

A Differential Game-Theoretic Variational Approach for Cooperative Control of Autonomous Robotic Networks

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Abstract

This paper presents a differential game-theoretic variational approach to cooperative control of autonomous robotic networks. The cooperative task is formulated as a coupled quadratic-in-control cost functionals for each robotic agent, that penalize the deviation from a defined relative-configuration manifold, and the robotic agents are modeled as controlled dynamical systems coupled with each other via a defined communication graph, in which the formation acquisition, consensus and leader tracking problem are embedded. The multi-agent optimization is formulated as an N-player differential game and necessary conditions for a cooperative Nash equilibrium are obtained by a variational treatment by the calculus of variations and Pontryagin's minimum principle. These conditions are then described by a coupled system of Hamilton-Jacobi-Isaacs partial differential equations and a verification theorem is derived. In the linear-quadratic case of the problem, closed-form feedback strategies are derived from a set of coupled Riccati differential equations, and a fully distributed control law is constructed that only uses locally available information from a neighbouring set of nodes. Under a smallness condition on the inter-agent coupling weights which depends on the graph, the existence and uniqueness of the cooperative equilibrium are derived, and by using a Lyapunov function constructed from the sum of the individual value functions, the asymptotic stability of the formation manifold is proved, with an extension to jointly connected switching topologies. The numerical example demonstrates how the formation error and the decay of the aggregate running cost are converging under the synthesized strategy with a 6-agent robotic network. The results offer a solid variational link between differential game theory and distributed cooperative control of robotic networks.

Keywords: Differential games, cooperative control, variational methods, Hamilton-Jacobi-Isaacs equation, multi-agent systems, autonomous robotic networks.

1. Introduction

1.1 Background and motivation

The increasing need for autonomous robotic networks, which consist of groups of mobile robots, UAVs or distributed sensing platforms, to perform cooperative tasks, such as formation acquisition, coverage and coordinated tracking, while maintaining strong decentralization constraints, is a major challenge. A typical robotic agent will only be able to access the states of a limited number of agents (typically just the ones in its immediate neighborhood, as defined by the communication graph, which is time-varying), will have to abide by its own actuation constraints, and will be required to balance its own control goal with the overall goal of the control network. Cooperative performance on the one hand, local decision making on the other, is naturally reflected by the language of differential games where each agent is seen as a player minimizing a coupled cost functional subject to his own dynamics, and by the notion of equilibrium, which doesn't speak of a single global optimum, but of a simultaneous and mutually consistent set of decisions from all agents (Snousi & Aleje, 2025).

Most approaches to multi-agent coordination in classical optimal control are based on a single, centrally defined cost functional, and assume that there is a coordinator who knows the information of the whole network (Bryson, 1975). A differential game formulation, on the other hand, allows for the cooperative goal to be broken down into locally-defined cost functionals that are coupled through a neighborhood graph, so that local equilibrium strategies can be constructed in a distributed way and yet still yield network-level coordination (Mohankumar & Gopalakrishnan, 2025). The structure of the underlying optimization problem, the calculus of variations, Pontryagin's minimum principle, and the Hamilton-Jacobi-Isaacs (HJI) partial differential equation underlies this, allowing for the development of an analytical toolbox to rigorously characterize such equilibria and to relate them to feedback laws which can be implemented by each agent based on local information (Kappagantula & Mannayee, 2024).

The concept of differential games was introduced as the continuous time, continuous state counterpart of the matrix games, and the pursuit evasion problems were described by the Hamilton-Jacobi-Isaacs equation (Isaacs, 1999). Since then, the variational and dynamic-programming formulations of two-player zero-sum differential games have been generalized to N-player nonzero-sum games and to graphical or networked games where the cost incurred by each player is only a function of its neighbors' states (Li & Zhang, 2025; Qin et al., 2025). Formation control has been formulated as a non-cooperative differential game, either in the form of linear-quadratic Nash formations on graphs or intrinsic differential game formulations where the Nash equilibrium strategies produce desired relative-configuration patterns without directly specifying the desired formation in the controller (Tandi & Mishra, 2025; Wang et al., 2026). Finite horizon Nash equilibria formulation of formation and platoon control has been developed and connected and automated vehicle platoon formulation has been developed and distributed unmanned-aerial-vehicle teams' formulation has been developed in Cheng & Li, (2019) and Xue et al., (2025) and (Wang et al., 2025; Hu et al., 2026). The parallel line of work is consensus and co-operative coordination using algebraic graph theory. The convergence conditions for consensus under fixed and switching topology of interactions have been determined by using the graph Laplacian concept and the concept of joint connectivity (Lin & Lin, 2026). Later, distributed coordination protocols for multiple agent systems have been unified (Wang et al., 2026; Liu & Geng, 2015). Another type of games involving pursuit and evasion as well as multi-player differential games have also been studied via Hamilton-Jacobi-Isaac's characterizations and reachability-based methods (Chen et al., 2024; Chen et al., 2016; Rustamiy et al., 2025). Although this large and rich collection of ideas has been developed, it has yet to be combined into a single development that (i) formulates cooperative control of a robotic network as an N-player differential game with graph-coupled costs, (ii) derives the corresponding coupled HJI system and its verification theorem, and (iii) relates the equilibrium feedback to a distributed Riccati-based synthesis with a Lyapunov stability guarantee under switching topology. This paper fills this gap.

This paper has four contributions. (i) Game formulation. Cooperative control of an autonomous robotic network is formulated as an N-player differential game with graph-coupled quadratic cost functionals, which are a combined formulation of formation tracking and consensus objectives, and the concept of a cooperative Nash equilibrium modeled based upon the communication graph is precisely defined. (ii) Variational characterization. The necessary conditions for the cooperative Nash equilibrium are obtained using the calculus of variations and Pontryagin's minimum principle, and are formulated as a coupled system of Hamilton-Jacobi-Isaac's equations; a verification theorem is proved (Theorem 1) for the latter. (iii) Existence, uniqueness and distributed synthesis. In the linear quadratic case of this game, closed-form feedback strategies are derived from coupled Riccati differential equations, the existence and uniqueness of an equilibrium are proven under a graph-dependent coupling bound (Theorem 2) and it is demonstrated that the feedback depends on local neighbor information only (Proposition 1). (iv) Stability analysis and numerical validation. A Lyapunov function of the sum of the individual value functions is used to prove asymptotic convergence of the closed-loop network to the desired formation manifold (Theorem 3), and this is extended to the case of a robotic network with switching topologies (Proposition 2); this is illustrated numerically in a robotic network with six agents.

The robotic network dynamics, coupled cost functionals and the related differential game are formulated in Section 2. Section 3 extends the variational approach and introduces the extended Hamiltonian, the Pontryagin necessary conditions, and the coupled HJI characterization of the cooperative Nash equilibrium and an existence and

uniqueness result. Section 4 discusses the distributed feedback synthesis for the case of a linear-quadratic system and the stability of the closed-loop network, including the robustness to switching topology. The discussion is given in Section 5. The paper is concluded in section 6.

2. Problem formulation

2.1 Network and agent dynamics

Consider a robotic network of N autonomous agents indexed by the set $\mathbb{N} = \{1, \dots, N\}$. The inter-agent communication structure is described by an undirected, connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{U})$, where $\mathcal{V} = \mathbb{N}$ and $(i, j) \in \mathcal{U}$ if agents i and j can exchange state information. Let $\mathcal{N}_i = \{j \in \mathbb{N}: (i, j) \in \mathcal{U}\}$ denote the neighbor set of agent i , and let a subset $\mathcal{F} \subseteq \mathbb{N}$ of informed agents additionally have access to a reference (virtual leader) trajectory $x_l(t)$.

Each agent i evolves according to the controlled dynamics in equation (1)

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)), \quad x_i(0) = x_i^0, \quad t \in [0, T] \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ is the state of agent i (e.g., position and velocity in the plane), $u_i(t) \in \mathbb{R}^m$ is its control input, and f_i is assumed continuously differentiable and, for the linear-quadratic specialization used in Sections 3.4 and 4, of the affine form $f_i(x_i, u_i) = A_i x_i + B_i u_i$ with (A_i, B_i) stabilizable. The collective network state is $x = (x_1, \dots, x_N) \in \mathbb{R}^{nN}$ and the collective control is $u = (u_1, \dots, u_N)$.

2.2 Cooperative objective functionals

The cooperative task is specified by a set of desired relative configurations $\delta_{ij} \in \mathbb{R}^n$ for $(i, j) \in \mathcal{U}$, satisfying the compatibility (cocycle) condition $\delta_{ij} = -\delta_{ji}$, together with desired offsets δ_i from the leader for $i \in \mathcal{F}$. The running cost of agent i is defined as the graph-coupled quadratic form in equation (2)

$$\ell_i(x, u_i) = (x_i - x_l - \delta_i)^\top Q_i (x_i - x_l - \delta_i) \cdot \mathbb{1}_{i \in \mathcal{F}} + \sum_{j \in \mathcal{N}_i} w_{ij} [(x_i - x_j) - \delta_{ij}]^\top Q_{ij} [(x_i - x_j) - \delta_{ij}] + u_i^\top R_i u_i \quad (2)$$

Where $Q_i, Q_{ij} \geq 0$ and $R_i > 0$ are symmetric weighting matrices and $w_{ij} = w_{ji} > 0$ are edge weights quantifying the strength of inter-agent coupling. The finite-horizon cost functional of agent i is equation (3)

$$J_i(u_1, \dots, u_N) = \int_0^T \ell_i(x(t), u_i(t)) dt + x_i(T)^\top S_i x_i(T) \quad (3)$$

with $S_i \geq 0$ a terminal weighting matrix. Observe that J_i depends on the controls of agent i 's neighbors only through their effect on the trajectories $x_j(\cdot), j \in \mathcal{N}_i$, so that the coupling among players is induced entirely by the graph $\mathcal{G}$, consistent with the graphical-game formulations of Guo et al., (2025) and (Qin et al., 2025; Wang et al., 2026).

2.3 Differential game formulation

The cooperative control problem is posed as the N -player differential game given by equation (4)

$$\mathcal{G}_{\text{diff}} = \langle \mathbb{N}, \{f_i\}, \{U_i\}, [0, T] \rangle \quad (4)$$

in which player i selects the control $u_i(\cdot)$ to minimize J_i subject to the dynamics (1) of all agents. A tuple of feedback strategies $u^* = (u_1^*, \dots, u_N^*)$, with $u_i^*(t) = \gamma_i^*(t, x(t))$, is a cooperative Nash equilibrium of $\mathcal{G}_{\text{diff}}$ relative to $\mathcal{G}$ if, for every $i \in \mathbb{N}$ and every admissible alternative strategy γ_i , given by equation (5)

$$J_i(\gamma_1^*, \dots, \gamma_i^*, \dots, \gamma_N^*) \leq J_i(\gamma_1^*, \dots, \gamma_i, \dots, \gamma_N^*) \quad (5)$$

and, in addition, each γ_i^* is required to be a function only of $(t, x_i, \{x_j\}_{j \in \mathcal{N}_i})$, i.e., admissible the strategies are limited to what is induced by the communication graph $\mathcal{G}$, in terms of the information structure. This restriction, which is universally adopted in graphical differential games, is what makes a two-player cooperative Nash equilibrium of the robotic network different from an unconstrained Nash equilibrium Guo et al., (2025) and Qin et al., (2025): It ensures that the two players' strategies are distributedly implementable. A variational characterization of u^* is then developed, and a constructive procedure is given for computing

3. Variational approach to the cooperative differential game

3.1 The extended hamiltonian

For each player i , introduce a costate (adjoint) trajectory $p_i(t) \in \mathbb{R}^n$ conjugate to $x_i(t)$, and define the player- i Hamiltonian equation (6)

$$H_i(x, u, p_i) = \ell_i(x, u_i) + p_i^\top f_i(x_i, u_i) \quad (6)$$

The calculus-of-variations treatment of the coupled optimization considers, for each i , the augmented functional obtained by adjoining the dynamic constraint (1) with $p_i(t)$ as a Lagrange multiplier function given by equation (7),

$$\mathcal{J}_i = x_i(T)^\top S_i x_i(T) + \int_0^T [H_i(x, u, p_i) - p_i^\top \dot{x}_i] dt \quad (7)$$

Taking the first variation of $\mathcal{J}_i$ with respect to $x_i(\cdot)$ and $u_i(\cdot)$ independently, and requiring stationarity for admissible variations δx_i vanishing at $t = 0$ (fixed initial state) and free at $t = T$, yields the Euler-Lagrange equations of the differential game given by equation (8):

$$\dot{p}_i = -\frac{\partial H_i}{\partial x_i}, \quad p_i(T) = 2S_i x_i(T), \quad 0 = \frac{\partial H_i}{\partial u_i} \quad (8)$$

Because ℓ_i also depends on x_j for $j \in \mathcal{N}_i$ through the coupling terms in (2), the same variational argument applied to x_j produces a cross-coupling contribution to the costate equation of every neighbor of i ; this is made explicit in the next subsection.

3.2 Necessary conditions via pontryagin's minimum principle

Because $R_i > 0$ and f_i is affine in u_i , H_i is strictly convex in u_i , and the stationarity condition in (8) is both necessary and sufficient for the pointwise minimization of H_i over u_i . Consequently, Pontryagin's minimum principle applies to each player, and the equilibrium control satisfies given as equation (9)

$$u_i^*(t) = \arg \min_{u_i} H_i(x^*(t), u_i(t), p_i(t)) = -\frac{1}{2} R_i^{-1} B_i^\top p_i(t). \quad (9)$$

Summing the contributions of ℓ_i and of every ℓ_k , k such that $i \in \mathcal{N}_k$, to the costate equation of agent i gives the coupled two-point boundary value problem (TPBVP)

$$\dot{x}_i = A_i x_i - \frac{1}{2} B_i R_i^{-1} B_i^\top p_i, \quad x_i(0) = x_i^0 \quad (10a)$$

$$\dot{p}_i = -A_i^\top p_i - 2Q_i(x_i - x_l - \delta_i) \cdot \mathbb{1}_{\{i \in \mathcal{B}\}} - 2 \sum_{j \in \mathcal{N}_i} w_{ij} Q_{ij}[(x_i - x_j) - \delta_{ij}] + 2 \sum_{k: i \in \mathcal{N}_k} w_{ki} Q_{ki}[(x_k - x_i) - \delta_{ki}], \quad p_i(T) = 2S_i x_i(T) \quad (10b)$$

One pair of equations (10a)–(10b) for each $i \in \mathbb{N}$ is a coupled TPBVP, and the solution to it will give the open-loop realization of the cooperative Nash equilibrium for a given initial condition. The coupling terms into $\dot{p}_i$ through neighbors k for which $i \in \mathcal{N}_k$ reflect precisely the graph-restricted information structure required in Section 2.3: agent i 's costate, and thus his optimal control, depends only on the graph-neighbor agents k and not on the network state as a whole.

3.3 Hamilton-Jacobi-Isaacs characterization of NASH equilibrium

Feedback, rather than open-loop, realization of the cooperative Nash equilibrium is obtained through dynamic programming. Define the value function of player i as equation (11)

$$V_i(t, x) = \min_{u_i(\cdot)} \left\{ \int_t^T \ell_i(x^*(s), u_i(s)) ds + x_i(T)^\top S_i x_i(T) : x^*(t) = x, u_{-i} = u_{-i}^* \right\} \quad (11)$$

where u_{-i}^* denotes the equilibrium strategies of all players other than i , held fixed. Standard dynamic-programming arguments show that, under the smoothness of V_i assumed throughout, V_i satisfies the Hamilton-Jacobi-Isaacs equation (12)

$$-\frac{\partial V_i}{\partial t} = \min_{u_i} \left\{ \ell_i(x, u_i) + \left(\frac{\partial V_i}{\partial x_i} \right)^\top f_i(x_i, u_i) + \sum_{j \neq i} \left(\frac{\partial V_i}{\partial x_j} \right)^\top f_j(x_j, v_j^*(t, x)) \right\}, \quad V_i(T, x) = x_i(T)^\top S_i x_i(T) \quad (12)$$

coupled across $i \in \mathbb{N}$ through the substitution of the other players' equilibrium feedback strategies v_j^* on the right-hand side. Equation (12) is the graphical-game analogue of the classical two-player HJI equation of Isaacs, specialized here to the N -player, nonzero-sum, graph-coupled setting.

Theorem 1

(Verification Theorem). Suppose, for each $i \in \mathbb{N}$, that a continuously differentiable function $W_i: [0, T] \times \mathbb{R}^{nN} \rightarrow \mathbb{R}$ satisfies the coupled HJI system (12) with terminal condition $W_i(T, x) = x_i^\top S_i x_i$, and that the minimizer on the right-hand side of (12) is attained at equation (13)

$$\gamma_i^\dagger(t, x) = -\frac{1}{2} R_i^{-1} B_i^\top \frac{\partial W_i}{\partial x_i}(t, x) \quad (13)$$

Then $V_i(t, x) = W_i(t, x)$ for all (t, x) , and $\gamma^\dagger = (\gamma_1^\dagger, \dots, \gamma_N^\dagger)$ is a cooperative Nash equilibrium of $\mathcal{G}_{\text{diff}}$ in the sense of equation (5).

Proof (sketch). Fix i and let $u_i(\cdot)$ be any admissible control for player i with the remaining players applying $\gamma_{-i}^\dagger$. Along the resulting trajectory $x(\cdot)$, the chain rule applied to $W_i(t, x(t))$ together with (12) evaluated with u_i , rather than at the minimizer, gives $\frac{d}{dt} W_i(t, x(t)) \geq -\ell_i(x(t), u_i(t))$, with equality if and only if $u_i(t) = \gamma_i^\dagger(t, x(t))$ for almost every t , since H_i is strictly convex in u_i . Integrating from t to T and using the terminal condition $W_i(T, x(T)) = x_i(T)^\top S_i x_i(T)$ yields $W_i(t, x) \leq \int_t^T \ell_i(x(s), u_i(s)) ds + x_i(T)^\top S_i x_i(T)$, with equality at $u_i = \gamma_i^\dagger$. Because this holds for every i simultaneously when all other players apply $\gamma_{-i}^\dagger$, $\gamma^\dagger$ satisfies the Nash inequality equation (5), and $W_i = V_i$ by definition of the value function.

3.4 Existence and uniqueness for the linear-quadratic game

For the linear-quadratic specialization $f_i(x_i, u_i) = A_i x_i + B_i u_i$, the coupled HJI system (12) admits a quadratic ansatz equation (14)

$$W_i(t, x) = \sum_{k \in \mathbb{N}} x_k^\top P_{ik}(t) x_k + (\text{cross terms}) + r_i(t) \quad (14)$$

where, restricting attention to the block-diagonal part relevant to the agent's own feedback gain, $P_{ii}(t) \triangleq P_i(t)$ solves the differential Riccati equation (15)

$$-\dot{P}_i = A_i^\top P_i + P_i A_i - P_i B_i R_i^{-1} B_i^\top P_i + \bar{Q}_i, \quad P_i(T) = S_i \quad (15)$$

with $\bar{Q}_i = Q_i \cdot \mathbb{1}_{\{i \in \mathcal{B}\}} + \sum_{j \in \mathcal{N}_i} w_{ij} Q_{ij}$ the aggregate local weighting matrix. If the neighbor states are considered as known exogenous signals that only appear in the affine (offset-tracking) part of V_i ; then the coupling among agents is not in the V_i rather than through P_i itself, and can be fully distributed synthesized as described in Section 4, using equation (15).

Theorem 2 (Existence and Uniqueness). Suppose (A_i, B_i) is stabilizable and $(A_i, \bar{Q}_i^{1/2})$ is detectable for every $i \in \mathbb{N}$, and suppose the coupling weights satisfy the graph-dependent bound as equation (16)

$$\max_{i \in \mathbb{N}} \sum_{j \in \mathcal{N}_i} w_{ij} \|Q_{ij}\| \cdot \|B_i R_i^{-1} B_i^\top\| < \frac{1}{4 \cdot \deg_{\max}(G)} \quad (16)$$

where $\deg_{\max}(G)$ is the maximum vertex degree of the communication graph. Then:

- Equation (15) admits a unique bounded solution $P_i(t) \geq 0$ on $[0, T]$ for every i ;
- The affine part of the coupled system obtained by substituting $P_i(t)$ is a linear system in the cross terms of Equation (14) with a strictly diagonally dominant coefficient matrix, and therefore admits a unique bounded solution on $[0, T]$; and
- The resulting feedback tuple $\gamma^\dagger$ of Theorem 1 is the unique cooperative Nash equilibrium of $\mathcal{G}_{\text{diff}}$ within the class of affine feedback strategies.

Proof (sketch). Part (a) follows from the standard existence-uniqueness theory for the differential Riccati equation under stabilizability and detectability of $(A_i, B_i, \bar{Q}_i^{1/2})$ (Forbes, 2017). For part (b), differentiating W_i in equation (14) and substituting into equation (12) shows that the cross-term coefficients solve a linear time-varying system whose coefficient matrix is the identity plus a matrix whose (i, j) block has norm bounded by $w_{ij} \|Q_{ij}\| \cdot \|B_i R_i^{-1} B_i^\top\|$ for $j \in \mathcal{N}_i$ and zero otherwise; condition (16) implies the coefficient matrix is strictly diagonally dominant on $[0, T]$, by a Gershgorin-disc argument, so it is invertible and the existence and uniqueness of a bounded solution follows from the variation of constants formula. Part (c) follows, because if there were two different feedback equilibria, then two solutions of the same linear cross-term system would satisfy the system, and they would have to be equal by the uniqueness of (b).

4. Distributed synthesis and stability analysis

4.1 Distributed feedback policy

Substituting the quadratic ansatz equation (14) into equation (13) shows that the equilibrium feedback strategy of agent i takes the explicit form equation (17)

$$\gamma_i^*(t, x) = -R_i^{-1} B_i^T \left[P_i(t) x_i - \sum_{j \in \mathcal{N}_i} w_{ij} Q_{ij} \delta_{ij} \cdot c_i(t) - Q_i \delta_i \cdot c_i(t) \cdot \mathbb{1}_{\{i \in \mathcal{F}\}} \right] \quad (17)$$

for an appropriate scalar time-weighting function $c_i(t)$ obtained from the affine part of equation (14), and depends on the network state x only through $x_i(t)$ itself and the desired offsets δ_{ij}, δ_i , which are known constants rather than measured neighbor states in the pure formation-tracking case. When Q_{ij} or Q_i additionally penalize instantaneous neighbor mismatch rather than only its constant offset, the affine term is replaced by a proportional-derivative combination of the form

$$\gamma_i^*(t, x) = -\sum_{j \in \mathcal{N}_i} \left[K_{ij}^p \left((x_i - x_j) - \delta_{ij} \right) + K_{ij}^d (\dot{x}_i - \dot{x}_j) \right] - \mathbb{1}_{\{i \in \mathcal{F}\}} \left[K_{il}^p \left((x_i - x_l) - \delta_i \right) + K_{il}^d (\dot{x}_i - \dot{x}_l) \right] \quad (18)$$

With gains $K_{ij}^p, K_{ij}^d, K_{il}^p, K_{il}^d$ obtained from $R_i^{-1} B_i^T$ and the corresponding blocks of $P_i(t)$ and Q_{ij}, Q_i . Equation (18), used in the numerical example of Section 5, makes explicit that the equilibrium strategy of every agent is computable from local measurements only.

Proposition 1

(Locality of the Equilibrium Feedback). Under the hypotheses of Theorem 2, the cooperative Nash equilibrium strategy γ_i^* of equation (17)– equation (18) is a function of $\left(t, x_i, \{x_j, \dot{x}_j\}_{j \in \mathcal{N}_i} \right)$ only, and is therefore admissible with respect to the information structure of Section 2.3.

Proof. This follows directly from the block structure established in the proof of Theorem 2: the coefficient matrix coupling the cross terms of W_i is nonzero only for $j \in \mathcal{N}_i$, so $\frac{\partial W_i}{\partial x_i}$, and hence γ_i^* through (13), depends on x_j only for $j \in \mathcal{N}_i$.

4.2 Consensus and formation convergence

Let $e_{ij} = (x_i - x_j) - \delta_{ij}$ for $(i, j) \in \mathcal{G}$ and $e_i = (x_i - x_l) - \delta_i$ for $i \in \mathcal{F}$ denote the formation-error coordinates, and let $e = (e_{ij}, e_i)$ collect all such errors. The desired formation manifold is $\mathcal{M} = \{x: e_{ij} = 0 \ \forall (i, j) \in \mathcal{G}, \ e_i = 0 \ \forall i \in \mathcal{F}\}$.

Theorem 3

(Asymptotic Stability of the Formation Manifold). Let γ^* be the cooperative Nash equilibrium strategy of Theorem 2 in the infinite-horizon ($T \rightarrow \infty$) limit, obtained from the algebraic counterparts of equation (15) with $\dot{P}_i = 0$, and suppose the leader-informed subgraph is such that every agent has a directed path from $\mathcal{F}$ in $\mathcal{G}$. Then the closed-loop network under (18) renders the formation manifold $\mathcal{M}$ globally asymptotically stable, and the aggregate cost $V(x) = \sum_{i \in \mathcal{N}} V_i(x)$ is a strict Lyapunov function for the error dynamics $\dot{e}$.

Proof (sketch). Along trajectories of the closed-loop network, differentiating $V = \sum_i V_i$ and using the algebraic form of equation (12) at the equilibrium gives $\dot{V}(x) = -\sum_i [\ell_i(x, \gamma_i^*)] \leq -\sum_i [e_{ij}^T \bar{Q}_{ij} e_{ij} + \gamma_i^{*T} R_i \gamma_i^*] \leq 0$, with $\dot{V} = 0$ if and only if $e = 0$, since $\bar{Q}_{ij} > 0$ on the edges of $\mathcal{G}$ and connectivity of the leader-informed subgraph, guaranteed by the standing assumption, rules out a nontrivial invariant set with $e \neq 0$ and $\dot{V} \equiv 0$ by a LaSalle argument applied to the connected graph Laplacian structure, in the spirit of Lin & Lin, (2026). Radial unboundedness of V follows from $P_i > 0$, which holds at the algebraic Riccati fixed point under the stabilizability and detectability hypotheses of Theorem 2. Global asymptotic stability of $\mathcal{M}$ then follows from the Lyapunov-LaSalle theorem.

4.3 Robustness under topology switching

In actuality, the communication graph of a mobile robotic network, $\mathcal{G}(t)$ evolves with time as mobile agents join and leave each other's sensing or communication range. $\mathcal{G}(t)$ switch among a finite set of graphs $\{\mathcal{G}_1, \dots, \mathcal{G}_M\}$ under the control of a piecewise-constant dwell-time-bounded switching signal, $\sigma(t)$.

Proposition 2

(Stability under Jointly Connected Switching). If each $\mathcal{G}_{\sigma(t)}$, for $\sigma \in \{1, \dots, M\}$, has a coupling bound as in (16) with the same set of weighting matrices, and the union of the graphs traversed over each interval of length τ_D (a uniform dwell bound) s connected with the leader reachable from each agent, then the common Lyapunov

function V of Theorem 3, computed using the topology-independent gains P_i obtained from the worst-case aggregate weight $\bar{Q}_i = \max_{\sigma} \bar{Q}_i^{(\sigma)}$, is non-increasing at switching instants, and the formation manifold M is globally asymptotically stable under arbitrary switching signals satisfying the dwell-time bound.

Proof. The switching-topology consensus results of Olfati-Saber and Murray are extended to the present coupled-cost, feedback-Nash setting: no jump in V will be induced at the switching times since V does not depend on the realization of $\mathcal{G}_{\sigma(t)}$ beyond the aggregate weights already dominated by the common gains P_i , already dominated by the common gains, and the same LaSalle argument from Theorem 3 applies to the union graph over each dwell interval.

5. Discussion

The simulated network also assumes the desired hexagonal shape, with only a mild transient near $t \approx 3\text{--}5$ s, caused by the interaction between the ring coupling among followers and the sparser subset of agents $\{R_1, R_3, R_5\}$; agents R_2, R_4, R_6 , not directly informed by the leader converge by the inter-agent edges only, as expected by Proposition 1 and the connectivity hypothesis of Theorem 3. The Lyapunov argument of Theorem 3 is not explicitly used in the simulation, but is corroborated by the monotone decay of the stage cost as the algebraic counterpart of the Riccati equation (15) is used to calculate the feedback gains. This numerical example is meant to illustrate the qualitative predictions of the theory and complete experimental validation with physical robotic hardware is left to further research as is a quantitative sensitivity analysis with respect to the coupling bound (16).

6. Conclusion

The present paper proposes a variational formulation in terms of a differential game theory for the cooperative control of autonomous robotic networks. The cooperative problems, including formation acquisition and leader tracking, have been considered by embedding them into quadratic cost functionals on a graph-based information structure for the problem of N -player differential games. By means of the variational method and Pontryagin's minimum principle, the necessary conditions for the existence of a cooperative Nash equilibrium have been derived and formulated in terms of a coupled Hamilton-Jacobi-Isaacs equation, for which a verification theorem has been proven. For the linear-quadratic problem, existence and uniqueness of the equilibrium have been proved under a graph-dependent coupling condition, leading to a fully distributed feedback controller through Riccati differential equations. The asymptotic stability of the target formation manifold was guaranteed by means of a Lyapunov function defined by a sum of the value functions of the agents extended to the setting of joint connectivity of the switching communication topologies, while the theoretical results were demonstrated numerically using a robotic network of six agents. As future work, will investigate the extension of the variational formulation for nonlinear agents, imposing state and input constraints on the problem using variational inequalities, weakening the condition of Theorem 2 numerically solving the coupled HJI system, and validating the distributed synthesis using robotic testbeds.

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