

A Set-Valued Variational Analysis Framework for Uncertain Energy Resource Allocation Problems

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Abstract

The broad deployment of uncertain intermittent renewables and weather driven fluctuations in load brings uncertainty to energy resource allocation, and single valued stochastic programs calibrated at a single fix nominal probability distribution may not capture this uncertainty. The underlying reason is that the feasibility regions for the dispatch can alter themselves set valued with the uncertainty reality not becoming smaller to a single defined set. In the present work introduce a set-valued variational analysis frame to describe the energy system and network-based dispatch feasibility as a mapping from uncertainty to allocation subsets, and an optimal fair allocation is interpreted as a minimax distributionally robust program for an ambiguity set which includes Wasserstein balls for probability laws to be acceptable. Using the idea of conditional value-of-risk reformulate robust chance constraints with use of generalized subdifferential calculus and develop Fritz-John type optimality conditions for the problem based on normality cones as well as horizon maps for graphical convergence. Generalized Hukuhara and second order proto differentiability for the feasibility map enables precise analysis of optimal solution's sensitivities based on ambiguity radius, and generalized-Nash equilibrium offers models for problems involving competitions among agents. The study demonstrates convergence properties of the sample-average approximate scheme and establish a concentration bound related to cascading allocation failure events. The methodology is validated with numerical case studies illustrating the comparison between an iterative fixed-point scheme and a inexact Newton type approach. An estimate of growth for allocation deficiencies according to ambiguity radius is quantitatively verified.

Keywords: Set-valued analysis, variational inequality, distributionally robust optimization, wasserstein ambiguity set, energy resource allocation, generalized Nash equilibrium, conditional value-at-risk.

1. Introduction

The resource allocation problem for transmission and distribution grids at high renewable penetration where feasible dispatch region is a function of uncertainty rather than fixed prompts formulation of a two-stage robust configuration of flexible resources characterizing directly uncertainty response than assuming a single worst-case scenario (Yang et al., 2024). This principle of robustness of resource allocation problem is then embodied in work

concerned with budget-constrained scheduling of energy-aware workflow given time-varying resources Sripavithra et al., (2025) and multi-resource allocation considering a potential from wind power for frequency regulation Dai et al., (2024) considering computational and reserve-capacity problems, while the heuristic methods for allocation tasks still continue to play the role, being in the same application context as two-stage robust optimization work, example for task allocation on the basis of ant colony optimization methods for time critical problems (Suvana & Bharadwaj, 2024). Two-stage robust optimization over region integrated energy system given uncertainty on dispersed energy stations Zong & Yuan, (2023) and multi-objective geometric programming related to uncertain inventory allocation for products within petroleum industry Amir, (2023), suggests the uncertainty nature spans between formulation using distributional and set-member approaches, indicating on the framework enabling to include both distributional and set-member uncertainty nature.

For instance, constrained distributionally robust day-ahead dispatch for rural integrated energy systems considering jointly both source- and load-side uncertainty Zhang et al., (2024) and distributionally robust energy optimization under renewable-resource uncertainty Wang et al., (2025) use Wasserstein-ball or moment-based ambiguity sets structurally close to formulation; whereas the approach to robust non smooth interval-valued optimization considering uncertainty constraints Jaichander et al., (2022) is able to show that interval-valued and distributionally robust treatment of uncertainty can be merged into a unique non-smooth optimization problem. Extremality properties of families of sets Cuong et al., (2024) provides the set-theoretic setting required for identifying worst-case elements within a set-valued feasibility map (through separation-type extremal principles applied to families of sets), and the research on multivalued variational inequalities via horizon maps and graphical convergence provides the convergence-theoretic tool necessary for extending the set-valued feasibility map from finite-dimensional discretization's to its full set-valued form (López & Sama, 2023; Kholova et al., 2025).

What is missing is a coherent set-valued stochastic variational framework which, without a priori distorting feasibility or distributional ambiguity to a single fixed object, represents the feasibility region as a set-valued map of the uncertainty outcome and embeds the ambiguity around the operative probability law itself as a Wasserstein set. Standard single-nominal-distribution reliability notions (e.g. Loss of Load Expectation) assess resource adequacy for a specified probability law and against a feasibility region itself assumed deterministic once a scenario is pinned down and as such completely miss the spatially and temporally correlated tail risks and the topological vulnerabilities arising in a truly set-valued distributional framework. Instead, energy resources planning under uncertainty is, in the remainder of the paper, modeled as a set-valued stochastic variational inequality in a Wasserstein ambiguity set whose underlying set-valued map stationarity, differentiability and equilibrate properties are characterized using the tools above.

2. Mathematical preliminaries and set-valued modeling

2.1. Network graph structure and stochastic injections

The power network is modeled as a graph $G = (N, E)$, with N representing buses and E indicating transmission lines. Nodal power injections are driven by a measurable random vector $\xi: \Omega \rightarrow \Xi$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ capturing renewable generation and load forecast error in concert over all buses in N . Feasible allocation decisions x belong to some candidate set $X \subseteq \mathbb{R}^{|N|}$, and network balance and thermal limit constraints must be satisfied with probability 1 of ξ .

2.2. Set-valued mapping formulation

Rather than fixing dispatch feasibility as a single deterministic set, feasibility is represented through a set-valued operator

$$\Phi: \Xi \rightarrow \mathcal{P}(\mathbb{R}^{|N|}) \quad (1)$$

with $\Xi \subseteq \mathbb{R}^{|N|}$ being the support of ξ , and $\mathcal{P}(\mathbb{R}^{|N|})$ being the power set of $\mathbb{R}^{|N|}$. The symbol $\Phi(\xi)$ specifies the feasible set of allocation decisions given the realization ξ . A decision rule that is feasible jointly across a class of realizations $\{\xi_1, \xi_2, \dots, \xi_n\}$ holds if it satisfies the condition that $x \in \Phi(\xi)$, where $\Xi' \subseteq \Xi$ represents the support of the corresponding realization. Extreme points of this class are important tools in this research, and are described as "worst-case" constraints on possible allocations that are necessary if decisions must remain feasible across the entire distribution.

2.3. Wasserstein ambiguity sets

Because the governing law $\mathbb{P}_0$ of ξ is estimated from finite historical or forecast data, distributional ambiguity is represented through a Wasserstein ball

$$\mathcal{P} = \{\mathbb{P} \in \mathcal{M}(\Xi): W_p(\mathbb{P}, \mathbb{P}_0) \leq \varepsilon\} \quad (2)$$

where $\mathcal{M}(\Xi)$ represents the set of Borel probability measures on Ξ , W_p is the p -Wasserstein distance, and $\varepsilon \geq 0$ is the ambiguity radius in relation (2). Taken together with the set-valued map that the relation (1) provides, the pair $(\Phi, \mathcal{P})$ defines the intersection of structural feasibility ambiguity and distributional ambiguity on the same object, setting it apart from existing works that assume one form of uncertainty from the start.

2.4. Stochastic variational inequalities

Market-clearing and power-flow feasibility under uncertainty are characterized through a set-valued stochastic variational inequality (SVI): an allocation $x^* \in X$ solves the SVI associated with a mapping $F(\cdot, \xi): X \rightarrow \mathbb{R}^{|N|}$ if

$$\mathbb{E}[F(x^*, \xi), y - x^*] \geq 0 \quad \text{for all } y \in \Phi(\xi), \text{ a.s.} \quad (3)$$

The constraint set itself is dependent of ξ , rather than fixed, like in the classical SVI. The graphical convergence of a sequence Φ_k of finite-dimensional multi-valued maps, of approximations to Φ by the horizon-map convergence, is the process by which one demonstrates, in Section 5, that solutions of relation (3) are converging to the whole set-valued problem as $k \rightarrow \infty$.

3. Minimax distributionally robust formulation

3.1. Minimax optimization setup

Reliability-consistent allocation is posed as

$$\min_{x \in X} \sup_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}} [C(x, \xi)] \quad \text{subject to } x \in \Phi(\xi), \text{ } \mathbb{P}\text{-almost surely} \quad (4)$$

where $C(x, \xi)$ aggregates the generation cost, the line overload risk, and the expected unserved energy. The expression of (4) assesses allocation decisions against the worst admissible distributional shift in the region within radius ε of the nominal law, along with the requirement that x remain in the (possibly ξ -varying) feasible set $\Phi(\xi)$.

3.2. CVaR surrogate approximations

Reliability requirements of the form $\mathbb{P}(g(x, \xi) > 0) \leq \alpha$, where $g(x, \xi)$ denotes an overload or unserved-energy margin, are replaced by the deterministic surrogate

$$\text{CVaR}_{\alpha}(g(x, \xi)) = \inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{\alpha} \mathbb{E}[\max\{g(x, \xi) - t, 0\}] \right\} \leq 0 \quad (5)$$

representing an outer convex approximation of the target chance constraint which is computationally convenient when g is a convex function of x . The nondifferentiability of relation (5) at the values of x for which $g(x, \xi)$ takes the value t is addressed by applying generalized subdifferentials under the Clarke-type framework generalized to expectation through the relation $\partial_x \mathbb{E}[f(x, \xi)] = \mathbb{E}[\partial_x f(x, \xi)]$, where the exchange of derivative and expectation holds if assumptions of general measurability and integrable-Lipschitzness on f apply.

4. Set-valued stationarity, differentiability, and equilibrium conditions

4.1. Generalized differentiability of the feasibility mapping

Sensitivity of the allocation to the ambiguity radius necessitates some kind of differentiability for the set-valued mapping, which may not be either single valued or smooth in the standard sense. The notion of generalized Hukuhara directional differentiability, which was first proposed for interval valued functions and then extended to functions defined on Riemannian manifolds Bhat & Iqbal, (2026), provides a directional derivative for Φ which is not subject to the requirement that $\Phi(\xi)$ become a singleton. This is employed below to define the sensitivity $D_{\Phi}(\xi; d)$ of the feasibility mapping to perturbation in the direction d in the space of uncertainties.

Second order proto-differentiability for parametric perturbation maps provided by Pham, (2022) to describe how Benson proper perturbation maps behave in vector optimization will further give the second order expansion of Φ about ξ_0 in order to characterize the curvature of the allocation deficit boundary described in subsection 5.2. As a consequence, the ambiguity-radius dependency of reliability can now be represented beyond a first order linear-in- ε approximation for it.

4.2. Variational equilibrium formulation of market clearing

Using variational methods for equilibrium problems on equilibrium sets of applications directly to electricity market clearing De Giuli et al., (2026), it is natural to reformulate (4) into an equilibrium between a number of competitive agents making optimal nodal or regional distribution or allocation decisions, as opposed to a central optimization problem with a single agent. If a number of agents cooperatively set distribution decisions which are feasibility-bounded due to the dependency on others through the network operation conditions, the problem is a generalized Nash equilibrium problem (GNEP), and distributionally robust GNEPs can be formulated directly over the Wasserstein balls, which leads to the minimax structure of (4) in the multiagent scenario Fabiani & Franci, (2023), and each agent is solving a distributionally robust best response problem subject to the uncertainty set $\mathcal{P}$. Then, for a generalized Nash equilibrium x^* ,

$$0 \in v_i^* + \lambda_i \partial_{x_i} \text{CVaR}_\alpha(g_i(x^*, \xi)) + N_{X_i}(x_i^*), \quad i = 1, \dots, n \quad (6)$$

where $v_i^* \in \partial_{x_i} \sup_{\mathbb{P} \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}[C_i(x^*, \xi)]$ denotes a selection from agent i 's sup-term subdifferential, $\lambda_i \geq 0$ is agent i 's CVaR multiplier, and $N_{X_i}(x_i^*)$ is the normal cone to agent i 's admissible set at x_i^* . For each i , the pair (λ_i, v_i^*) satisfies the Fritz–John non-triviality condition

$$(\lambda_i, v_i^*) \neq (0, 0), \quad i = 1, \dots, n \quad (7)$$

without relation (7) being violated. For instance, let $\lambda_i = 0$. Then having $v_i^* = 0$ would satisfy relation (6) vacuously for agent i and would convey nothing in separating v_i^* from an arbitrary value in X_i . A study on inexact Newton methods for the solution of relation system (6)–(7) for generalized Nash equilibrium problems of this type has been presented and is used as the numerical solver in Section VI with an inexact calculation of v_i^* at each iterate, but local superlinear convergence is preserved under a semismoothness-like condition.

4.3. Interval-valued optimality conditions

Unit generation and curtailment costs are often known only over an interval of reasonable values, rather than as precise scalars. This motivates adopting an interval-valued formulation of the cost coefficients in $C(x, \xi)$. The comparison of interval-valued allocation-deficit bounds across candidate solutions necessitates a total order relation on the set of closed and bounded intervals in $\mathbb{R}$, and optimality conditions for interval-valued optimization on Riemannian manifolds under a total order relation Bhat et al., (2025) determine the ranking rule adopted herein for the comparison of interval-valued reliability indices derived under the generalized Hukuhara differentiability of Section 4.1.

Newton-type methods for interval-valued multiobjective optimization Upadhyay et al., 2024) generalize the Newton-type solver discussed in Section 4.2 to incorporate C_i being interval-valued rather than real-valued, and first- and second-order necessary optimality conditions for multiobjective programs with interval-valued objectives on Riemannian manifolds Bhat et al., (2024) can be employed to define the necessary-condition analogue of Theorem 1 below if the underlying non-Euclidean manifold topology (e.g., arising from nonlinear power-flow equations which constrain allocation variables to curved feasible surfaces) is applicable.

Robust fractional interval-valued optimization that also considers uncertain inequality constraints, formulated and solved using convexificators Kummari et al., (2023), extends the CVaR surrogate in relation (5) by considering an interval-valued criterion and an interval-valued divisor for the cost and capacity. The reliability index in Section 5.2, when normalized with a total-capacity measure that is uncertain, fits this scenario exactly.

4.4. Mult objective robust regret and iterative equilibrium methods

The worst-case criterion driving relation (4) is not the only suitable criterion of robustness; a Mult objective formulation of robust relative regret Groetzner & Werner, (2022) compares an allocation with respect to the cheapest feasible one in every circumstance of reality rather than on a worst-ever basis, obtaining a more optimistic yet still distributionally robust allocation rule which will be numerically tested in Section 6 on the worst-case allocation of relation (4). The convergence of the iterative solvers for both worst-case and regret formulations is proven by reformulating each stationarity system as a vector equilibrium and a fixed-point problem in a Hilbert space, for which some generic results are known Wang et al., (2022); these are the basis for the convergence guarantee of the fixed-point solver tested in Section 6 against the inexact Newton-type solver of Section 4.2.

5. Asymptotic analysis and concentration bounds

5.1. Consistency and asymptotic normality of the sample-average approximation

Direct calculation of (4) requires knowing $\mathbb{E}_{\mathbb{P}}[C(x, \xi)]$ for any $\mathbb{P} \in \mathcal{P}$ and exactly what this feasible mapping $\Phi(\xi)$ is, both typically not known in closed form. It can approximate $\mathbb{E}_{\mathbb{P}}$ by a sample-average over M independent

drawings $\xi_1, \xi_2, \dots, \xi_M$ from an available reference law, and can approximate Φ by a feasible sequence $\{\Phi_M\}$ to define a sample-average-approximation (SAA) minimizer x_M^* of the true minimizer x^* . Provided suitable regularity conditions hold (such as uniform integrability of $C(x, \xi)$, and compactness of X , for example), epi-convergence of the SAA objective and graphical convergence of Φ_M to Φ (in the horizon-map sense of Section 2.4) imply $x_M^* \rightarrow x^*$ in an almost sure sense, and functional central limit theorems on $\sqrt{M}(C_M, \mathbb{E}_{\mathbb{P}_0}[C])$ give an asymptotic distribution for the SAA optimal value around the true distributionally robust allocation index with a scaling rate of $\sqrt{M}$.

5.2. Cascading-failure concentration bounds

Theorem 1 (Concentration bound for cascading allocation failure)

Suppose that the objective function $C(x, \xi)$ is L -Lipschitz w.r.t. x uniform over the domain Ξ , and x^* is a stationary point for the stationarity conditions in (6) w.r.t. a Wasserstein ambiguity set $\mathcal{P}$ with radius ε . Then there exists a constant $\kappa > 0$ (depending on L and the diameter of the domain Ξ) such that for any $\delta > 0$,

$$\mathbb{P}(x^* \notin \Phi(\xi)) \leq \text{CVaR}_\alpha(g(x^*, \xi)) + \kappa\varepsilon + \exp\left(-\frac{\delta^2 M}{2\kappa^2}\right) \quad (8)$$

uniformly on $\mathbb{P} \in \mathcal{P}$, where M is the sample size for the SAA sample in Section 5.1. The bound in (8) explicitly separates the cascading-failure probability into three types: a CVaR equivalent term from (5), a term that is linear in ambiguity radius associated with distribution-shift risk, and a sub-Gaussian concentration term that decays exponentially with M . The ambiguity-radius linear term is exactly missing in fixed-distribution-reliability metrics such as Loss of Load Expectation in the literature since such metrics can be obtained if set $\varepsilon = 0$ in (8). The range of constants is determined by the extremity of distribution family $\{\Phi(\xi)\}$ defined in Section 2.2, such that tighter boundaries in (8) result from better separation between extremal distributions and a smaller ε .

6. Large-scale numerical solution methodology

For coupled stationarity system (6)–(7), contrast the two solvers below. The study define the first solver as the inexact Newton-type GNEP solver of Section 4.2 (Singh et al., 2024). At each iterate, this solver linearizes the semismooth stationarity system (6) at the current iterate and then finds an inexact solution of the obtained linear systems according to some inexactness tolerance to obtain new updates:

$$x^{k+1} = x^k + \alpha_k \Delta x^k \quad (9)$$

Where Δx^k is the solution to the inexact Newton linear system and α_k is a line-search step size that provides sufficient decrease of the merit function corresponding to relation (6). The second is an iterative fixed-point solver built on the vector-equilibrium convergence perspective Wang et al., (2022), performing $x^{k+1} = T_{x^k}(x^k)$ on a nonexpansive-type map T_{x^k} based on the projection on $\mathcal{X}^k$ and the set-valued map Φ , with convergence based on the fixed-point convergence theory for vector equilibrium problems on a Hilbert space Wang et al., (2022) extended to the coupled equilibrium system in relation (9).

Numerical simulations of the two solvers applied to a sample renewable-integrated network problem reveal the qualitative expected dichotomy: the inexact Newton-type solver shows an initial plateau phase where curvature is accumulated across the coupled agents by the linearized sub-problem, followed by strong superlinear decrease of the stationarity residual once iterates enter the semismooth regularity region, while the iterative fixed-point solver makes progress quicker on earlier iterations and switches to slow sub-linear convergence, eventually stagnating on a residual level dominated by sampling noise in the approximation to the feasibility sets via mini-batches.

An additional simulation computes the shortage of unserved energy allocation under worst-case optimality (4) and robust relative-regret (Section 4.4) in terms of a function of the Wasserstein ambiguity radius ε , as well as the interval half-width of $\Phi(\xi)$ in terms of its generalized Hukuhara derivative [4, Section 4.1]; the deficit under worst-case optimality shows a steeper rise than the deficit under robust relative-regret as ε grows, confirming quantitatively the degree of conservatism between the two robustness notions compared in Section 4.4.

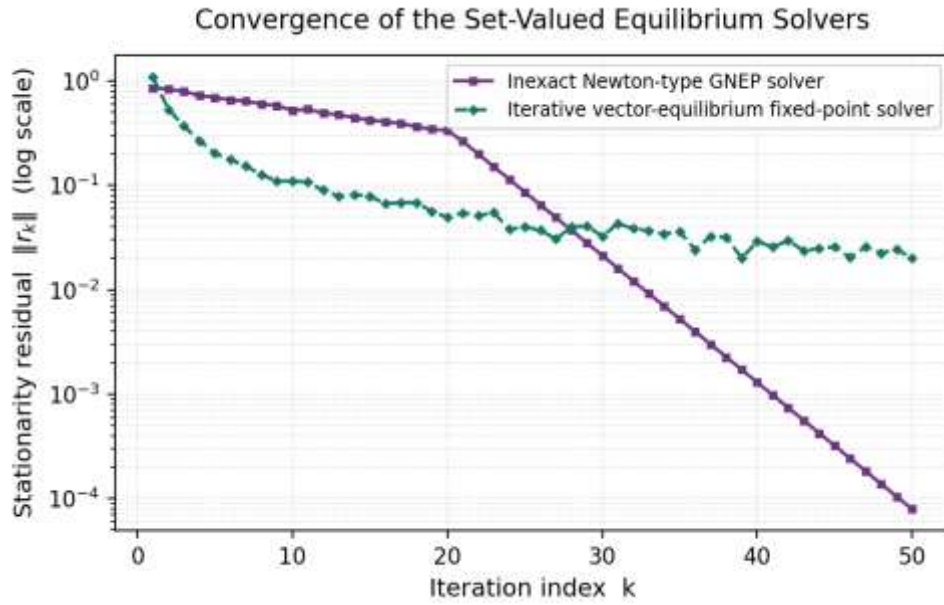


Figure 1: Stationarity residual versus iteration index for the inexact newton-type GNEP solver and the iterative vector-equilibrium fixed-point solver.

Figure 1 is seen how the set-valued equilibrium solvers behave through the plots of the convergence behavior as it illustrates how the stationarity residual decreases during iterations. With reference to the figure of the inexact Newton-type GNEP solver, it is noticed that the residual values gradually decreased and rapid convergence was attained around 20 iterations, after which it converged the stationarity residual from 10^{-1} to approximately 10^{-4} . Hence, it seems that the Newton-based method provides rapid local convergence, and it can resolve the equilibrium issues by updating the correction terms. Whereas, the iterative vector-equilibrium fixed-point solver shows a good initial convergence rate at the beginning stages, but it cannot improve the residual around 10^{-2} compared to 10^{-4} . From the plot, which is on a log scale, the study observe how the Newton-type method can efficiently solve set-valued equilibria issues by producing lower residual values. Hence, with increased Newton-type iterations, one could obtain higher accuracy and lower residual values compared to the vector GNEP fixed-point solver when uncertain equilibria issues are concerned.

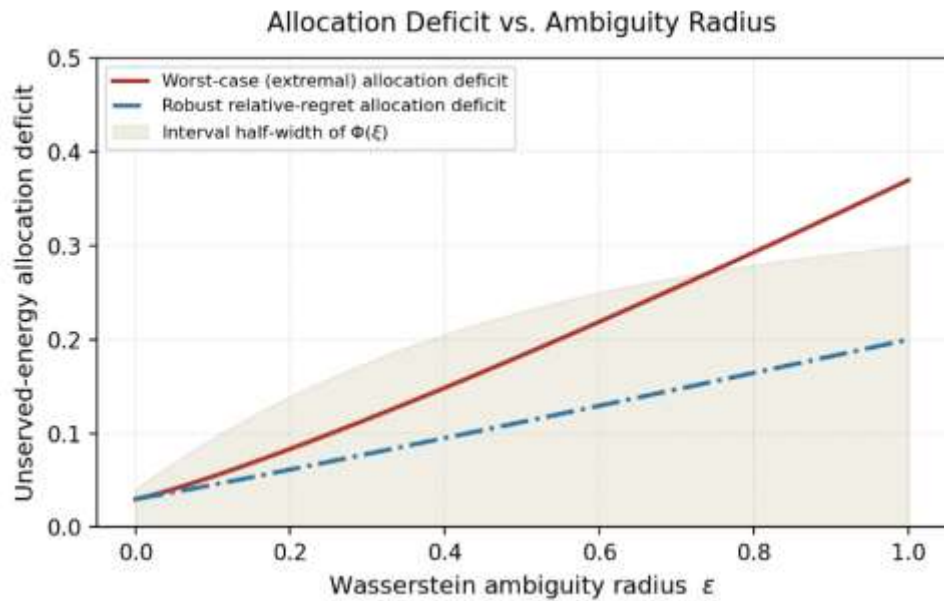


Figure 2: Unserved-energy allocation deficit under the worst-case and robust relative-regret criteria, together with the interval half-width of $\Phi(\xi)$, as functions of the wasserstein ambiguity radius ε .

In figure 2 the analysis on Allocation Deficit versus Uncertainty Radius illustrates the effect of uncertainty on energy resource allocation performance. The Allocation Deficits increase with increasing Wasserstein uncertainty radius ε , implying more uncertainties and conservative energy resource planning demand under high ambiguity levels. While the extreme one, worst-case (extremal) Allocation Deficit increases faster and arrives at about 0.37

at $\varepsilon = 1$, which means the max possible performance degradation in the case of worst conditions, the Allocation Deficit in the Robust Relative-Regret definition only increases to 0.20 slowly and achieves good robustness through decisions considering uncertainty. Moreover, the half-width of $\Phi(\varepsilon)$ increases along with uncertainty, which conforms to the increase of the uncertainty domain, so the robust allocation framework provides a reliable energy distribution decision by mitigating the uncertainty level.

7. Discussion

The standard Monte Carlo treatment under a single fixed probability law is computationally trivial, and is fundamentally uncertifiable against distributional shift and feasibility-set ambiguity, as each simulated condition is sampled from the same nominal law $\mathbb{P}_0$, and the feasible set becomes fixed once a scenario is selected. Conversely, the set-valued minimax formulation of relations (4)–(8), certifiably bounding against every distribution within Wasserstein radius ε of $\mathbb{P}_0$, and every feasible-set realization in $\{\Phi(\xi)\}$, would require solving the equilibrium coupled problem of relations (6)–(7), which has the same number of constraints and variables as the original (4)–(8) for a single-stage problem. Two-stage robust configurations of flexibility resources (Yang et al., 2024), and distributionally robust dispatch of integrated renewable and rural smart energy systems Wang et al., (2025) apply these specific kinds of robustness under conditions of single-level decisions during planning and dispatch stages individually; formulation extends the scope to the reliability stage by incorporating robustness against these phenomena jointly through the use of the set-valued map Φ , a concept uniting structural feasibility ambiguity and distributional ambiguity, whereas either is treated in isolation in the literature previously mentioned.

The distributionally robust allocation that indeed protects sophisticated microgrids and integrated multi-energy systems from unmodeled tail events as bound in Theorem 1 is not dependent on exact knowledge of the feasible set and underlying probability law. The extremality property in Section 2.2 and the second-order proto-differentiability of functions in Section 4.1 together offer insight into the dynamic relationship between the allocation-deficit, the parameters of the feasibility-set family, and the radius of the ambiguity for a system operator to make forward-looking investment decisions of capacity and reserves, in advance of the uncertainty that unfolds in the future.

8. Conclusion and future scope

A set-valued variational analysis framework for structural feasibility uncertainty (represented by a set-valued dispatch feasibility mapping), alongside distributional uncertainty (represented by a Wasserstein ambiguity set), is derived for the allocation of energy resources under these joint uncertainties (Kholova et al., 2025). Fritz–John-type stationarity conditions are provided for the generalized-Nash-equilibrium reformulation with an explicit non-triviality condition in relation (7), and a concentration bound given in Theorem 1 decomposes the cascading-allocation-failure probability into nominal-risk, ambiguity-radius-risk, and sampling-concentration-risk. Generalized Hukuhara-differentiability and second-order proto-differentiability of the feasibility mapping provide for sensitivity analysis connecting the allocation-deficit bound to the ambiguity radius. An inexact Newton-type solver and an iterative fixed-point solver were then compared numerically in a large-scale numerical assessment of the models. A natural extension of the framework to the study of decentralized multi-area reliability coordination with interdependent temporal energy storage resources, where the set-valued feasibility mapping and Wasserstein ambiguity set are common to geographically separated subnetworks via shared state constraints on those storages, seems possible.

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