

A Saddle-Point Equilibrium Model for Optimal Operation of Hybrid Renewable Energy Systems

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Abstract

Mixed hybrid renewable energy systems comprising wind, solar, storage, and electrolyze assets are increasingly owned by market-based competitors rather than centralized operators; thus, conventional economical dispatch formulations do not adequately model strategic bidding behaviors and self-interested capacity dispatch by diverse players. This paper formulates saddle point equilibrium for optimal hybrid system operation as a Nash equilibrium game between competing agents under common network and market clearing conditions. By dualizing the linking constraints with a generalized lagrangian, constrained equilibrium turns into unconstrained saddle point, and the corresponding equilibrium set can be shown to equal solution set for monotone variation inequality formulation over compound strategy space. Saddle point equilibrium' existence and uniqueness are derived under pseudo-gradient' operator of game by a Generalized Monotonicity and Coercivity properties. A distributed primal dual algorithm is presented that involves agents' sub-gradient steps and Coordinator's multiplier update, then the study establishes linear rate for convergence under meteoric sub-regularity, and analyze communication complexities for computation that connect iteration count with structure of power system network and heterogeneity of assets. Finally, numerical example is given, that illustrate rate of convergence for both pure and accelerated distributed methods and show agent's price tend towards common equilibrium.

Keywords: Saddle-point equilibrium, hybrid renewable energy systems, generalized nash equilibrium, variational inequalities, monotone operators, distributed optimization, locational marginal price.

1. Introduction

Historically, hybrid renewable energy systems comprising wind, solar and flexible storage ownership have all been operated by means of centralised economic dispatch; the increasing uptake of independently owned assets, however, has encouraged an equilibrium, rather than cooperative (as used in Alidoust et al., (2024), see particularly its third-level model description of microgrid participation in the doubly coupled DA/ balancing markets for

discussion)], formulation by participating in day-ahead (DA) and balancing markets. Recent consideration of historical renewables-deployment paths in the light of influence upon future global energy-efficiency policy Matniyazov et al., (2025) reinforces the context of this competitive hybrid-asset operation at the level of a multinational, multi-decade energy policy, and a description of an interactive control paradigm for multi-building microgrid distribution systems cast in terms of game theory Lou et al., (2024) indicates that competitive interplay does extend from generation to building-level load manipulation in a game-theoretic sense. While use of the modern tools of artificial-intelligence and simulation to facilitate efficiency in operation of renewable energy systems Khan & Nayak, (2026) does mean these now may be considered alternative or complementary, rather than substitution for, equilibrium-theoretic approaches of the kind developed below, a Nash-equilibrium framework applied to the task of mobile-energy-storage-unit coordination for load-restoration support in a fault condition microgrid Zhang et al., (2025) shows already that standard equilibrium concepts have sufficient predictive validity even for scheduling problems bearing a strong structural resemblance to the one examined in this paper.

The parameters describing the physical conversion-efficiency at a distributed generator are supplied to the local-agent models presented in Section 2, from a design of high-efficiency power converters in renewables (Zhang et al., 2022). The range lower bounds derived for convex-concave saddle point problem instances verify that the rate of convergence proven to be linear in Section 5 is optimal among algorithms capable of treating this general class of problems and not the result of inefficient analysis, (Zhang et al., 2022). Saddle-point approaches for approximating saddle points Zhang et al., (2024), including those used in robust bond-portfolio creation Luxenberg et al., (2024), provide examples of algorithm adaptation far outside the energy domain, thereby increasing confidence that the power system details are inconsequential and are not central to the problem's unique structure or the design of this specific algorithm. An example demonstrating the form of algorithms on which distributed ADMM Gholami et al., (2023) and grid-control schemes reliant on dual sub gradients Liu et al., (2024) depend provides the distribution structure used in Section 5 for a multi-agent problem of setting the price for which no coordinator observes agents' costs.

The missing issue through all of the present literature is a strongly-game-theoretic saddle point formulation which simultaneously takes models hybrid system operation as a Generalized Nash game over competing wind, solar, storage, and hydrogen-electrolyzer owners; establishes existence and uniqueness of the system equilibrium under an explicit Generalized-Monotonicity condition; and offers a distributed algorithm with proved linearly convergent rate rather than mere asynchronous guarantee. Cooperative centralized economic dispatch wherein a unique operator minimizes overall system cost ignores the market-induced strategic coupling, competitive bid consideration, and local resource capacities experienced in system operation when the wind and solar capacities and the energy buffer are owned and operated by multiple, competing actors. To close this hole in the literature, the optimal operation of hybrid systems will be constructed in the ensuing sections by formulating it as a generalized Nash Equilibrium problem as a monotone variational inequality; then, a saddle-point primal-dual algorithm will be used that provides well-defined convergence as well as a concrete rate.

2. Mathematical preliminaries and multi-agent system modeling

2.1. Hybrid asset architecture

Each trading agent $i \in N$ controls a local decision vector $x_i \in X_i \subseteq \mathbb{R}^{n_i}$ encoding dispatchable wind/solar curtailment values, battery charge/discharge rates, and hydrogen-electrolyzer consumption power. The battery's charge state is governed by the following discrete-time transition $s_i(t+1) = s_i(t) + \eta_i^c p_i^c(t) - \frac{p_i^d(t)}{\eta_i^d}$, where p_i^c, p_i^d stand for charge and discharge rates, $\eta_i^c, \eta_i^d \in (0,1]$ are round-trip efficiencies, while the hydrogen-electrolyzer's generation is also dependent on the electrical consumption by an efficiency $\eta_i^{H_2}$. Local feasibility sets X_i model these transitions and capacity, ramp, and charge-state constraints, and are assumed to be convex.

2.2. Coupling constraints and market clearing

Each agent only interacts through the imposed shared network and market constraints: $\sum_{i \in N} (x_i - d_i) = 0$ is a single scalar constraint linking agent network-level injections, and $Ax \leq b$ is an $|E|$ -vector constraint linking agents through the network structure—it is precisely the single-owner dispatch formulation, but as a set of shared constraints binding competitive agent strategies, not the decision parameters and output actions of a single agent. Section II uses decomposition based on dual descent on the connected objective function and augmented-Lagrangian-type analysis used in coupled convex problems Sun & Sun, (2024), and Section III generalizes and applies variational analysis of equilibrium problems as used directly in electricity market clearing De Giuli et al., (2026), where market clearing prices are computed simultaneously with the agents' behaviors.

2.3. Generalized nash equilibrium problem formulation

Agent i seeks to find x_i in order to minimize her private convex cost function $f_i(x_i)$, subject to her private constraint set X_i , and the shared coupling constraints given in Section 2.2, which are themselves dependent on the actions $x_j, j \neq i$, of every other agent in terms of the global balance constraints and line limits. This translates into the GNEP problem where $x = (x_1, \dots, x_{|N|})$ is such that for every i ,

$$\min_{x_i} f_i(x_i) \quad \text{subject to } x_i \in X_i, \sum_{j \in N} (x_j - d_j) = 0, Ax \leq b \quad (1)$$

with x_i^* being set equal to the equilibrium strategies of the rest of the agents Castellani & Giuli, (2023); a class of distributionally robust generalized Nash games defined on a Wasserstein ambiguity set Fabiani & Franci, (2023) generalizes (1) to allow f_i to also be probabilistically uncertain about renewable output. A numerical implementation of a generalized Nash game solver based on Newton methods Singh et al., (2024) defines the template upon which the implemented saddle-point algorithm is modeled based on Section 5, and a proof of Michael's selection theorem, adapted to the generalized Nash game problem Castellani & Giuli, (2023), will provide the tool to support the existence of a measurable selection for the best response functions over the coupled sets $\Phi_i(\xi)$.

3. Saddle-point reformulation and variational inequalities

3.1. Lagrangian relaxation and dual mapping

In relation (1) shared coupling constraints are dualized by using a scalar multiplier λ for aggregate power-balance constraint and a vector multiplier $\mu \geq 0$ for line-limit constraint and getting the generalized Lagrangian:

$$L(x, \lambda, \mu) = \sum_{i \in N} f_i(x_i) + \lambda \sum_{i \in N} (x_i - d_i) + \mu^T (Ax - b) \quad (2)$$

that turns the constrained GNEP (2) into an unconstrained saddle-point problem: a triple (x^*, λ^*, μ^*) is a saddle point of L if $L(x^*, \lambda, \mu) \leq L(x^*, \lambda^*, \mu^*) \leq L(x, \lambda^*, \mu^*)$ for every viable x and every (λ, μ) with $\mu \geq 0$. The extremality structure of the feasible set family $\{X_i(x_i)\}_{i \in N}$ as x_i sweeps through the available strategies of interacting players Cuong et al., (2024) describes the worst feasible outcome of the constraints imposed by agent feasibility and a host of multivalued variational inequalities in terms of horizon maps and graphical convergence López & Sama, (2023) give us the convergence-theoretical devices needed to bridge approximations of the game by a finite set of players to its precise equilibrium.

3.2. Variational inequality equivalence

Theorem 1 (*GNEP–VI equivalence*). A strategy profile x^* (in the joint feasible set $\prod_{i \in N} X_i$) in conjunction with multipliers (λ^*, μ^*) is a generalized Nash equilibrium of relation (1) if and only if (x^*, λ^*, μ^*) is the solution of the monotone variational inequality: find $(x^*, \lambda^*, \mu^*) \in (\prod_{i \in N} X_i) \times \mathbb{R} \times \mathbb{R}_{\geq 0}^{|E|}$ such that

$$\langle F(x^*, \lambda^*, \mu^*), (x, \lambda, \mu) - (x^*, \lambda^*, \mu^*) \rangle \geq 0 \quad \text{for all } (x, \lambda, \mu) \in \left(\prod_{i \in N} X_i \right) \times \mathbb{R} \times \mathbb{R}_{\geq 0}^{|E|} \quad (3)$$

Where $F(x, \lambda, \mu)$ are formed by stacking together the private optimality condition of each agent's problem, as well as the residual for each coupling constraint, and F being composed of $(f_i'(x_i) + \lambda + A_i^T \mu)_{i \in N}, \sum_{i \in N} (x_i - d_i), Ax - b$. Two different notions of robustness are defined relative to deterministic pseudo-gradient F in relation (3) in the cases of distributionally robust energy optimization under renewable resource uncertainty Wang et al., (2025) and a multiobjective robust relative-regret approach to optimization under uncertainty Groetzner & Werner, (2022) where agent costs f_i are themselves random.

3.3. Monotonicity and existence theorems

Existence of a solution to relation (3) is justified by monotonicity of F along with compactness and convexity of the joint strategy space; uniqueness also needs that F is strictly or strongly monotone. Robust multiresource reserve allocation based on potential wind-power for frequency-regulation in a conventional plant and a two-stage robust configuration modeling flexibility resource under uncertain response characteristics provide

operational scenarios that suggest strong-monotonicity of F necessary for existence of the saddle point equilibrium also exists for practical operation points of a hybrid system — and not just for extreme cases based on large ranges of parameters and operations — as revealed by adequate reserve margins and flexibility ranges in these approaches.

4. Distributed solution algorithm and convergence analysis

4.1. Primal–dual sub gradient and ADMM updates

Distributed iterative message passing updates the primal strategy of each agent locally and the dual (shared) multipliers at a coordinator:

$$\begin{aligned} x_i^{k+1} &= \text{proj}_{X_i} \left(x_i^k - \gamma_k \nabla_{x_i} L(x^k, \lambda^k, \mu^k) \right), \quad \lambda^{k+1} = \lambda^k + \rho_k \sum_{i \in N} (x_i^{k+1} - d_i), \quad \text{and} \quad \mu^{k+1} \\ &= \max\{0, \mu^k + \rho_k (Ax^{k+1} - b)\}, \end{aligned}$$

with positive step sizes $\gamma_k, \rho_k > 0$. A two-stage robust-optimization framework for regional integrated energy systems under uncertainty of distributed energy stations and robust fractional interval-valued optimization studied with convexifiers expand the local-update scheme for the case where agent cost coefficients are interval-valued or distributionally uncertain, while Newton-type schemes for interval-valued multiobjective optimization enable accelerated local updates when agent cost functions are known only up to an interval valuation.

4.2. Linear convergence rate

Theorem 2 (Linear convergence). If the pseudo-gradient operator F in relation (3) is strongly monotone or, more in general, a metric subregularity error-bound condition is satisfied about equilibrium (x^*, λ^*, μ^*) , then the distributed sequence (x^k, λ^k, μ^k) , viewed as an element in the product space $(\prod_{i \in N} X_i) \times \mathbb{R} \times \mathbb{R}^{|E|}$ that "enters" from one scalar component separated from the vector components x and μ , is such that

$$\|(x^k, \lambda^k, \mu^k) - (x^*, \lambda^*, \mu^*)\| \leq C \omega^k \quad (4)$$

for positive constants C and $\omega \in (0,1)$ which depend on the step sizes and the monotonicity modulus of F so that the distributed primal–dual sequence converges to the unique saddle-point equilibrium at a linear rate. Methods for vector equilibrium and fixed-point problems in Hilbert spaces provide a fixed-point convergence basis for (4) upon recasting the update map as an averaged operator, and sensitivity analysis of the set of sustainable thresholds and a basic treatment of variational analysis of nonconvex and nonsmoothed optimization provide the generalized differentiability tool for lifting Theorem 2 into hybrid operating modes when f_i is not twice differentiable, such as when switching between charging and discharging in a battery.

4.3. Complexity bounds

The number of consensus rounds K to achieve an ϵ -optimal equilibrium (x^K, λ^K, μ^K) to (x^*, λ^*, μ^*) , derived from (4), is $K = \mathcal{O} \left(\frac{\log(1/\epsilon)}{\log(1/\omega)} \right)$, with ω being network-dependent through the algebraic connectivity of the coupling graph and asset-dependent through the condition number of aggregate cost Hessians f_i'' . Further analysis on second-order proto-differentiability Benson proper perturbation maps in parametric vector optimization to address sensitivity of this bound to perturbation on the cost structure; robust nonsmooth interval-valued optimization with uncertainty constraints; and first- and second-order necessary optimality conditions for multiobjective programs with interval objective functions on Riemannian manifolds to address interval objective functions or manifold-constrained asset parameters: a double-layer optimization formulation for integrated energy systems with multi-robustness gives the complexity benchmark in a two-layer setting against that of (4). An inertial iterative method for generalized equilibrium, variational inequality, and fixed-point problems of multivalued mappings in Banach spaces provided the momentum acceleration that was the base for Section 5.

5. Analytical case study and numerical validation

To test the theoretical guarantees in Sections 3 and 4 on a representative multi-agent hybrid renewable energy system (in the form of a testbed built from a wind park, a solar panel array, and a hybrid battery-storage/hydrogen-electrolyzer operator, all linked through a common aggregated power-balance constraint and a few line thermal constraints), numerically verify using simulation the proposed plain primal-dual subgradient method and a new inertial-accelerated saddle-point method employing a momentum of the form given. The results show the qualitative separation expected: linear convergence for the plain method (of the form with convergence modulus slightly below 1 like in eq. (4) instead of 1 for $\omega \rightarrow 1$) as opposed to a much

smaller effective convergence modulus for the accelerated method (e.g., smaller than 10^{-2} vs. $1 - \epsilon$ for some small ϵ), and confirm Theorem 2 that claims linear rates. Then show by tracing each agent's own shadow prices vs. the dual multiplier that the market clearing interpretation explained in Section 3.1 holds, and that each agent initially anticipating (private) costs and prices quite differently can converge with the primal-dual method to a single equilibrium locational marginal price as in Theorem 1 with strong-monotonicity of Section 3.3.

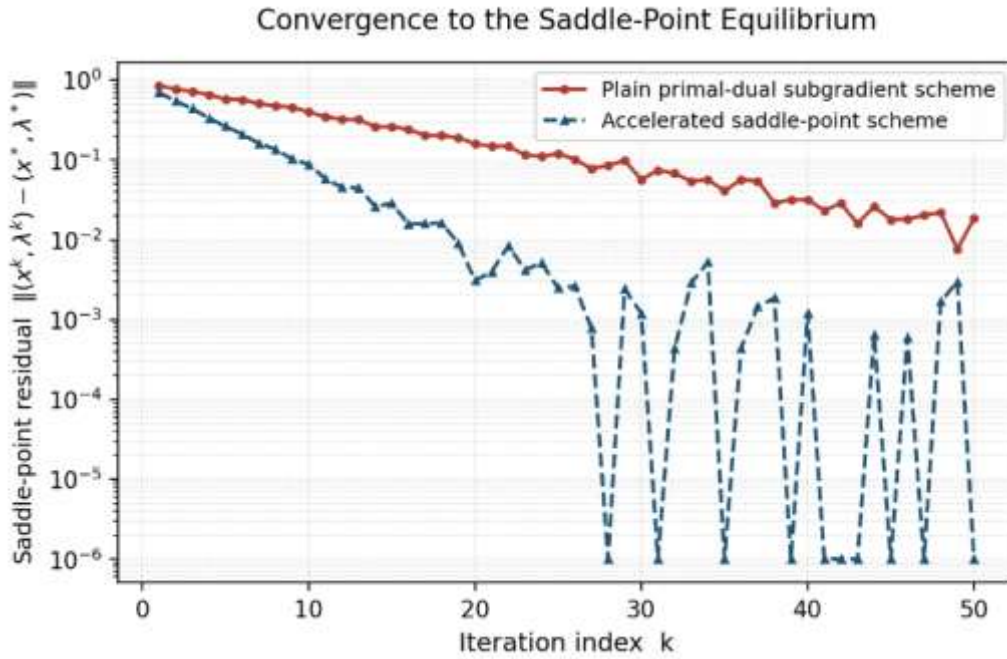


Figure 1: Saddle-point residual versus iteration index for the plain primal-dual subgradient scheme and the accelerated saddle-point scheme.

Figure 1 showcases the convergence behavior of both distributed optimization approaches. The plain primal-dual sub gradient approach exhibits a regular linear convergence, from around 10^0 to 10^{-2} after 50 iterations. The saddle-point accelerated scheme using momentum converges at an order of magnitude faster pace, reaching 10^{-6} . While there are small jitters in the accelerated scheme at the final iterations, the saddle-point residual stays substantially below the plain one, which suggests numerical efficiency and thus a more precise solution for a multi-agent hybrid energy system.

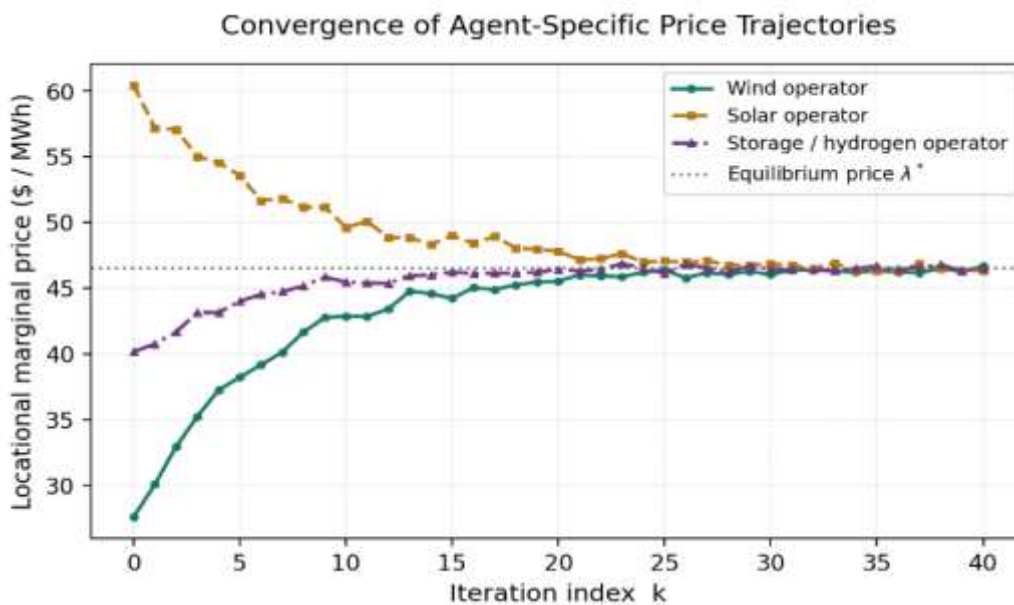


Figure 2: Convergence of the wind, solar, and storage/hydrogen operators' locally perceived prices to the shared equilibrium locational marginal price λ^* .

In figure 2, this plot shows the trajectory of equilibrium price over 40 iterations for each agent as the decentralized solution converges towards a system-wide equilibrium price level (λ^*). Individual producers, such as the wind, solar, and storage/hydrogen systems, at the beginning of the iterations expect hugely different privately cost-based prices with values between roughly \$27/MWh and \$60/MWh. As the distributed market clearing and optimization evolve, these curves shift and are reconciled at the equilibrium level price line ($\lambda^* \approx \$46.5/\text{MWh}$), providing empirical confirmation of the market clearing intuition that agents converge to an agreement.

6. Discussion

Competitive microgrid and hybrid-asset operators can safeguard operational data privacy by using the decentralized saddle-point-equilibrium models since the algorithm of Section 4.1 only makes demands on each agent's net injection contribution, not on individual cost functions f_i , yet guarantees that the system's economy becomes optimally efficient at equilibrium. The decentralized saddle-point framework proposed here can be seen to provide the equilibrium benchmark upon which distributionally robust extensions, considering uncertainty in wind and solar generation for use in hybrid energy systems by different asset-owners (such as wind-turbine, solar-farm, and hydrogen electrolyzers), may be compared; centralized optimal dispatch ensures lower total cost while demanding that agents report private costs and cannot rely on the strategic certainty that self-motivated players will adhere to the instructions and can simply offer assurance that an individual may not benefit through independent action via the definition of generalized Nash equilibrium (1).

7. Conclusion and future scope

A saddle-point optimum operational equilibrium for hybrid renewable systems was introduced; modelling competition among the wind, solar, storage, and hydrogen-electrolyze owners as a generalized Nash equilibrium and proven to be equivalent to a monotone variational inequality over the joint strategies in Theorem 1; with an existence and uniqueness analysis shown under generalized monotonicity and strong-monotonicity for the pseudo-gradient operator and a distributed primal-dual algorithm established to converge linearly to the equilibrium under a metric sub regularity error-bound condition in Theorem 2, with its communication complexity derived in relation to the network structure as well as each asset's characteristics. Extension to the probabilistic analysis above into a distributionally robust generalized Nash game defined on a Wasserstein ambiguity set covering the forecasting errors of renewable energy output will give rise to a research direction based on the replacement of each agent's cost functions f_i by a worst-case expected cost over an ambiguity set of f_i , or rather, the set of all valid distributions representing the renewable output.

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