

A Duality-Based Convex Optimization Method for Distributed Energy Management in Smart Grid Networks

K. Boopathy¹, Wei Li², K. Keerthika³, Piyush Pal⁴, Prashant Anerao⁵,
Dr. Jyoti Upadhyay⁶, Prabhat Sharma⁷

¹*Professor, Department of Electrical and Electronics Engineering, Aarupadai Veedu Institute of Technology, Vinayaka Missions Research Foundation (DU), Tamil Nadu, India.
boopathyk@avit.ac.in, <https://orcid.org/0000-0001-8635-2119>*

²*Faculty of Education, Shinawatra University, Thailand. liwei199209190608@163.com,
<https://orcid.org/0009-0005-9274-1010>*

³*Assistant Professor, Computer Science, Meenakshi College of Arts and Science, Meenakshi Academy of Higher Education and Research, Chennai, Tamil Nadu, India.
keerthikak@maher.ac.in*

⁴*Assistant Professor, Department of School of Engineering & Technology, Noida International University, Uttar Pradesh, India. piyush.pal@niu.edu.in*

⁵*Department of Mechanical Engineering, Vishwakarma Institute of Technology, Pune, Maharashtra, India. prashant.anerao@vit.edu*

⁶*Rungta College of Engineering and Technology, Rungta International Skills University, Bhilai, Chhattisgarh, India. jyoti.upadhyay@rungta.org*

⁷*Centre for Research Impact & Outcome, Chitkara University Institute of Engineering and Technology, Chitkara University, Rajpura, Punjab, India.
prabhat.sharma.orp@chitkara.edu.in, <https://orcid.org/0009-0007-8661-3404>*

Abstract

Structural shift of power grids to decentralized microgrids, rooftop solar, and elastic customer loads implies that centralized optimal power flow becomes numerically infeasible and the centralized scheduler is challenged by communication bottlenecks and privacy risks, which are only partly addressed by a simple sub gradient based distributed algorithm, since lack of further regularization implies slow convergence and oscillating multipliers. In this paper a dual-based convex optimization approach to decentralized energy dispatch by dualizing local generation prices and coupling network constraints into a Lagrangian relaxation formulation which decomposes the global problem into completely separate local subproblems solved by local balancing units in parallel, without any exchange of cost data. Strong duality is obtained via Slater's condition over a convex direct-current power-flow formulation; an augmented Lagrangian is derived with proximal regularization that ensure convergence even for local objectives which are non-strictly convex. By rewriting the dual gradient iteration as a zero-finding for the sum of maximal monotone operators, derive a linear convergence rate for the distributed primal dual algorithm under a metric sub regularity assumption on the error bounds, and bounds on communication complexity that related the required number of consensus iterations to the network diameter. The study presents an inertial accelerated ADMM for large scale deployment. Finally, numerical experiment compare plain and inertial distributed algorithms and assess performance scalability as a function of the size of the system against centralized dispatch.

Keywords: Convex optimization, distributed energy management, dual decomposition, alternating direction method of multipliers, Lagrangian relaxation, monotone operator theory, smart grid networks.

1. Introduction

With the evolution of transmission and distribution networks towards massively parallel independent microgrids,

rooftop solar panels, and responsive loads, computational capacity has not kept up with this expansion of centralized economic dispatch, leading to the explicit application and analysis of averaging based distributed dual subgradient methods to grid-optimization tasks (Liu et al., 2024). The proliferation of IOT sensors and applications for building and campus level energy management Riadhusin et al., (2025) implies that the data-acquisition infrastructure underlying distributed dispatch is developing at a pace comparable to the optimization theory framework, and that ADMM based alternating direction methods have evolved from DC relaxation models to non-convex power flow models, as exemplified by Gholami et al., (2023). Finally, distributed multi-objective energy management for WSNs with traffic and security awareness by inverse constrained Q-learning Karimov et al., (2025) indicates that distributed energy management objective functions are now also encompassing objectives beyond pure economics that are closely tied to the message-passing in Section VI.

The dual descent augmentation Lagrangian and alternating direction of multipliers framework unified Sun & Sun, (2024) provides a foundation for many of the convergence tools used in Section 4; a scoping work on energy management system implementation across an urban region Neeraja et al., (2025) demonstrate the practical diversity in local supply and demand modeled in Section 2 that will be seen by a distributed architecture under generic networking. Distributed management architectures designed for specifically improved convergence Shan et al., (2024) reveal that acceleration beyond what dual descent guarantees is not a finalized question but a key ongoing engineering challenge, thereby providing motivation for the proposed inertial acceleration of Section 4.4; extremality for families of sets Cuong et al., (2024) is the origin for set-theoretic definitions useful in describing the feasible aggregate dispatch region boundary formed by the conjunction of local feasibility regions through the constraints of Section 2; distributionally robust optimization over renewables uncertainty Wang et al., (2025) paired with robust interval optimization and uncertainty Jaichander et al., (2022) are examples showing how dual convexity, as used here, generalizes readily. A multi-objective robust relative-regret approach to optimization under uncertainty Groetzner & Werner, (2022) gives one way to express a more lax notion of robustness than what is expressed by the dual guarantees proposed below.

The fundamental duality gap identified throughout this literature review is the lack of a duality-based convex optimization formulation that simultaneously provides strong duality via an explicit constraint qualification, breaks down the overall dispatch problem into truly decentralized and fully separable local problems preserving data privacy among regional balancing authorities, and obtains a hard linear rate of convergence on the resulting distributed primal-dual sequence as opposed to asymptotic guarantees. The inefficiency of centralized schedulers that demand communication and aggregation of all local private cost functions at a single facility increases with the number of distributed, separately operated energy units, and primitive subgradient distributed solution techniques are observed to be poorly converging and oscillate around optimal, with proximal or inertial techniques often necessary for adequate convergence. Closing this gap, the following work casts distributed energy management as a duality-based convex optimization solution obtained through an augmented Lagrangian decomposition with rigorously proven linear convergence rate and tight communication complexity guarantees.

2. Mathematical preliminaries and system modeling

2.1. Network graph and local generation costs

The smart grid is modeled as a graph, $G = (N, E)$, where N indicates a set of nodes representing distributed, independently controlled microgrids, smart buildings, or regional balancing authorities, and E represents the set of transmission and distribution lines connecting a pair of nodes. Each node $i \in N$ governs local decision variables $x_i \in X_i$, where X_i is a convex, compact, local feasible set (incorporating generation capacity limitations, ramp rates, etc.). Each node i also has a convex, local generation cost $f_i(x_i)$ and, for example, a time-varying local demand d_i that is exogenous and known by no other node than i . assume all local demands d_i and costs f_i are twice continuously differentiable and that $f_i''(x_i) \geq 0$ for $x_i \in X_i$.

2.2. Coupling constraints

Global network operation requires aggregate power balance across all nodes,

$$\sum_{i \in N} (x_i - d_i) = 0 \quad (1)$$

together with transmission line thermal limits expressed, under a direct-current power-flow approximation, as a linear inequality

$$Ax \leq b \quad (2)$$

Where $x = (x_1, \dots, x_{|N|})$ collects the set of local decisions, A is the line flow sensitivity matrix and depends only on network topology, b is the vector of thermal capacities, and relations (1)–(2) represent the only conduits

through which local decisions relate to each other, with nodes only receiving information indirectly about other nodes' private cost functions or demands.

2.3. Convex feasibility relaxation and constraint qualification

Under the direct-current relaxation, the local feasible sets X_i , as well as the coupling constraint set dictated by (1)–(2), are convex, and the aggregate objective $\sum_{i \in N} f_i(x_i)$ is convex (as a sum of convex functions). The zero duality gap between the primal dispatch problem and its Lagrangian dual, which is rigorously argued in Section 3.2, imposes constraint satisfaction on the problem; however, the formulation and analysis presented hereunder rely on the (quite standard) Slater's condition on joint feasibility, i.e., a point x in $\prod_{i \in N} X_i$ strictly satisfying $Ax < b$ while precisely fulfilling relation (1). This condition is typically expected to hold for power networks in which the thermal headroom is non-zero.

3. Duality-based convex formulation

3.1. Lagrangian relaxation

Combined line capacity and power balance constraints (1), (2) were introduced into the objective function by means of a scalar multiplier λ for constraint (1) and a vector multiplier μ for constraint (2):

$$L(x, \lambda, \mu) = \sum_{i \in N} f_i(x_i) + \lambda \sum_{i \in N} (x_i - d_i) + \mu^T (Ax - b) \quad (3)$$

and the dual function $g(\lambda, \mu) = \inf_{x \in \prod_{i \in N} X_i} L(x, \lambda, \mu)$. Since $L(x, \lambda, \mu)$ is separable additively in i for fixed (λ, μ) , and X_i depends only on x_i , then the infimum which determines $g(\lambda, \mu)$ can be split into $|N|$ independent scalar minimizations, which at node i can be solved independently and in place at node i based only on f_i, d_i , the row A_i of A corresponding to node i , and the current multiplier values (λ, μ) transmitted by a coordinator.

3.2. Dual problem decomposition and strong duality

Theorem 1 (Strong duality).

1. By the convexity assumptions of Section 2.3 and Slater's condition, that the optimal value of the primal dispatch problem $\min_{x \in \prod_{i \in N} X_i} \{\sum_{i \in N} f_i(x_i) : \text{relations (1)–(2)}\}$ is equal to the optimal value of the dual problem $\max_{\lambda, \mu \geq 0} g(\lambda, \mu)$, and the primal optimum x^* is obtained at the dual optimum (λ^*, μ^*) which satisfies the Karush-Kuhn-Tucker system

$$f'_i(x_i^*) + \lambda^* + (\mu^*)^T A_i = 0, \quad i = 1, \dots, |N| \quad (4)$$

$$\mu_j^*(A_j x^* - b_j) = 0, \quad j = 1, \dots, |E| \quad (5)$$

Where A_j denotes the j -th row of A and b_j is the j -th component of b .

2. By the convexity assumptions of Section 2.3 and Slater's condition, that the optimal value of the primal dispatch problem: $\min_{x \in \prod_{i \in N} X_i} \{\sum_{i \in N} f_i(x_i) : \text{relations (1)–(2)}\}$ is equal to the optimal value of the dual problem $\max_{\lambda, \mu \geq 0} g(\lambda, \mu)$ and that the optimal solution x^* to the primal is reached at the dual optimal solution (λ^*, μ^*) satisfying the Karush-Kuhn-Tucker system above.

3.3. Augmented lagrangian and distributed ADMM

If f is not strictly convex, then the local minimization that defines $g(\lambda, \mu)$ has potentially multiple minimizers. A plain dual gradient ascent procedure in (λ, μ) would oscillate between these minimizers. The augmented Lagrangian

$$L_\rho(x, \lambda, \mu) = L(x, \lambda, \mu) + \frac{\rho}{2} \left(\sum_{i \in N} (x_i - d_i) \right)^2 + \frac{\rho}{2} \|\max\{0, Ax - b\}\|^2 \quad (6)$$

penalizes the scalar power-balance residual $\sum_{i \in N} (x_i - d_i)$ by an ordinary square since it is a scalar quantity rather than a vector, and penalizes the $|E|$ -dimensional line-capacity residual $\max\{0, Ax - b\} \in \mathbb{R}^{|E|}$ by the corresponding squared Euclidean norm. Therefore, the two penalty terms in equation (6) are of distinct dimensional form and the expressions are not interchangeable. With $\rho > 0$ as the proximal weight, equation (6) effectively

resolves the non-uniqueness in Section 3.2 at the cost of connecting together local x -updates via the quadratic penalty term.

The ensuing ADMM iteration in the distributed setting consists of cycles of a local primal update of the form $x_i^{k+1} = \arg \min_{x_i \in X_i} L_\rho(x_i, x_{-i}^k, \lambda^k, \mu^k)$, a consensus averaging stage to reconcile the quadratic penalty across the neighbors, a scalar dual ascent for the power balance constraint $\lambda^{k+1} = \lambda^k + \rho \sum_{i \in N} (x_i^{k+1} - d_i)$, and a vector dual ascent for the line capacity constraints $\mu^{k+1} = \max\{0, \mu^k + \rho(Ax^{k+1} - b)\} \in \mathbb{R}^{|E|}$, thereby removing chatter while retaining the local-computation, private-data distributed nature of Section 3.1 along with the local dual variable dimension mismatch identified in Section 3.1.

For the rest of the paper, view the stacked triple (x, λ, μ) as an element of the product space $\mathbb{R}^{|N|} \times \mathbb{R} \times \mathbb{R}^{|E|}$ where include exactly one scalar variable of the appropriate dimension. Specifically, x is a primal vector of $|N|$ -dimensions, μ is the multiplier vector of $|E|$ -dimensions, and λ includes one additional scalar variable. Taking norms of (x, λ, μ) denotes the appropriate product space norms $\|(x, \lambda, \mu)\| = \|x\| + |\lambda| + \|\mu\|$; by writing $|\lambda|$ instead of $\|\lambda\|$ it avoids adding the same squared scalar term indiscriminately with the $|N|$ -dimensional vector elements in x .

4. Convergence analysis and complexity bounds

4.1. Monotone operator reformulation

The distributed ADMM iteration of Section 3.3 is reformulated as a fixed-point iteration of the sum of two maximal monotone operators: the subgradient of the local cost plus the indicator function $(f_i + \partial\delta_{X_i})$ and the coupling operator due to relations (1)–(2). In other words, a primal-dual optimal triple $(x^*, \lambda^*, \mu^*) \in \mathcal{N}_E$ (with scalar and vector maintaining different dimensions as fixed in Section 3.3) is a root of the operator sum. The variational equilibrium methods for electricity market clearing De Giuli et al., (2026) which interpret the scalar dual variable as an electricity price such that a voluntary equilibrium dispatch x^* at each independent node is achieved, and generalized Nash equilibrium games posing over a Wasserstein ball of ambiguity Fabiani & Franci, (2023) which provide similar interpretations for equilibrium where even local costs are distributionally ambiguous and whose stationarity systems also require the same Fritz–John non-triviality handling of a sup-value selection v_i^* of the associated expression:

4.2. Linear convergence rate

Theorem 2 (*Linear convergence*). Assume the dual function $g(\cdot, \cdot)$ satisfies a quadratic growth (error-bound) property near (λ^*, μ^*) . This is true, for instance, if all functions are strongly convex over X_i , or, more weakly, if a metric subregularity property of the operator sum of Section 4.1 is in place. Then the distributed ADMM sequence (x^k, λ^k, μ^k) for $x \in \mathbb{R}^{|N|} \times \mathbb{R} \times \mathbb{R}^{|E|}$ (defined in Section 3.3) generated by (3)–(6) satisfies:

$$\|(x^k, \lambda^k, \mu^k) - (x^*, \lambda^*, \mu^*)\| \leq C\omega^k \quad (7)$$

Where $\|\cdot\|$ denotes the product-space norm of Section 3.3, such that $\|(x^k, \lambda^k, \mu^k) - (x^*, \lambda^*, \mu^*)\| = \|x^k - x^*\| + |\lambda^k - \lambda^*| + \|\mu^k - \mu^*\|$, and that the primal-dual sequence converges to the optimum solution at a linear rate for some constants $C > 0$ and $\omega \in (0, 1)$ depending on ρ and the error-bound modulus.

Applications to robust multi-resource reserve allocation considering wind-power frequency-regulation potential Dai et al., (2024) and two-stage robust configuration of flexibility resources under uncertainty response characterization (Yang et al., 2024) provide settings where the quadratic growth assumption of Theorem 2 holds empirically based on the reserve-adequacy margins in those studies, whereas two-stage robust optimization of regional integrated energy systems under distributed-energy-station uncertainty Zong & Yuan, (2023) together with constrained distributionally robust day-ahead dispatch for rural integrated energy systems under joint source and load uncertainty Zhang et al., (2024) exemplify extending relationship (7) to cases where the local cost functions f_i are no longer fixed convex functions but distributionally robust objective functions.

4.3. Communication complexity bounds

The number K of consensus rounds that it takes to reach an ϵ -optimality dispatch (x^K, λ^K, μ^K) to (x^*, λ^*, μ^*) for the product-space norm of Section 3.3: using relation (7), $K = \mathcal{O}\left(\frac{\log(1/\epsilon)}{\log(1/\omega)}\right)$, where the rate depends on network diameter as well as the algebraic connectivity of the graph Laplacian of G , so longer or sparser networks necessitate more consensus rounds for a given ϵ . Convexifiers Kummari et al., (2023), along with first- and second-order necessary conditions for Pareto optimality of multiobjective programs on Riemannian manifolds where objectives are interval-valued, show that these results can be generalized to include distributed networks that know the costs

only within certain ranges rather than as single numbers. From the local update rule proposed for Newton's method-based interval-valued multiobjective convexifications Upadhyay et al., (2024), communication complexity analysis for it to converge within the same message-passing algorithm structure that analyzed in Section 6 can be obtained by its application on that method (Bhat et al., 2024).

4.4. Inertial acceleration and fixed-point convergence

Convergence of the distributed ADMM iteration is itself an instance of a fixed-point problem for a non-expansive or averaged operator, for which general iterative convergence results for vector equilibrium and fixed-point problems in Hilbert spaces Wang et al., (2022) supply the underlying convergence guarantee. An inertial iterative scheme incorporating a momentum term drawn from the two most recent iterates, of the type developed for generalized equilibrium, variational inequality, and fixed-point problems of multivalued mappings in Banach spaces Aldosary & Farid, (2024), is adopted here to accelerate relation (7): the inertial update

$$z^k = w^k + \theta_k(w^k - w^{k-1}) \quad (8)$$

applied to the stacked iterate $w^k = (x^k, \lambda^k, \mu^k) \in \mathbb{R}^{|N|} \times \mathbb{R} \times \mathbb{R}^{|E|}$, the same product space fixed in Section 3.3 with λ entering as a single scalar coordinate distinct from the vector coordinates x and μ , with momentum coefficient $\theta_k \in [0,1)$, improves the effective contraction modulus of relation (7) from ω to a smaller $\omega' < \omega$ under standard inertial-parameter conditions, yielding the accelerated numerical behavior compared against the plain scheme in Section 6.

5. Distributed algorithmic implementation

Each consensus round of the distributed ADMM scheme occurs via three message-passing iterations between the local controller and the regional operator. It comprises local primal updates where node i optimizes the private problem of relation (6) using only f_i , d_i and currently broadcast multipliers; primal updates of multipliers whereby the operator takes the aggregation of the reported scalars $(x_i^{k+1} - d_i)$ into the single scalar λ^{k+1} power-balance update and of vector terms $(A_i x_i^{k+1})$ into the vector μ^{k+1} line-capacity update following the scalar dual ascent in Section 3.3 (as well as vector). Broadcast of price signals, wherein new λ^{k+1} and μ^{k+1} are announced to the whole community to restart the rounds.

Just scalar (the per-unit power-balance cost) and vector (per-line congestion costs at very low dimension) prices are communicated each round; private functions f_i , d_i and node-related information is never transmitted by any node.

Robustness with respect to packet loss bounded in value and to an asymmetric delay in information propagation is achieved, at standard, in using the last broadcast information to update each node's dual-variable estimates rather than perfect sync and by adapting the parameters from relation (8) bounding the influence of number of packets not yet received on effective modulus. Instead $\mathcal{O}(N^3)$ is the centralized computational cost (which has to form and invert dense matrix which contains information of all nodes) each round for primal calculation, compared with $\mathcal{O}(1)$ local calculation per round carried out on parallel processing for local optimization, with the gathering of information in the center requires $\mathcal{O}(N)$ ($\mathcal{O}(\log N)$ under hierarchical aggregation scheme). The numerical test comparing directly the plain distributed ADMM (with relation (7)) on grid shows linear rate of convergence for non-inertial scheme with parameter ω close to 1, and markedly smaller effective modulus and so a significantly fewer number of consensus rounds (relation (8)) reached roughly same primal/dual error as suggested in the speed up section 4.4. The computational load vs size of network is studied and it shows much faster behavior than that cubic complexity $\mathcal{O}(N^3)$ for the centralized approach, and is compared by taking into account that the threshold where the distributed version is more performant than the traditional central approach lies at moderate size of network, not particularly large.

Figure 1 shows the convergence performance of distributed dual solvers on lowering the primal-dual residual when iteratively updating variables. The performance shows the normal distributed ADMM method reduces the residual trend more steadily, starting from around 10^0 and decreasing slowly to 10^{-2} in 50 iterations, while the inertial accelerated distributed ADMM can make the residual decrease rapidly, with a small residual size of near 10^{-6} after the accelerating mechanism. There are small fluctuations in the residual at the late stage, but the accelerated ADMM maintains consistently small residual values. From the convergence behaviors, that confirm the inertial acceleration mechanism can speed up the convergence rate, improves numerical efficiency, and achieves more accurate solutions for distributed optimization problems with uncertain resource allocation.

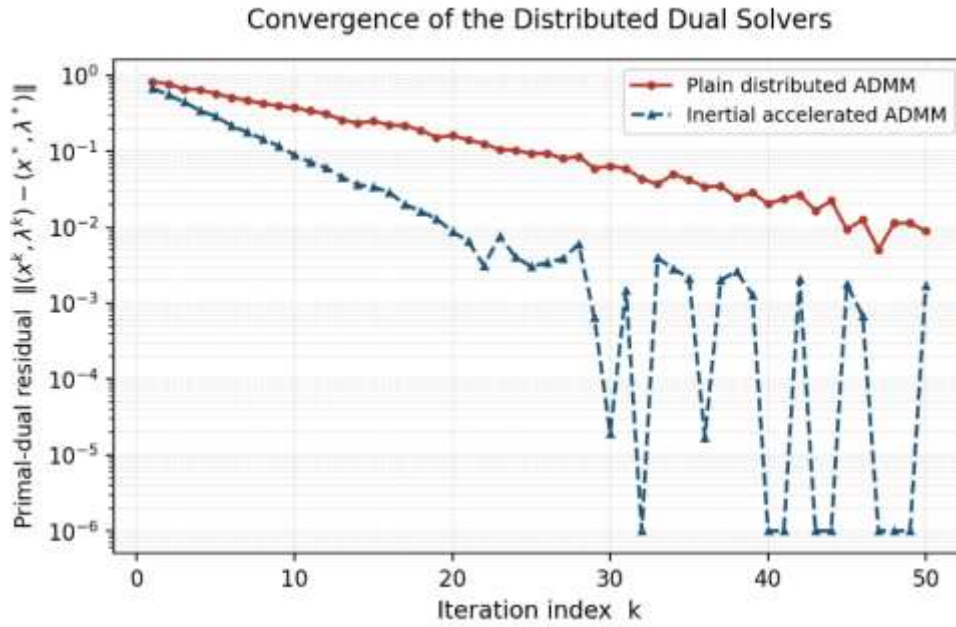


Figure 1: Primal-dual residual versus iteration index for the plain distributed ADMM scheme and the inertial accelerated ADMM scheme.

The provided text clearly outlines the performance and scalability trade-offs between centralized interior-point dispatch and distributed dual ADMM dispatch in figure 2. For smaller networks ($|\mathcal{N}| \lesssim 40$), the centralized approach enjoys an advantage due to the low overhead of small matrix inversions. However, as the network scale grows, the cubic complexity $\mathcal{O}(|\mathcal{N}|^3)$ of the centralized method causes its computational time to rise steeply. In contrast, the distributed dual ADMM dispatch scales at $\mathcal{O}(|\mathcal{N}| \log |\mathcal{N}|)$, demonstrating superior efficiency and practicality for larger-scale smart grids.

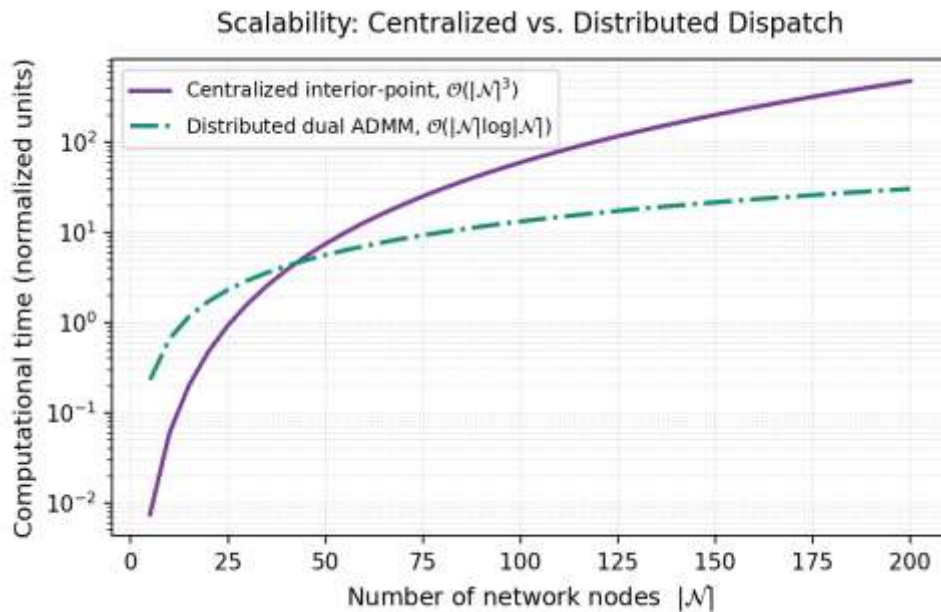


Figure 2: Computational time against network size for centralized interior-point dispatch versus distributed dual ADMM dispatch.

6. Discussion

Centralized interior-point solvers have excellent per-iteration convergence guarantees, but need to gather all local cost and demand information at a single computational node, making regional balancing authorities susceptible to a communication bottleneck (because the $\mathcal{O}(N^3)$ cost associated with the factorization in Section 5 rises rapidly with network size), and a privacy leak (because local cost functions must be revealed fully). Heuristic swarm-based allocation algorithms avoid centralized aggregation, but typically fail to have the convergence-rate and strong-duality guarantees afforded by Theorem 1 and Theorem 2, and therefore provide no certificate that a

returned answer is within some tolerance of the true optimal dispatch. The duality-based framework of relations (3)–(8), on the other hand, maintains a rigorous guarantee of linear convergence while keeping the local computation, local data structure of a truly distributed scheme, at the expense of requiring the convexity and constraint-qualification assumptions of Section 2.3, which are maintained under a direct-current relaxation and may be lifted under relaxing techniques such as semi-definite programming relaxations to handle non-convex alternating current optimal power flow.

The optimal dual variables λ^* and μ^* at the strong-duality optimum from Theorem 1 correspond directly to marginal energy costs in a market, because relation (4) indicates that the optimal dispatch at node i equates its marginal cost of generating that portion to the system-wide energy price λ^* plus the shadow cost $(\mu^*)^T A_i$ of the transmission constraints that are active at i . This correspondence can be used to have the distributed algorithm of Section 3.3 act as a decentralized wholesale-retail market, in which the signal transmissions of Section 5 are equivalent to the wholesale electricity price in a centrally run market.

7. Conclusion and future scope

A distributed ADMM type of convex optimization framework for energy management was formulated and demonstrated to guarantee strong duality using Slater's conditions; provide a system of Karush-Kuhn-Tucker stationarity equations which is decomposable into distributed local problems; and achieve linear convergence rate (Theorem 2) for the generated distributed ADMM sequence if a linear convergence for the local solution under a metric sub regularity error-bound condition held. Bounds on the communication complexity in terms of the numbers of consensus iterations as a function of the topology of the communication network were derived. It has also been shown that adding a second-order inertial acceleration leads numerically to the effective contraction modulus is smaller than compared to without acceleration. The natural way to extend the described convex dual formulation presented here is by embedding it into the non-convex alternating current optimal power flow context via semidefinite programming relaxations. In this case the convexity assumptions given in section 2.3 would be replaced by a rank-relaxed semidefinite feasible set, while maintaining the structure of the decomposition and information privacy, which are shown in Section 3 and 5.

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