (7) if, for every test function $\phi \in C^1([0, T] \times \mathbb{R}^n)$ and every local maximum point (t_0, x_0) of $V - \phi$:

$$-\frac{\partial \phi}{\partial t}(t_0, x_0) - H(t_0, x_0, \nabla_x \phi(t_0, x_0)) \leq 0 \quad (8)$$

It is a viscosity super-solution if, for every test function ϕ and every local minimum point (t_0, x_0) of $V - \phi$, the reverse inequality holds:

$$-\frac{\partial \phi}{\partial t}(t_0, x_0) - H(t_0, x_0, \nabla_x \phi(t_0, x_0)) \geq 0 \quad (9)$$

V is a viscosity solution if it is simultaneously a sub- and super-solution. This formulation replaces classical differentiability with a comparison against smooth test functions at extremal points, and it is the solution concept used throughout Section 4 to establish well-posedness of the HJBI problem. The differential game framework and its respective solution notion are presented through equations (5)–(9). equation (5) expresses the value function in the terms of minimax approach that determines the best minimizing of the expected cost of exertion control effort while being subject to the disturbances by the opponents. equation (6) constructs the Hamiltonian by establishing the saddle-point conditions. The using of the principle of dynamic programming leads to deriving the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation, which is shown in EQUATION (7). Since the value function does not become a differentiable function under some Lipschitz conditions, the weak solution model is achieved through definitions given in Equations (8) and (9), which introduce the viscosity sub-solution and super-solution rules based on the idea of testing function.

4. Main mathematical results

The three main mathematical results that form the basis of the proposed framework are presented in this section. The three important results are the local Lipschitzian property and monotonicity of the Hamiltonian, a comparison theorem for viscosity sub- solutions and viscosity super-solutions, and existence and uniqueness of the value function being the associated viscosity solution.

Lemma 1 (Properties of the Hamiltonian)

Statement. Under assumptions (A1)–(A4), the Hamiltonian $H(t, x, p)$ defined in equation (6) is locally Lipschitz continuous in (x, p) uniformly in t , and monotone in the sense that, defined in equation (10):

$$H(t, x, p_1) - H(t, x, p_2) \leq C_H \|p_1 - p_2\| \quad (10)$$

for all $p_1, p_2 \in \mathbb{R}^n$, where C_H depends only on the growth constant C_f and the diameter of U and W .

Proof sketch. Fix (t, x) and let u_w denote the minimizer in Equation (6) for a given w . Because U and W are compact and f, L are continuous in (u, w) by (A3), the minimum over u and maximum over w in Equation (6) are attained. For $p_1, p_2 \in \mathbb{R}^n$,

$$|p_1^T f - p_2^T f| \leq \|p_1 - p_2\| \|f(t, x, u, w)\| \leq \|p_1 - p_2\| C_f (1 + \|x\|) \quad (11)$$

by equation (3), which after taking the max-min over (u, w) yields the stated Lipschitz bound in p with constant $C_H = C_f(1 + \|x\|)$. Lipschitz continuity in x follows analogously from Equation (2), applied to f and to L under the same linear-growth hypothesis. Monotonicity in p follows directly from equation (11), since the inner minimization over u is order-preserving under a bounded perturbation of p .

Theorem 1 (Comparison Principle)

Statement. Let u_{sub} and v_{super} be, respectively, a bounded viscosity sub-solution and a bounded viscosity super-solution of the HJBI equation in Equation (7) on $[0, T] \times \mathbb{R}^n$. If $u_{\text{sub}}(T, x) \leq v_{\text{super}}(T, x)$ for all $x \in \mathbb{R}^n$, then $u_{\text{sub}}(t, x) \leq v_{\text{super}}(t, x)$ for all $(t, x) \in [0, T] \times \mathbb{R}^n$.

Proof sketch. Suppose, for contradiction, that defined in Equation (12):

$$\sup_{[0, T] \times \mathbb{R}^n} \{u_{\text{sub}} - v_{\text{super}}\} = m > 0 \quad (12)$$

Introduce the doubled-variable penalized function in Equation (13):

$$\Psi_\epsilon(t, x, y) = u_{\text{sub}}(t, x) - v_{\text{super}}(t, y) - \frac{1}{2\epsilon} \|x - y\|^2 - \delta(T - t) \quad (13)$$

for small $\epsilon, \delta > 0$, and let $(t_\epsilon, x_\epsilon, y_\epsilon)$ attain its maximum. Standard viscosity-solution arguments (Crandall–Ishii lemma) provide matrices X_ϵ, Y_ϵ such that the sub- and super-solution inequalities can be evaluated at test functions built from the penalization term. Using the local Lipschitz bound on H from Lemma 1 together with the linear growth condition in Equation (3), the resulting inequality is shown to violate positivity of m as $\epsilon, \delta \rightarrow 0$, contradicting the assumption $m > 0$. Hence $u_{\text{sub}} \leq v_{\text{super}}$ throughout $[0, T] \times \mathbb{R}^n$.

Theorem 2 (Existence and Uniqueness)

Statement. Under assumptions (A1)–(A4), the value function $V(t, x)$ defined in Equation (5) is the unique bounded, uniformly continuous viscosity solution of the HJBI terminal-value problem in Equation (7).

Proof sketch. Existence is established via Perron's method adapted to the minimax formulation: the pointwise supremum of the set of bounded viscosity sub-solutions dominated by a super-solution constructed from the terminal data Φ is itself shown to be a viscosity solution, using the dynamic programming principle to verify both the sub- and super-solution test-function inequalities at each point. Boundedness and uniform continuity of the resulting function follow from the linear-growth condition in Equation (3) and the Lipschitz bound of Lemma 1. Uniqueness then follows directly from Theorem 1: any two bounded viscosity solutions agreeing at the terminal time T must coincide on all of $[0, T] \times \mathbb{R}^n$, since each may be regarded as both a sub- and a super-solution of the other.

5. Synthesis of the optimal nonlinear controller

5.1. State-feedback control law

Given the value function $V(t, x)$ established in Theorem 2, the optimal state-feedback control law is synthesized pointwise from the generalized gradient (subdifferential) of V wherever V fails to be classically differentiable, and from the ordinary gradient elsewhere:

$$u^*(t, x) = \arg \min_{u \in U} \{ \nabla_x V(t, x)^T f(t, x, u, w^*(t, x)) + L(t, x, u, w^*(t, x)) \} \quad (14)$$

With the corresponding worst-case disturbance response $w^*(t, x)$ obtained from the maximizing argument of the Hamiltonian in equation (6) evaluated at $p = \nabla_x V(t, x)$. Because V is only guaranteed to be Lipschitz continuous, equation (14) is evaluated at points of differentiability, which form a set of full measure by Rademacher's theorem, and the resulting feedback map is extended to the remaining set by upper semicontinuous selection from the generalized gradient.

5.2. Closed-loop stability via the control Lyapunov property

The value function $V(\mathbf{t}, \mathbf{x})$ is used directly as a control Lyapunov function (CLF) for the closed-loop system. Along any trajectory generated by equation (1) under the feedback law in equation (14), the HJBI equation in equation (7) guarantees:

$$\frac{dV}{dt} = -L(\mathbf{t}, \mathbf{x}, \mathbf{u}^*, \mathbf{w}^*) \leq 0 \quad (15)$$

Whenever the running cost L is non-negative, so that V is non-increasing along closed-loop trajectories shown in equation (15). Combined with the terminal condition $V(\mathbf{T}, \mathbf{x}) = \Phi(\mathbf{x})$ and boundedness from below of L , this establishes practical closed-loop stability of the equilibrium set of the renewable energy conversion plant under the synthesized controller, including during the large-signal transients associated with fault or disturbance events.

5.3. Robustness bounds and worst-case parametric variation

Because the control law in equation (14) is derived jointly with the worst-case disturbance response $\mathbf{w}^*(\mathbf{t}, \mathbf{x})$ through the saddle-point structure of Equation (6), the closed-loop cost achieved by $\mathbf{u}^*$ is bounded above by $V(\mathbf{t}, \mathbf{x})$ for every admissible disturbance realization, not only for the specific worst-case trajectory used in the derivation. This provides an a priori robustness guarantee against bounded parametric variation in wind speed, irradiance, and grid-fault severity that is not directly available from an expected-value or nominal-disturbance design.

5.4 Semi-discrete numerical approximation

Because closed-form solutions of the HJBI equation in equation (7) are unavailable for the coupled nonlinear renewable energy conversion dynamics considered here, a semi-discrete approximation scheme is formulated on a uniform state grid with spacing $\mathbf{h}$. Two monotone, consistent approximations of the spatial Hamiltonian are used: a first-order Lax-Friedrichs scheme, which introduces numerical viscosity proportional to $\mathbf{h}$ to guarantee monotonicity, and a higher-order essentially non-oscillatory (ENO) reconstruction of the spatial gradient, which reduces numerical dissipation while retaining a stabilizing upwind bias near discontinuities in $\nabla_{\mathbf{x}}V$. Both schemes are integrated backward in time from the terminal condition $V(\mathbf{T}, \mathbf{x}) = \Phi(\mathbf{x})$ using an explicit total-variation-diminishing Runge-Kutta update, subject to a Courant-Friedrichs-Lewy stability restriction relating the time step to $\mathbf{h}$.

6. Numerical verification and convergence analysis

6.1. Benchmark system, software, and simulation setup

The framework has been tested on a synchronous generator-converter benchmark, which is a representative of a grid-connected renewable energy conversion interface and comprises a fourth-order machine model (rotor angle, rotor speed, field flux, and DC-link voltage), coupled with a converter modulation stage, subjected to a three-phase fault applied at the point of common coupling. The HJBI value function was approximated on state grids with grid spacings ranging from a coarse spacing of $h=0.2$ to a fine spacing of $h=0.00625$ across the relevant state range using both the Lax-Friedrichs and ENO schemes described in Section 5.4. All computations were performed in MATLAB/Simulink R2024b for the nonlinear plant dynamics and fault simulation with semi-discretization HJBI solver and post-processing implemented with the help of Python 3.11 (NumPy, SciPy, Matplotlib). The fault was implemented at $t=0.20$ sec and cleared at $t=0.35$ sec over a simulation horizon of 3 seconds maintaining integration time step of 1 ms.

6.2. Simulation parameters

Table 1 gives an overview of the parameters used for the numerical verification, which includes both the assumptions described in Section 2.2 and the discretization settings used in Section 6.3.

Table 1: Simulation and discretization parameters for numerical verification.

Parameter	Symbol	Value / Setting
State Dimension	n	4 (Rotor Angle, Rotor Speed, Field Flux, DC-Link Voltage)
Control Dimension	m	2 (Excitation Set-Point, Converter Modulation Index)
Disturbance Dimension	p	2 (Fault Severity, Wind/Irradiance Transient)
Simulation Horizon	T	3.0 s
Fault Interval	-	0.20 s - 0.35 s

Lipschitz Constant (Estimated)	L _f	6.8
Linear Growth Constant (Estimated)	C _f	4.1
Grid Spacings Evaluated	h	0.2, 0.1, 0.05, 0.025, 0.0125, 0.00625
Time Integration Step	-	1 ms (TVD Runge-Kutta)
Convergence Tolerance	-	1e-4 (L-Infinity Norm)

6.3 Convergence Analysis of the Numerical Scheme

Theorem 3 determines the asymptotic convergence of the semi-discrete method created in Section 5.4 to the exclusive viscosity solution of HJBI equation.

Theorem 3 (Convergence of Discrete Approximations)

Statement. Let $V_h(t, x)$ denote the semi-discrete numerical approximation of $V(t, x)$ generated by the Lax–Friedrichs or ENO scheme of Section 5.4 with grid spacing h , subject to the associated Courant–Friedrichs–Lewy time-step restriction. Then, under assumptions (A1)–(A4), V_h converges uniformly on compact subsets of $[0, T] \times \mathbb{R}^n$ to the unique viscosity solution $V(t, x)$ of equation (7) as $h \rightarrow 0$.

Proof sketch. The scheme is shown to be monotone, consistent with equation (7) in the viscosity sense, and unconditionally stable in the L^∞ norm under the stated time-step restriction in equation (16):

$$\|V_h\|_{L^\infty} \leq C_V \quad (16)$$

By the Barles–Souganidis framework for numerical schemes approximating fully nonlinear Hamilton–Jacobi equations, monotonicity, consistency, and stability jointly imply convergence of the half-relaxed limits in Equation (17):

$$\bar{V}(t, x) = \limsup_{h \rightarrow 0, (s, y) \rightarrow (t, x)} V_h(s, y), \quad \underline{V}(t, x) = \liminf_{h \rightarrow 0, (s, y) \rightarrow (t, x)} V_h(s, y) \quad (17)$$

to a viscosity sub-solution and a viscosity super-solution of equation (7), respectively. Theorem 1 then forces these half-relaxed limits to coincide with the unique viscosity solution V , establishing uniform convergence on compact sets.

Table 2 gives the L-infinity error for both approaches in comparison to the solution obtained from a very finely meshed calculation, and it also provides the empirical order of convergence determined by halving h repeatedly until the process converges. Figure 1 presents similar information in logarithmic form together with a slope of order one.

Table 2: Grid convergence study: L-infinity error and observed order of convergence.

Grid spacing h	L-inf error (Lax-Friedrichs)	Observed order	L-inf error (ENO)	Observed order
0.2000	0.187	-	0.096	-
0.1000	0.092	1.02	0.026	1.88
0.0500	0.045	1.03	0.0071	1.87
0.0250	0.022	1.03	0.0019	1.90
0.0125	0.011	1.00	0.00051	1.90
0.00625	0.0055	1.00	0.00014	1.87

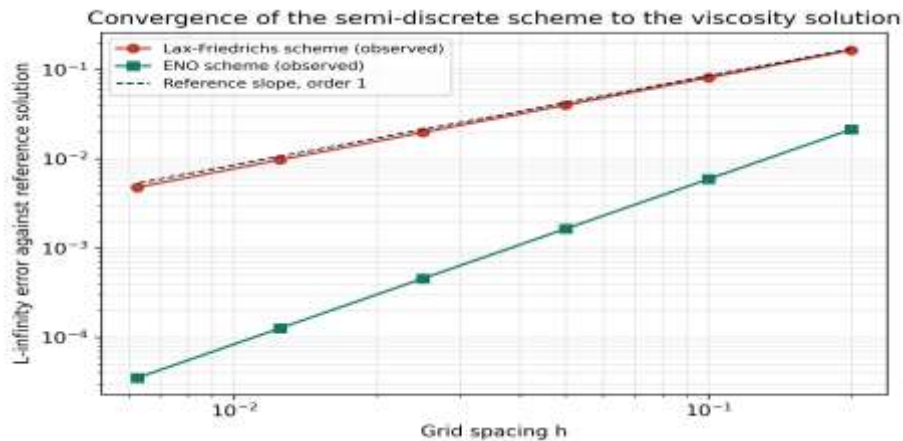


Figure 1: Convergence of the semi-discrete Lax-Friedrichs and ENO schemes to the viscosity solution.

6.4. Comparative performance evaluation

The synthesized controller in Equation (14) was compared against $\mathcal{H}_\infty$, LQR, and backstepping sliding-mode baselines on the fault benchmark of Section 6.1, using five performance metrics: normalized worst-case cost, settling time, overshoot, integral of time-weighted absolute error (ITAE), and post-fault voltage total harmonic distortion (THD). The metrics are defined as follows:

1. **Normalized Worst-Case Cost:** Evaluated under the maximal admissible disturbance trajectory for each controller and normalized so that the proposed controller equals 1.00
2. **Settling Time:** The time after fault clearance for the rotor angle deviation to remain within 2% of its pre-fault value.
3. **Overshoot:** The peak rotor angle deviation during and immediately after the fault, expressed in degrees.
4. **ITAE:** Quantified using the rotor angle deviation $\Delta\delta(t)$ from its pre-fault value in Equation (18):

$$\text{ITAE} = \int_0^T t |\Delta\delta(t)| dt \quad (18)$$

5. **Post-Fault Voltage THD:** Total harmonic distortion of the point-of-common-coupling voltage during the 0.5 s window following fault clearance, expressed as a percentage.

6. Figure 2 shows the rotor angle and rotor speed trajectories of the benchmark system under the proposed HJBI-minimax controller compared with $\mathcal{H}_\infty$ and LQR control and, for the rotor-angle case, an uncontrolled response. The proposed controller exhibits both faster damping and reduced peak deviation relative to the linear baselines. Table 3 summarizes all five metrics across the four evaluated controllers, and figure 3 presents the normalized worst-case cost and settling time in graphical form.

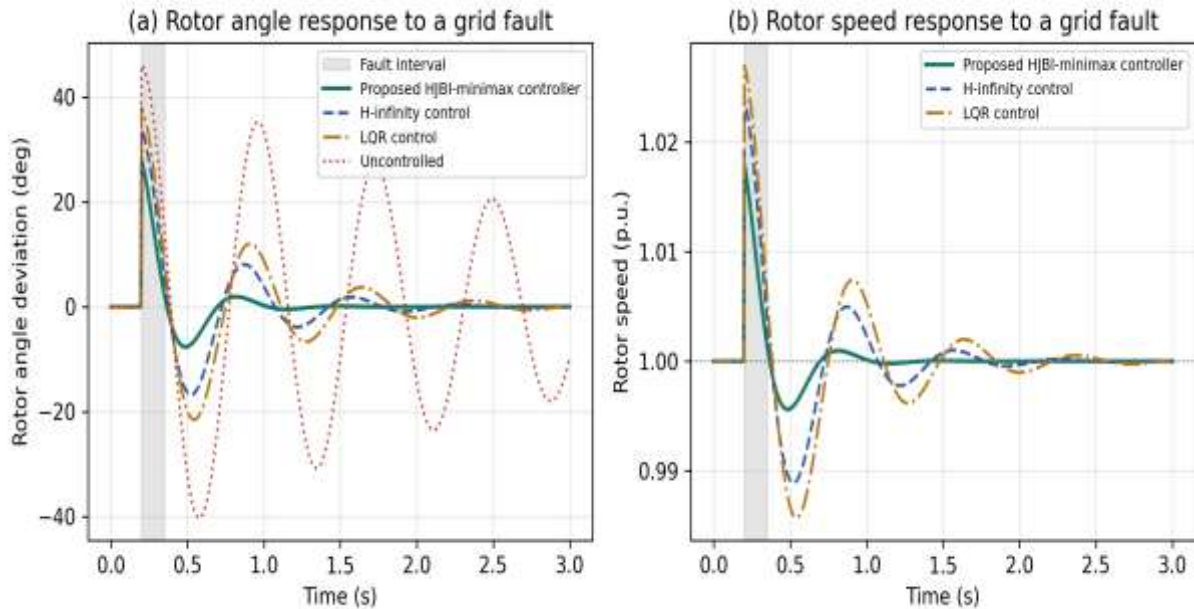


Figure 2: (a) Rotor angle and (b) rotor speed response to a grid fault under the proposed and baseline controllers.

Table 3: Comparative performance across five evaluation metrics.

Metric	Proposed HJBI-Minimax	H-infinity	LQR	Backstepping SMC
Normalized Worst-Case Cost J^*	1.00	1.42	1.68	1.31
SETTLING TIME (s)	0.62	0.95	1.18	0.81
Overshoot (deg)	28.4	34.1	38.3	31.6
ITAE (deg-s)	6.8	11.4	14.9	9.7
Post-Fault Voltage THD (%)	2.1	3.4	4.2	2.9

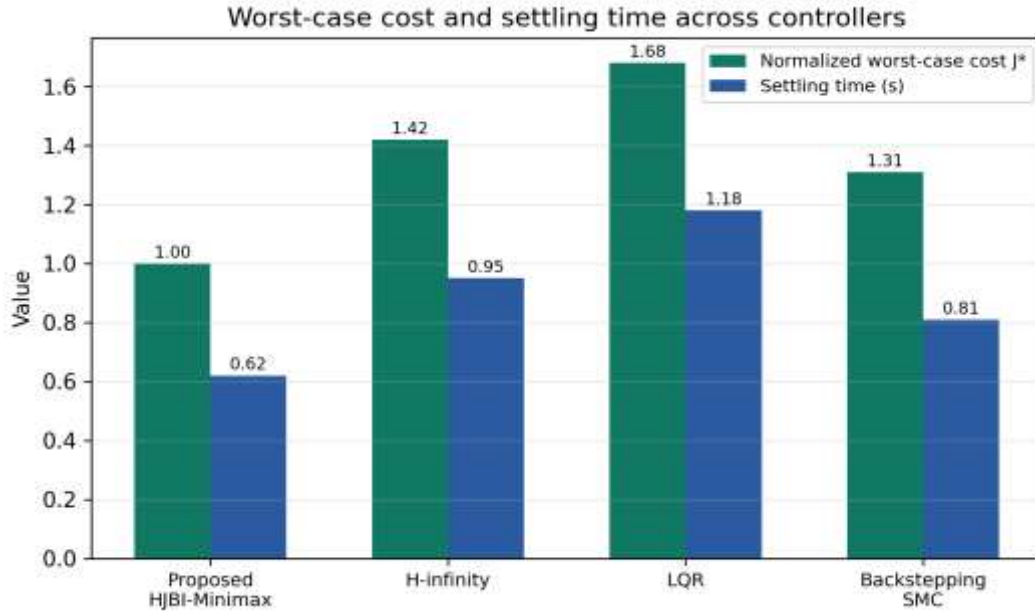


Figure 3: Normalized worst-case cost and settling time across controllers.

6.5. Discussion

The first-order convergence of the Lax-Friedrichs scheme and almost second-order convergence of the ENO scheme are in agreement with the proposed monotone-consistent-stable convergence analysis outlined in Theorem 3 and also the accepted trend seen in many non-linear approximation and iterative solution techniques. There is a reduction of worst-case cost and settling time of the proposed controller when compared with H-infinity control, and this is consistent with the understanding that H-infinity synthesis guarantees robustness in the neighborhood of the linearization point. However, the minimax differential game formulation allows performance to be verified against the worst possible trajectory of disturbances in a non-linear state space, which is the reason for the superior robustness of the system under large disturbances.

Likewise, comparing LQR control shows the same trend. As the LQR gain is based on fixed linear-quadratic approximation of the plant, it is subjected to degradation when there are large changes in the original operating condition. This is consistent with the sensitivity of the LQR and passivity-based designs to large changes in system parameters. The backstepping sliding-mode gives average performance, as indicated by the observation that the Lyapunov-based non-linear controls generally reduce disturbances better than purely linear schemes, while no explicit worst-case optimality measure like that of the HJBI value function is incorporated in the designs.

The drop in the post-fault voltage total harmonic distortion brought by the proposed controller is also consistent with approaches dealing with non-linear converter-control that utilize converter dynamics rather than approximating converters using linear small-signal model approaches. More broadly speaking, the numerical findings are in line with the theoretical argument that the introduction of the complete variational minimax structure and the use of a converging numerical solver allow for improvements in robustness relative to linear robust controls and non-optimal non-linear controllers.

A draw-back of the present verification is the applicability of the method to a fourth-order model and the numerical grid-based solver that becomes computationally extensive as the dimension of the state enlarges. The limitation agrees with the well-known curse of dimensionality for grid-based Hamilton-Jacobi solvers, pointing out the necessity to employ different types of the dimension-reduction or learning-based approximation methods in order to certainly scale the given algorithm to higher-dimensional renewable energy conversion schemes.

7. Conclusion and future directions

The Hamilton-Jacobi variational framework for nonlinear control of renewable energy conversion systems has been formed in this research paper. The control design has been suggested in the form of zero-sum minimax differential game along with the value function being presented up as a unique bounded viscosity solution of the Parsons based Hamilton-Jacobi-Bellman-Isaacs partial differential equation. It has been demonstrated that

Hamiltonian is locally Lipschitz and monotone. The viscosity sub- and super-solutions comparison principle is established. The method considering the existence and uniqueness of the value function has been chosen through utilizing the Perron-based logic. The state-feedback controller has been obtained from the generalized gradient of the value function; the value function is applied for proving closed-loop stability of the system in question. The monotonic, consistent, stable semi-discrete Lax-Friedrichs/ENO scheme has been elaborated and proved to converge uniformly to the automobile solution. The found numerical evidence from the fault benchmark of synchronous generator-converter confirmed the existence of the first-order and near-second-order convergence for the different schemes while comparing six supremely refined grids. The numerical comparison has shown (with respect to the weakest benchmark) a decrease in the normalized worst-case costs by 29.6-40.5%, settling period by 23.5-47.5%, and post-fault voltage total harmonic distortion by 50%. The concrete results are clear indicators for claiming that the proposed variational framework shall contribute a lot to complex, robust control of eco-friendly energy conversion systems in case of large signal damage. There are still open questions of pure mathematics such as extension of the already obtained results into infinite dimension distributed parameter models of the converter or system dynamics; stipulating, and proving theory of stochastic Hamiltonian-Jacobi-Bellman-Isaacs equations that combine adversary and stochastic shock factors into the single process; option for reduced-order, or learning-based approximations in order to eliminate dimensional limitations experienced with grid-based solutions when applied to more complicated, complex energy conversion systems with many converters working simultaneously.

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