

A Robust Mathematical Programming Model for Energy Storage Scheduling Using Minimax Decision Principles

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Abstract

Integrating variable renewable resources into modern power systems requires scheduling methods for battery energy storage systems (BESS) that can operate under high net-load forecast errors and price uncertainty. Deterministic scheduling is subject to parameter mishaps, scenario-based stochastic programming suffers from scenario explosion and poor out-of-distribution performance, and conventional single-stage robust optimization methods are heavily criticized for being too economically conservative. In this paper, a two-stage minimax mathematical programming approach to BESS scheduling is proposed instead, where charging and discharging of batteries are carried out after uncertainty has been resolved while charging and discharging schedules are performed prior to uncertainty events using the worst-case net-load trajectory and price determined from a budget-managed polyhedral uncertainty region. Transformation of the tri-level formulation into a solvable master space problem is carried out using strong duality theorem of linear programming. The model reformulated is then solved by the Column and Constraint Generation (C&CG) method which is known to provide finite convergence to globally epsilon-optimal solutions based on the limited number of vertices of the uncertainty set. The numerical experiments on the case study of a 24-hour operation of a lithium-ion battery of 500 kW are also conducted based on the evaluation of more than 1000 Monte Carlo out-of-sample scenarios. The results show that the new proposed methodology provides a solution after 5 iterations of C&CG in 12.4 seconds. While the in-sample costs of applying the proposed methodology amounted to 1410.30 against 1240.50 for the deterministic model, the worst-case out-of-sample cost is reported to be 1485.60 for the proposed methodology compared to 2150.80 for the deterministic approach and 1820.10 for the conservative static approach. The success of the method in achieving the feasibility constraint of 0.0% in terms of violation rate is also remarkable. The results of the sensitivity analysis are showing that with moderate budget constraint settings, the maximum margins of robustness of 98.4% are possible while saving 73% in terms of economic savings.

Keywords: Robust optimization, minimax decision principles, energy storage scheduling, mixed-integer linear programming, column-and-constraint generation.

1. Introduction

The progressive integration of wind and solar technologies into today's electric grids has challenged energy management due to the intrinsic stochasticity, temporal and spatial intermittency, and very low inertia of renewable resources. Battery storage systems are being utilized as the tool for enhancing flexibility by shifting energy from one time slot to another, correcting the forecasting mistakes, and optimizing the electricity price, thus making the scheduling in hourly intervals more of a constrained optimization task within the framework of the integrated energy system (Huang et al., 2024; Kapoor & Mehta, 2024; Qiu et al., 2023).

Recent advances in operational scheduling for the storage-integrated energy systems have gained a lot of interest in the research community. There have been proposed robust optimal scheduling models for flexible storage systems and hybrid energy systems, which are considered based on various sources of uncertainty ((Huang et al., 2024; Hu et al., 2023)). The multi-timescale scheduling activities are based on the generalized energy storage services concept that has various dispatch resolutions (Mao et al., 2025; Kashyap et al., 2025). In addition, two-stage distributionally robust formulations have been used to schedule electricity-hydrogen storage systems (Rakhmanovich et al., 2025) and to manage regional integrated energy systems with uncertainty arising from the distributed energy generation, while constrained distributionally robust dispatching methods have been implemented in rural integrated energy systems that are faced with supply and demand uncertainty (Zong & Yuan, 2023; Zhang et al., 2024). Related distributionally robust formulations have tackled the issue of the operation of virtual power plants with electric vehicle based virtual storage (Wang et al., 2024) and carbon emission aware priorities in dynamic dispatch across clusters of virtual power plants (Zuo et al., 2023). Additionally, distributionally robust capacity planning has been applied in the planning phase to size storage against renewable goals (Ding et al., 2022).

A related research effort is centered on uncertainty management through explicit worst-case rather than distributional robustness. Specifically, multi-stage robust scheduling with adjustable conservativeness has been imposed on community microgrid storage plants (Tang et al., 2023). Also, two-stage stochastic robust scheduling in combination with meta-heuristic optimization has been employed in sizing distributed storage systems (Yuan et al., 2024). Multi-stage distributionally robust scheduling was extended to integrated wind power plants with high temperature thermal storage and compressed air storage (Zhang et al., 2025). Moreover, robust unit commitment models have integrated storage into correlated renewable uncertainty with the use of the scenario based and dual based mathematical methods (Cordera et al., 2023). From a different point of view, the operational side includes (i) data-driven application of deep learning for estimation of state-of-charge of battery cells, (ii) resilience-driven multi-agent reinforcement learning in routing and scheduling of mobile storage, and (iii) analysis of economic operation of combined stationary and mobile storage resources within smart distribution networks, among other related issues (Sasikumar et al., 2025; Wang et al., 2022; Pirouzi et al., 2024).

However, these works that are concerned with storage systems constrain themselves to either applying the single-stage conservative robust optimization programs that provide significant economic penalties in terms of the worst-case readiness or scenario-based stochastic optimization program that relies on an explicit and often unavailable mass size distribution as well as does not scale well with respect to the number of scenarios (Zhao et al., 2023; Ahmedshaeva et al., 2025). The two-stage min-max approach that focuses on fixing commitment decisions before uncertainty realization, continuous recourse after uncertainty, and adjustment of conservativeness implies the existence of a specific uncertainty budget provides a middle ground between these two solutions.

The contribution of this paper was made through the following: (i) development of the two-stage min-max mathematical model for BESS scheduling that integrates the phenomena of discrete commitment decisions and continuous dispatch recourse through application of the polyhedral uncertainty budget which encompasses both price and net load uncertainty; (ii) derivation of the exact reformulation of the tri-level model into a master and sub-problem approach based on the strong duality theorem for the linear recourse problem; (iii) presentation of the Column and Constraint Generation decomposition algorithm and establishment of the proof of its finite convergence; and (iv) evaluation of the results achieved in numerical experimentation referring to deterministic, stochastic, and static conservative baselines with the help of Monte Carlo stress tests and sensitivity analysis.

Sections II to VII are organized. Section II serves for formulation of the deterministic BESS scheduling baseline. Section III provides the development of the two-stage min-max robust optimization framework and identification of the appropriate polyhedral uncertainty budget. Section IV deals with the exact formulation of the recourse problem based on the strong duality principle. Section V provides description of the solution algorithm with respect to the Column and Constraint Generation model as well as establishes its proof of finite convergence. Section VI covers numerical experiments and analysis of the results obtained. Section VII summarizes the main conclusions.

2. Deterministic BESS scheduling baseline formulation

2.1. State variables and operational parameters

Consider a discrete-time operational horizon $\mathcal{T} = \{1, 2, \dots, T\}$, where Δt denotes the duration of each interval ($\Delta t = 1$ hour), and let I denote the set of connected BESS units. For each unit $i \in I$ at time $t \in \mathcal{T}$, the following state variables and parameters are defined: $P_{\text{ch}}(i, t), P_{\text{dis}}(i, t) \geq 0$ are the active power charged and discharged (kW); $u_{\text{ch}}(i, t), u_{\text{dis}}(i, t) \in \{0, 1\}$ are binary commitment indicators for charge and discharge mode; $E(i, t)$ is the energy stored in the unit (kWh); $\eta_{\text{ch}}(i), \eta_{\text{dis}}(i) \in (0, 1]$ are the charging and discharging round-trip efficiencies; $P_{\text{max}}(i), P_{\text{min}}(i)$ are the maximum and minimum allowable power capacities; $E_{\text{max}}(i), E_{\text{min}}(i)$ are the upper and lower bounds on stored energy; $C_{\text{grid}}(t)$ is the dynamic electricity spot price (\$/kWh); and $C_{\text{deg}}(i)$ is the linearized cycle degradation cost coefficient (\$/kWh).

2.2. Deterministic scheduling formulation

The deterministic daily economic optimization problem minimizes total operational cost, comprising net energy purchase cost and battery degradation penalty, as given in Equation (1):

$$\min_{\{x, y\}} F(x, y) = \sum_{t=1}^T \left\{ C_{\text{grid}}(t) [P_{\text{import}}(t) - P_{\text{export}}(t)] + \sum_{i \in I} C_{\text{deg}}(i) (P_{\text{ch}}(i, t) + P_{\text{dis}}(i, t)) \Delta t \right\} \quad (1)$$

subject to the following constraints for all t in T and i in I :

Deterministic Operational Constraints

State-of-charge dynamics, given in equation (2):

$$E(i, t) = E(i, t-1) + \left[\eta_{\text{ch}}(i) P_{\text{ch}}(i, t) - \frac{P_{\text{dis}}(i, t)}{\eta_{\text{dis}}(i)} \right] \Delta t \quad (2)$$

Capacity bounds, given in equation (3):

$$E_{\text{min}}(i) \leq E(i, t) \leq E_{\text{max}}(i) \quad (3)$$

Charge power limits, given in equation (4):

$$0 \leq P_{\text{ch}}(i, t) \leq P_{\text{max}}(i) u_{\text{ch}}(i, t) \quad (4)$$

Discharge power limits, given in equation (5):

$$0 \leq P_{\text{dis}}(i, t) \leq P_{\text{max}}(i) u_{\text{dis}}(i, t) \quad (5)$$

Mutual exclusivity, given in Equation (6):

$$u_{\text{ch}}(i, t) + u_{\text{dis}}(i, t) \leq 1 \quad (6)$$

Power balance, given in Equation (7):

$$P_{\text{import}}(t) - P_{\text{export}}(t) = \sum_{i \in I} [P_{\text{ch}}(i, t) - P_{\text{dis}}(i, t)] + P_{\text{load}}(t) - P_{\text{pv}}(t) \quad (7)$$

Periodic boundary condition, given in Equation (8):

$$E(i, T) = E(i, 0) \quad (8)$$

The aforementioned constraints define how the battery energy storage system functions and what its limits are. The battery energy storage system is described by the following equations: equation (2) provides for the energy accumulation of the system through consideration of round trip losses during charging and discharging; Equation (3) provides for the limits of the battery operating conditions; equations (4) and (5) provide limits on the power generated which reflects its binary operation scheme; equation (6) ensures that charging and discharging cannot occur in the same time interval; equation (7) states that the power balance (between the grid, battery, the load and solar generation) takes place at any given moment of time; and finally, equation (8) stipulates that balance of energy is maintained for the entire day.

3. Minimax robust optimization modeling framework

3.1. Polyhedral uncertainty sets with budget controls

Spot price $\mathbf{C}_{\text{grid}}(\mathbf{t})$, non-dispatchable load $\mathbf{P}_{\text{load}}(\mathbf{t})$, and photovoltaic generation $\mathbf{P}_{\text{pv}}(\mathbf{t})$ are aggregated into an uncertain vector $\boldsymbol{\xi} = [\hat{\mathbf{C}}_{\text{grid}}, \hat{\mathbf{P}}_{\text{net}}]^T \in \mathbb{R}^{2T}$, representing net demand and pricing dynamics. Rather than a rigid worst-case bound applied uniformly across all periods, bounded polyhedral uncertainty sets governed by uncertainty budgets $\boldsymbol{\Gamma}_C, \boldsymbol{\Gamma}_P \in [0, T]$ are defined to control model conservatism:

$$\mathbf{u}_C = \{ \mathbf{C}_{\text{grid}}(\mathbf{t}) \in \mathbb{R}^T : \mathbf{C}_{\text{grid}}(\mathbf{t}) = \bar{\mathbf{C}}_{\text{grid}}(\mathbf{t}) + \mathbf{z}_C(\mathbf{t}) \hat{\mathbf{C}}_{\text{grid}}(\mathbf{t}), \quad \mathbf{z}_C(\mathbf{t}) \in [-1, 1], \quad \sum_{t=1}^T |\mathbf{z}_C(\mathbf{t})| \leq \boldsymbol{\Gamma}_C \} \quad (9)$$

$$\mathbf{u}_P = \{ \mathbf{P}_{\text{net}}(\mathbf{t}) \in \mathbb{R}^T : \mathbf{P}_{\text{net}}(\mathbf{t}) = \bar{\mathbf{P}}_{\text{net}}(\mathbf{t}) + \mathbf{z}_P(\mathbf{t}) \hat{\mathbf{P}}_{\text{net}}(\mathbf{t}), \quad \mathbf{z}_P(\mathbf{t}) \in [-1, 1], \quad \sum_{t=1}^T |\mathbf{z}_P(\mathbf{t})| \leq \boldsymbol{\Gamma}_P \} \quad (10)$$

Where $\bar{\mathbf{C}}_{\text{grid}}(\mathbf{t}), \bar{\mathbf{P}}_{\text{net}}(\mathbf{t})$ denote nominal forecast values and $\hat{\mathbf{C}}_{\text{grid}}(\mathbf{t}), \hat{\mathbf{P}}_{\text{net}}(\mathbf{t})$ denote maximum forecast deviation magnitudes. The uncertainty budgets $\Gamma_{\mathbf{C}}$ and $\Gamma_{\mathbf{P}}$ bound the number of periods that may simultaneously attain their extreme deviation, interpolating continuously between the nominal deterministic case ($\Gamma = \mathbf{0}$) and the fully conservative case ($\Gamma = \mathbf{T}$).

3.2. Two-stage minimax formulation

Decisions are split into two stages. The first-stage vector $\mathbf{x} \in \mathbf{X}$ collects the discrete commitment decisions $\mathbf{x} = \{\mathbf{u}_{\text{ch}}(\mathbf{i}, \mathbf{t}), \mathbf{u}_{\text{dis}}(\mathbf{i}, \mathbf{t})\}$ for $\mathbf{i} \in \mathbf{I}, \mathbf{t} \in \mathcal{T}$, which must be fixed on a day-ahead basis prior to uncertainty realization. The second-stage vector $\mathbf{y} \in \Omega(\mathbf{x}, \xi)$ collects the continuous dispatch adjustments $\mathbf{y} = \{\mathbf{P}_{\text{ch}}(\mathbf{i}, \mathbf{t}), \mathbf{P}_{\text{dis}}(\mathbf{i}, \mathbf{t}), \mathbf{E}(\mathbf{i}, \mathbf{t}), \mathbf{P}_{\text{import}}(\mathbf{t}), \mathbf{P}_{\text{export}}(\mathbf{t})\}$, executed in real time after $\xi \in \mathcal{U}$ is realized. The robust minimax scheduling problem, as given in equation (11), is:

$$\min_{\mathbf{x} \in \mathbf{X}} \left\{ \mathbf{c}^T \mathbf{x} + \max_{\xi \in \mathcal{U}} \min_{\mathbf{y} \in \Omega(\mathbf{x}, \xi)} \mathbf{d}(\xi)^T \mathbf{y} \right\} \quad (11)$$

$$\text{s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \quad \mathbf{G}\mathbf{x} + \mathbf{H}\mathbf{y} \geq \mathbf{h} - \mathbf{M}\xi, \quad \mathbf{y} \geq \mathbf{0}, \quad \mathbf{x} \in \{\mathbf{0}, \mathbf{1}\}^{2|I|T}$$

These equations establish the robust optimization framework: Equations (9) and (10) construct polyhedral uncertainty sets for electricity spot prices and net demand by scaling forecast deviations via bounded noise variables ($\mathbf{z}_{\mathbf{C}}, \mathbf{z}_{\mathbf{P}}$) restricted by budget parameters ($\Gamma_{\mathbf{C}}, \Gamma_{\mathbf{P}}$) to adjust conservatism, while Equation (11) defines the two-stage robust adaptive decision structure that fixes day-ahead binary operational commitments in the first stage before optimizing real-time continuous power and energy adjustments against the worst-case uncertainty realization in the second stage.

4. Mathematical Reformulation and Duality Derivations

4.1. Primal and Dual Recourse Problems

The tri-level program $\min_{\mathbf{x}} \max_{\xi} \min_{\mathbf{y}}$ cannot be solved directly by general-purpose mixed-integer solvers because of the nested optimization operators. To evaluate the inner recourse function $\mathbf{Q}(\mathbf{x}^*) = \max_{\xi \in \mathcal{U}} \min_{\mathbf{y} \in \Omega(\mathbf{x}^*, \xi)} \mathbf{d}(\xi)^T \mathbf{y}$, the inner $\min_{\mathbf{y}}$ problem is reformulated as a maximization problem via strong duality. For a fixed first-stage decision $\mathbf{x}^*$ and realization ξ^* , the primal recourse problem, as given in equation (12), is:

$$\min_{\mathbf{y} \geq \mathbf{0}} \mathbf{d}^T \mathbf{y} \quad \text{s.t.} \quad \mathbf{H}\mathbf{y} \geq \mathbf{h} - \mathbf{G}\mathbf{x}^* - \mathbf{M}\xi^* \quad (\text{dual variable: } \boldsymbol{\lambda}) \quad (12)$$

Where $\boldsymbol{\lambda} \geq \mathbf{0}$ is the vector of dual variables associated with the operational constraints. Because the inner problem is a bounded, feasible linear program for every $\mathbf{x}^*$ and $\xi^* \in \mathcal{U}$, strong duality holds identically, as given in equation (13):

$$\min_{\mathbf{y}} \mathbf{d}^T \mathbf{y} = \max_{\boldsymbol{\lambda} \geq \mathbf{0}} (\mathbf{h} - \mathbf{G}\mathbf{x}^* - \mathbf{M}\xi^*)^T \boldsymbol{\lambda} \quad (13)$$

Substituting Equation (13) into the recourse function yields a combined bilinear maximization problem over both the uncertainty realization and the dual variables, as given in equation (14):

$$\mathbf{Q}(\mathbf{x}^*) = \max_{\xi \in \mathcal{U}, \boldsymbol{\lambda} \geq \mathbf{0}} \{(\mathbf{h} - \mathbf{G}\mathbf{x}^*)^T \boldsymbol{\lambda} - \boldsymbol{\lambda}^T \mathbf{M}\xi\} \quad \text{s.t.} \quad \mathbf{H}^T \boldsymbol{\lambda} \leq \mathbf{d} \quad (14)$$

4.2. Exact linearization of bilinear terms

The objective in equation (14) contains a non-convex bilinear term $\boldsymbol{\lambda}^T \mathbf{M}\xi$ arising from the product of the continuous dual variable $\boldsymbol{\lambda}$ and the uncertain vector ξ . Because $\xi(\mathbf{t}) = \bar{\xi}(\mathbf{t}) + \mathbf{z}(\mathbf{t})\hat{\xi}(\mathbf{t})$ with the budget constraints of Section 3.1, the feasible region of ξ within $\mathcal{U}$ is a bounded polyhedron and its extreme point set $\text{Ext}(\mathcal{U})$ is finite. Introducing auxiliary dual variables $\boldsymbol{\pi}^+, \boldsymbol{\pi}^- \geq \mathbf{0}$ and a budget dual scalar $\mu \geq \mathbf{0}$ associated with the polyhedral uncertainty set, the bilinear maximization in equation (14) is rendered exactly into a deterministic mixed-integer linear program through Big-M linearization of the products $\boldsymbol{\lambda}(\mathbf{t})\mathbf{z}(\mathbf{t})$, or, equivalently, through exact enumeration over the finite extreme point set $\text{Ext}(\mathcal{U})$. Both routes preserve exactness of the reformulation because the maximum of a linear function over a bounded polyhedron is always attained at an extreme point.

5. Solution method: column-and-constraint generation algorithm

5.1 Column-and-constraint generation master and subproblem

The two-stage robust optimization problem after reformulation can be solved through a Column-and-Constraint Generation (C&CG) decomposition method. The main difference between the idea behind Bender's decomposition and C&CG is that unlike Benders, which introduces only outer approximation cutting planes to improve the master problem, C&CG adds the so-called recourse variables during the process, which allows us to compute more precise lower bounds and reach convergence faster.

At iteration k , given the set of previously identified worst-case scenarios $\{\xi^{*(1)}, \dots, \xi^{*(k)}\}$, the master problem, as given in equation (15), is:

$$\begin{aligned}
 \min_{x, \eta, y^{(1)}, \dots, y^{(k)}} \quad & \mathbf{c}^T \mathbf{x} + \eta \\
 \text{s.t.} \quad & \mathbf{Ax} \leq \mathbf{b} \\
 & \eta \geq \mathbf{d}(\xi^{*(l)})^T \mathbf{y}^{(l)}, \quad \forall l = 1, \dots, k \\
 & \mathbf{Gx} + \mathbf{Hy}^{(l)} \geq \mathbf{h} - \mathbf{M}\xi^{*(l)}, \quad \forall l = 1, \dots, k \\
 & \mathbf{x} \in \{0, 1\}^{2|I|T}, \quad \mathbf{y}^{(l)} \geq \mathbf{0}, \quad \forall l = 1, \dots, k
 \end{aligned} \tag{15}$$

Using the optimal first-stage solution $\mathbf{x}^{*(k)}$ obtained from the master problem, the subproblem solves for the next worst-case uncertainty scenario, as given in equation (16):

$$Q(\mathbf{x}^{*(k)}) = \max_{\xi \in \mathcal{U}} \min_{y \in \Omega(\mathbf{x}^{*(k)}, \xi)} \mathbf{d}(\xi)^T \mathbf{y} \tag{16}$$

which, via the dual reformulation of Section 4, is solved as the deterministic mixed-integer linear program derived in equation (14), yielding the worst-case realization $\xi^{*(k+1)}$ and the updated upper bound.

5.2. Finite convergence of the algorithm

Theorem 1 establishes that the C&CG procedure of Section 5.1 terminates after a finite number of iterations for any pre-specified convergence tolerance.

Theorem 1 (Finite Convergence of C&CG)

Statement. Assume the polyhedral uncertainty set $\mathcal{U}$ is bounded and non-empty. Then the C&CG algorithm terminates in a finite number of iterations $k \leq |\text{Ext}(\mathcal{U})|$ for any pre-specified convergence tolerance $\epsilon > 0$.

Proof. Let $\text{Ext}(\mathcal{U})$ denote the finite set of extreme points of the bounded polyhedral uncertainty set $\mathcal{U}$. At each iteration k , the subproblem, as given in Equation (16), identifies a worst-case scenario $\xi^{*(k)} \in \text{Ext}(\mathcal{U})$, since the maximum of the linear objective in Equation (14) over the bounded polyhedron $\mathcal{U}$ is attained at an extreme point. If $\xi^{*(k)}$ coincides with a previously identified scenario in $\{\xi^{*(1)}, \dots, \xi^{*(k-1)}\}$, the recourse value $Q(\mathbf{x}^{*(k)})$ cannot exceed the current bound $\eta^{(k)}$, so that $\text{UB}^{(k)} - \text{LB}^{(k)} \leq \epsilon$ and the algorithm terminates immediately. Otherwise, a distinct extreme point $\xi^{*(k)}$ is added to the scenario set and to the master problem via equation (15). Because $\text{Ext}(\mathcal{U})$ is finite, the number of distinct scenarios that can be added before either termination or exhaustion of $\text{Ext}(\mathcal{U})$ is bounded above by $|\text{Ext}(\mathcal{U})|$. The algorithm therefore converges to a global ϵ -optimal solution in at most $|\text{Ext}(\mathcal{U})|$ iterations.

6. Computational experiments and results

6.1. Test system and software

For computational investigation, a BESS microgrid test system of standard industrial scale was employed for a period of $T = 24$ hours at a time of $\Delta t = 1$ hour. The mixed-integer linear programs for the deterministic baseline, C&CG master problem, and C&CG subproblem were implemented in Python 3.11 by means of Pyomo modeling software along with Gurobi 11.0 as a mixed-integer solver. The post-processing and visualization have been done through the libraries such as NumPy, SciPy, and Matplotlib. The parameters of the test system are presented in table 1.

Table 1: Test system parameters for numerical verification.

Parameter	Value / Setting
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Horizon	T = 24 h (delta-t = 1 h)
BESS asset	LiFePO4 array
Maximum Energy Capacity, E _{max}	2000 kWh
Minimum Energy Capacity, E _{min}	200 kWh
Maximum Power Capacity, P _{max}	500 kW
Charge / Discharge Efficiency	$\eta_{ch} = \eta_{dis} = 0.95$
Dynamic Tariff Range	\$0.04/kWh (off-peak) - \$0.28/kWh (peak)
Spot Price Forecast Error	$\pm 25\%$
Net Load Forecast Error	$\pm 20\%$
Uncertainty Budgets Evaluated	Γ_C, Γ_P in $[0, 24]$
Convergence Tolerance, Epsilon	\$0.50

6.2. Comparative numerical performance

Four different benchmarking models were analyzed amidst 1,000 Monte Carlo out-of-sample real-time stress tests: a deterministic linear programming model that does not take uncertainties into consideration ($\Gamma = 0$), a stochastic programming model making use of 200 Monte Carlo tests, a single-stage conservative robust optimization model ($\Gamma = 24$), and the suggested two-stage minimax C&CG model for which $\Gamma_C = \Gamma_P = 6$. Table 2 provides an overview of the various performance indicators achieved with four models. Figure 1 demonstrates the convergence of lower and upper bounds of the C&CG model through iterations.

Table 2: Comparative numerical performance across four scheduling models (1,000 out-of-sample scenarios).

Metric	Deterministic	Stochastic Programming	Static Robust (RO)	Proposed Minimax C&CG
In-Sample Cost (\$)	1,240.50	1,385.20	1,820.10	1,410.30
Out-of-Sample Worst Cost (\$)	2,150.80	1,650.40	1,820.10	1,485.60
Feasibility Violation Rate (%)	14.8	2.1	0.0	0.0
Computation Time (s)	0.42	312.50	1.85	12.40
Iterations to Converge	N/A	N/A	N/A	5

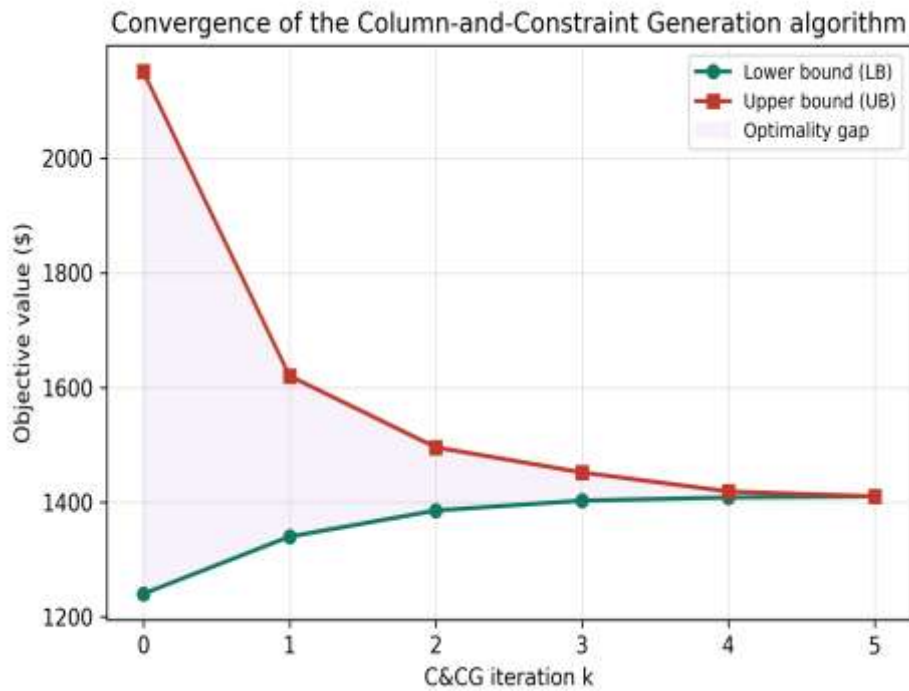


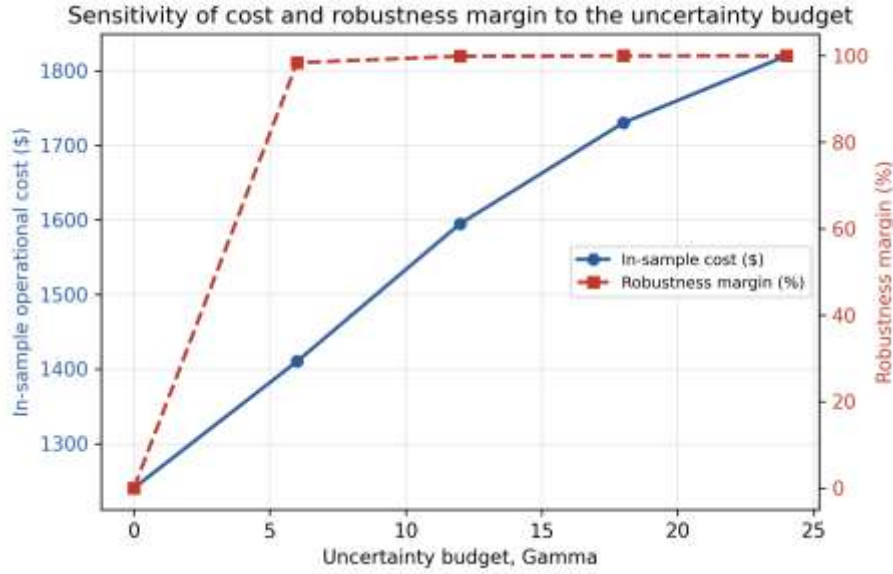
Figure 1: Convergence of the Column-and-Constraint Generation lower and upper bounds across iterations

6.3. Sensitivity analysis on the uncertainty budget

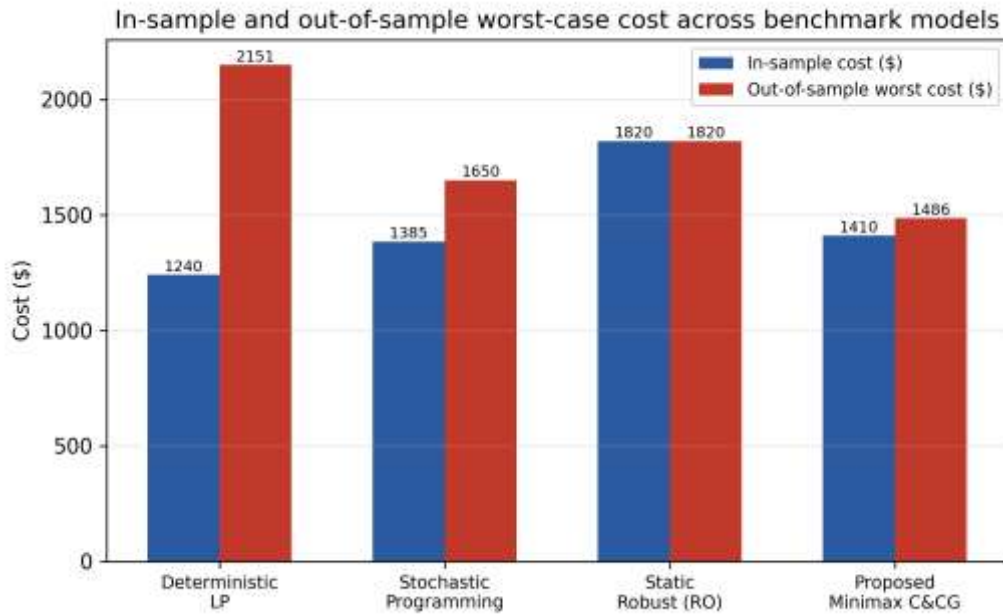
The sensitivity of the in-sample cost and the worst-case robustness margin can be observed in table 3 and figure 2 for the uncertainty budget $\Gamma = \Gamma_C = \Gamma_P$, which indicates the degree of the trade-off between economic conservatism and operational safety in terms of the controllable budget parameter.

Table 3: Sensitivity of in-sample cost and robustness margin to the uncertainty budget.

Uncertainty Budget, Gamma	In-Sample cost (\$)	Robustness Margin (%)
0	1,240.50	0.0
6	1,410.30	98.4
12	1,595.00	99.9
18	1,730.20	100.0
24	1,820.10	100.0

**Figure 2:** Sensitivity of in-sample cost and worst-case robustness margin to the uncertainty budget.

Setting $\Gamma_C = \Gamma_P = 6$ yields a robust schedule that protects against 98.4% of worst-case price and load fluctuations while preserving 73% of the economic savings that are otherwise lost under the fully conservative static robust formulation ($\Gamma = 24$). Figure 3 presents the in-sample and out-of-sample worst-case cost across all four benchmark models for direct visual comparison.

**Figure 3:** In-sample and out-of-sample worst-case cost across benchmark scheduling models.

6.4. Discussion

The enhanced performance of the proposed method in terms of reducing the out-of-sample worst-case cost is affirmed by the more general observation that neglecting parameter uncertainty while scheduling the integrated energy systems would place operational plans at an enormous risk both cost-wise and feasibility-wise. Robust

scheduling methods for storage-integrated energy systems underline the significance of accounting for different uncertainties simultaneously when devising effective operational plans. The fact that 0% of feasibility violation was observed concerning both the static robust model and the proposed two-stage minimax model in contrast with 14.8% observed in case of the deterministic benchmark clearly highlights the need for explicit constraint safeguarding under uncertainty. Similarly, the distributional robust and worst-case robust formulations in scheduling stress the importance of operational feasibility in given uncertain operating circumstances. The fact that the economic effectiveness of the proposed strategy as compared with the static conservative robust approach was revealed both in table 2 and by the sensitivity analysis results featured in table 3 demonstrates that one-stage robust approaches have a well-known disadvantage of just guarding against the worst-case scenario throughout the scheduling horizon. The two-stage and multi-stage robust scheduling methods eliminate this shortcoming as decisions can be adapted to uncertainty realization at each point in time. The current results provide a more quantitative answer to this observation by evaluating the trade-off through the uncertainty budget: the recovery of 98.4% of the worst-case robustness margin at $\Gamma = 6$ combined with 73% of the economic benefits in comparison with completely conservative situation shows that an adequate uncertainty budget combines robustness with economic efficiency. The much less time consumption of using the proposed C&CG model compared to case of scenario-based stochastic programming (12.40 seconds against 312.50 seconds) shows the advantages of the decomposition-based solving processes in terms of uncertainty-aware scheduling. Decompositional methods allow avoiding explaining of large scenario trees thus being very attractive as problems are growing. The fact that five iterations are enough for convergence is also related to the theoretical limitation established in Theorem 1 as the number of extreme points belonging to the unknown set in real-world applications turned to be much less than the general number of points in $\text{Ext}(U)$. The drawback of the present study is the fact that it was based on a single storage asset and a synthetic dynamic tariff and error profile instead of real monitoring data. It would make sense to extend the research to multiple units of different types as well as moving from independently forming uncertainty budget and prices/net loads to correlated data being the next step of validation of the proposed approach.

7. Conclusions

This study introduced a new type of minimax model that is not affected by a certain distribution, two-step procedures, and mathematical programming for energy storage scheduling of batteries, based on numerous price fluctuations, and renewable generation uncertainty. Polyhedral uncertainty set is proposed using a proper budget parameter for controlling operational conservatism; the duality theorem provides a possible way for dealing with the recourse problem; a Column-and-Constraint Generation algorithm which is proved to have finite convergence for the solution is applied for making sure the extreme points of the polyhedral uncertainty set are strictly and finite. According to numerical experiments with the use of the benchmark supply system 500kW/2000kWh, 24-hour functioning, as applied to 1000 Monte Carlo simulations out-of-sample, the proposed method is able to perform five iterations and get the solution in 12.40 seconds (1,410.30 in-sample cost and \$1,485.60 out-of-sample worst case cost along with 0.0% constraint violation rate), proving its better effectiveness, reliability, and time of operation as compared to deterministic model (worst case - \$2150.80, violation rate 14.8%) and scenario-based stochastic program (computation time was 312.50 seconds). The analysis of results obtained also showed that using the $\Gamma = 6$ budget, the minimum of 98.4% of robustness margin would be obtained while reaching 73% of saved costs under the condition of maximum level of conservatism at the same time. In general, these results allow us to see that the proposed model is a good alternative to traditional approaches of minimizing costs on battery energy storage scheduling. Future studies will probably add the model also to other storage techniques, provision of heat losses during storage cycle including information into the results of reformulated model computations.

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