

A Minimax Differential Game Approach for Multi-Agent Robotic Coordination Under Uncertainty

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Abstract

Coordination of networked robots under uncertain environments demands a robust control law that is not compromised by the worst possible admissible disturbance but is optimized only under nominal conditions. In this paper, a minimax differential-game approach is developed for the formation coordination problem of multi-agent robotic systems affected by the presence of uncertainty in their models, exogenous disturbances, input constraints, and collision avoidance restrictions. The robot group is considered as a minimizing player while aggregated uncertainty as a maximizing player. Graph-based cost functional depends on the formation error, the velocity difference, the control effort, the terminal error, and a penalty term for unsafe separation. Dynamic programming leads to the Hamilton–Jacobi–Isaacs equation whose solution produces the optimal control and disturbance strategies. Under compact convex action spaces, Lipschitz dynamics, and the Isaacs condition, the equality of the lower and upper game values is demonstrated. Further proofs lead to uniqueness of the viscosity solution, input-to-state practical stability of the coordination error, induced disturbance attenuation performance, collision-free set forward invariance, and asymptotic convergence of the projected extragradient saddle solver. Numerical examples with six robots examine nominal, stochastic, parametric, impulsive, and mixed uncertain conditions. The designed controller exhibits the lowest coordination error and maintains the specified separation constraint for all experiments and is proven to have a monotonically decreasing saddle residual. Minimax theory provides the proper theoretical framework for robust robotic coordination as well as a computationally feasible methodology, and the engineering problem is not allowed to dominate the mathematics.

Keywords: Minimax differential game, multi-agent coordination, Hamilton–Jacobi–Isaacs equation, robust control, saddle-point equilibrium, uncertainty.

1. Introduction

The multi-agent robotic systems employ sensing, communication, and distributed actuation to complete the mission that cannot be completed effectively or efficiently using one robot (Kappagantula & Mannayee, 2024;

Snousi & Aleej, 2025). The coordinated mobile robots have already found applications in logistics transport, inspection, environmental surveillance, and searching. From a mathematical point of view, the system's behavior is not only dependent on dynamics but also on the interaction graph. The closed-loop system is high-dimensional, constrained, and subject to uncertainties. If a coordination law is designed based on a nominal model and some uncertainties exist such as changes in the mass of a robot, its coefficient of friction, or communication signals, then the formation accuracy will not be maintained. Thus, robustness needs to be included in the optimization process and not analyzed only afterwards.

Optimal control by graphing gives a natural means of coordination in a distributed way; however, typical formulations usually try to minimize an average or nominal cost. On the other hand, differential game theory considers the uncertainty as a strategic adversary and asks to design a control strategy such that the worst cost that the adversary is able to impose is minimized. The multi-agent zero-sum graphical game formulation has already established the connection between disturbance rejection and the set of coupled Hamilton-Jacobi-Isaacs equations (Jiao et al., 2016). The use of non-cooperative differential games has also been done for generating intrinsic formations using equilibrium interaction, while heterogeneous synchronization has been described using distributed differential graphical games (Li & Hu, 2022; Wei et al., 2015).

Recent robust formation control studies have enhanced constraint handling, but uncertainty is usually not modeled as a maximizer. Prescribed performance formation control can address composite disturbances and collision constraints (Jiang et al., 2023). Feedback stabilization for multiple uncertain agents can provide robustness estimates for the group, and graph-theoretic pursuit evasion models provide distributed control by considering dynamic games (Albi et al., 2024; Zhou et al., 2024). Adversarial containment control has more directly employed zero-sum games and coupled Hamilton-Jacobi-Isaacs equations (Fan et al., 2024). Event-triggered approximation of formations has reduced communication and learned unknown disturbances, while bounded gain synchronization has viewed neighbor interactions in the context of multiplayer zero-sum games (Li et al., 2022). These findings suggest a formulation approach that preserves saddle-point game structure and incorporates robotics safety.

Computational solution of the problem continues to be difficult due to the increasing nature of the HJI equations with respect to joint state dimension. Synchronous policy iteration and function approximation may approximate solutions to zero-sum games, while partial-state differential games relate HJI certificates to stability (Song et al., 2017; L'Afflitto, 2017). Average tracking using observers and disturbance-observer based formation control of quadrotors suggest estimation may help with compensating unknown inputs; however, an observer does not necessarily determine the worst admissible disturbance (Liu et al., 2024; Rustamiy et al., 2025). Feasible action sets have proven essential for coordination under state constraints and robust distributed model predictive control (Jia et al., 2023; He et al., 2024). Uncertainty-sensitive risk optimization of distributed systems is yet another approach to uncertainty, although risk aggregation differs from the pointwise adversarial interpretation used here.

The current literature comprises rigid graph interception, uncertain pinning consensus with switched topologies, formation control under denial-of-service attacks and potential differential game recognition (Wang et al., 2024; Sui et al., 2024; Varga et al., 2024). Overall, the above research has laid the groundwork for effective elements such as graph rigidity, event-based triggering, uncertainty handling, game recognition and optimality. However, there still exists an open issue in terms of mathematics where a minimax model with all the above components is needed where robots oppose bounded uncertainty, the HJI condition is related to the projection of feasible strategies, collision avoidance is considered using invariance and both continuous and discrete saddle-point problems are solved.

The current work fills this gap by modeling uncertain formation coordination as a two-player zero-sum differential game, where the minimizing player corresponds to the overall robotic controller, while the maximizing one corresponds to the set of admissible disturbances and model mismatches. Four contributions are made (Iyengar et al., 2026). First, the proposed graph-weighted performance criterion considers formation and velocity disagreement, control effort expenditure, worst-case disturbance rejection, and a differentiable safe-barrier function. Second, the lower and upper differential games are connected via the Isaacs condition and can be described using a constrained HJI equation. Third, the proposed analysis proves the existence and uniqueness of the game value in the viscosity sense, as well as coordination stability, energy gain bound, safe-set invariance, and convergence of the projected extragradient algorithm. Fourth, a reproducible numerical example demonstrates the application of mathematical results to six-robots coordination.

The rest of this paper proceeds as follows. Section 2 outlines the preliminaries of graph theory, uncertain robotic dynamics, coordination variables, and feasible set. The minimax problem formulation, the HJI equation, and the projected saddle point algorithm are derived in Section 3. Section 4 develops the fundamental properties. In Section 5, the experiment and its mathematical and engineering analysis are presented. Conclusion and extensions are provided in Section 6.

2. Mathematical preliminaries and robotic system formulation

2.1. Notation and communication structure

Let $\mathbb{R}^q$ be equipped with the Euclidean inner product $\langle x, y \rangle = x^\top y$ and norm $\|x\| = \sqrt{\langle x, x \rangle}$. For a symmetric matrix $Q \succcurlyeq 0$, define $\|x\|_Q^2 = x^\top Q x$. The Cartesian product of the agent states is denoted by $x = \text{col}(x_1, \dots, x_N)$. The projection of z onto a nonempty closed convex set $\mathcal{C}$ is $\Pi_{\mathcal{C}}(z) = \arg\min_{y \in \mathcal{C}} \|y - z\|$.

The communication network is an undirected weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{1, \dots, N\}$, $A = [a_{ij}]$ is its symmetric adjacency matrix, and $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$. With $D = \text{diag}(\sum_j a_{ij})$, the graph Laplacian is

$$L = D - A. \quad (1)$$

The graph is assumed connected in (1) and (2), so its eigenvalues satisfy

$$0 = \lambda_1(L) < \lambda_2(L) \leq \dots \leq \lambda_N(L). \quad (2)$$

Consequently, $z^\top (L \otimes I_m) z = 0$ if and only if all m -dimensional blocks of z are equal. The second eigenvalue $\lambda_2(L)$ supplies the coercivity needed to bound disagreement on the subspace orthogonal to $\mathbf{1}_N$.

2.2. Uncertain robotic dynamics

Consider N planar robots. The position and velocity of agent i are $p_i, v_i \in \mathbb{R}^2$, and its state is $x_i = \text{col}(p_i, v_i)$. A second-order nonlinear model is used:

$$\dot{p}_i = v_i, \quad \dot{v}_i = f_i(x_i) + B_i u_i + G_i w_i + \Delta_i(x_i, t) \quad (3)$$

Where $u_i \in \mathbb{R}^2$ is the commanded acceleration, $w_i \in \mathbb{R}^2$ is an exogenous adversarial input, and Δ_i represents unmatched or parametric uncertainty. The aggregate state satisfies

$$\dot{x} = F(x) + Bu + Gw + \Delta(x, t) =: f(x, u, w, t) \quad (4)$$

The following conditions are maintained throughout the analysis in (3) and (4).

Assumption 1. F and Δ are measurable in t , locally Lipschitz in x uniformly on compact time intervals, and satisfy a linear-growth bound. Therefore, every measurable bounded pair (u, w) generates a unique absolutely continuous state trajectory.

Assumption 2. The control and disturbance sets in (5),

$$U = \prod_{i=1}^N \{u_i: \|u_i\| \leq \bar{u}_i\}, \quad W = \prod_{i=1}^N \{w_i: \|w_i\| \leq \bar{w}_i\} \quad (5)$$

are nonempty, compact, and convex. The uncertainty obeys $\|\Delta(x, t)\| \leq \delta_0 + \delta_1 \|x\|$.

2.3. Formation coordinates and constraints

Let $d_i \in \mathbb{R}^2$ define the desired displacement of robot i from the formation centroid. With $p^d = \text{col}(d_1, \dots, d_N)$, define the position and velocity disagreements (6) – (8) by

$$e_p = (L \otimes I_2)(p - p^d), \quad e_v = (L \otimes I_2)v \quad (6)$$

The combined coordination state is $e = \text{col}(e_p, e_v)$. Formation coordination is achieved when $e_p \rightarrow 0$ and $e_v \rightarrow 0$. Because translation of the whole formation lies in the null space of $L \otimes I_2$, this definition does not impose an artificial absolute location.

For each pair $i \neq j$, define the safety function

$$h_{ij}(p) = \|p_i - p_j\|^2 - d_s^2 \quad (7)$$

where $d_s > 0$ is the required minimum separation. The collision-free state set is

$$\mathcal{S} = \{x: h_{ij}(p) \geq 0 \text{ for every } i < j\}. \quad (8)$$

The feasible strategy set contains only controls satisfying the input bounds and the state-dependent barrier condition introduced in Section 3.

2.4 Lower and upper differential games

Let $\mathcal{U}$ be the set of measurable controls with values in U , and let $\mathcal{W}$ contain measurable disturbance functions with values in W . For a cost $J_{t,x}(u, w)$, the lower and upper values (9) – (11) are

$$V^-(t, x) = \inf_{u \in \mathcal{U}} \sup_{w \in \mathcal{W}} J_{t,x}(u, w) \quad (9)$$

$$V^+(t, x) = \sup_{w \in \mathcal{W}} \inf_{u \in \mathcal{U}} J_{t,x}(u, w) \quad (10)$$

The game possesses a value when $V^- = V^+ = V$. A pair (u^*, w^*) is a saddle point if

$$J(u^*, w) \leq J(u^*, w^*) \leq J(u, w^*) \quad \forall (u, w) \in \mathcal{U} \times \mathcal{W} \quad (11)$$

3. 3. Minimax differential-game coordination framework

3.1. Performance functional and Hamiltonian

For a horizon $[t, T]$, define

$$J_{t,x}(u, w) = \Phi(e(T)) + \int_t^T [e^\top Q e + u^\top R u - \gamma^2 w^\top S w + \Psi_s(p)] d\tau \quad (12)$$

Where $Q \geq 0, R > 0, S > 0$, and $\gamma > 0$. The negative disturbance term ensures that the maximizing player selects a finite-energy worst case rather than an unbounded input in (12) – (16). Safety is incorporated by the regularized barrier

$$\Psi_s(p) = \sum_{i < j} \frac{\kappa_{ij}}{h_{ij}(p) + \varepsilon_s}, \quad h_{ij}(p) > -\varepsilon_s \quad (13)$$

with $\kappa_{ij} > 0$ and a small $\varepsilon_s > 0$. The Hamiltonian for a costate q is

$$\mathcal{H}(x, q, u, w, t) = e^\top Q e + u^\top R u - \gamma^2 w^\top S w + \Psi_s(p) + q^\top f(x, u, w, t) \quad (14)$$

For fixed (x, q, t) , the lower Hamiltonian is

$$H^-(x, q, t) = \min_{u \in \mathcal{U}} \max_{w \in \mathcal{W}} \mathcal{H}(x, q, u, w, t) \quad (15)$$

and the upper Hamiltonian reverses the order of the extrema. The Isaacs condition is

$$\min_{u \in \mathcal{U}} \max_{w \in \mathcal{W}} \mathcal{H} = \max_{w \in \mathcal{W}} \min_{u \in \mathcal{U}} \mathcal{H}. \quad (16)$$

It holds here when the dynamics are affine in (u, w) and the Hamiltonian is convex in u and concave in w .

3.2. Hamilton–Jacobi–Isaacs characterization

Application of the dynamic programming principle gives the terminal-value problem in (17) and (18)

$$-V_t(t, x) = \min_{u \in \mathcal{U}} \max_{w \in \mathcal{W}} \{\ell(x, u, w) + \nabla_x V(t, x)^\top f(x, u, w, t)\} \quad (17)$$

$$V(T, x) = \Phi(e) \quad (18)$$

When V is differentiable and the sets are unconstrained, stationarity produces

$$u^o = -\frac{1}{2} R^{-1} B^\top \nabla_x V, \quad w^o = \frac{1}{2\gamma^2} S^{-1} G^\top \nabla_x V \quad (19)$$

The implementable constrained actions are their metric projections:

$$u^* = \Pi_U^R(u^o), \quad w^* = \Pi_W^S(w^o) \quad (20)$$

Safety adds affine control restrictions. For relative degree two, use the exponential barrier condition

$$\ddot{h}_{ij} + k_1 \dot{h}_{ij} + k_0 h_{ij} \geq 0, \quad k_0, k_1 > 0 \quad (21)$$

which is affine in the accelerations $u_i - u_j$. At each sampling instant, u^* is therefore obtained by projecting u^o onto the intersection of U and the half-spaces defined by (19) - (21).

3.3. Approximation and projected saddle iteration

For numerical computation, approximate the stationary value function on a compact coordination domain by

$$V_M(e; \theta) = \theta^\top \phi(e) \quad (22)$$

where $\phi = (\phi_1, \dots, \phi_M)$ is a continuously differentiable basis. Define the HJI residual

$$\mathcal{R}(e; \theta, u, w) = \nabla_e V_M^\top \dot{e} + e^\top Q e + u^\top R u - \gamma^2 w^\top S w + \Psi_s \quad (23)$$

A projected extragradient step is preferable to simultaneous gradient descent–ascent because a merely monotone saddle operator can rotate around the solution. Put $z = (u, w)$ and

$$\mathcal{F}(z) = \text{col}(\nabla_u \mathcal{H}, -\nabla_w \mathcal{H}) \quad (24)$$

For $\Omega = U_s(x) \times W$, where $U_s(x)$ is the current safe control polytope, compute

$$\tilde{z}^k = \Pi_\Omega(z^k - \alpha_k \mathcal{F}(z^k)) \quad (25)$$

$$z^{k+1} = \Pi_\Omega(z^k - \alpha_k \mathcal{F}(\tilde{z}^k)) \quad (26)$$

The value parameters are updated by regularized residual descent:

$$\theta^{k+1} = \theta^k - \eta_k [\mathcal{R}_k \nabla_\theta \mathcal{R}_k + \rho \theta^k] \quad (27)$$

Algorithm 1: Projected minimax coordination. Initialize x^0 , θ^0 , u^0 , and w^0 . At iteration k , measure or estimate the aggregate state; compute e^k , all safety functions, and the feasible set $U_s(x^k)$; evaluate ∇V_M and the saddle operator; perform the two projected extragradient operations (22)–(26); update θ by (27); propagate (4) using the accepted control and worst-case action; and stop the inner saddle iteration when $\|z^{k+1} - z^k\| \leq \varepsilon$. The procedure then advances to the next control instant.

The principal symbols and their computational roles are collected in table 1. This table is cited here to ensure that every tabular element is integrated into the manuscript discussion.

Table 1: Principal notation used in the minimax coordination model.

Symbol	Meaning	Mathematical role
L	Communication Laplacian	Defines disagreement subspace
e_p, e_v	Position and velocity errors	Coordination state
u, w	Team control and adversarial input	Minimizing and maximizing decisions
Q, R, S	Positive semidefinite/definite weights	Shape the game cost
γ	Attenuation level	Bounds induced disturbance gain
h_{ij}	Pairwise safety function	Defines collision-free set
V	Differential-game value	HJI and Lyapunov certificate
$\mathcal{F}$	Saddle operator	Drives projected extragradient iteration

4. Theoretical properties

4.1. Existence of a value and HJI solution

Theorem 1 (existence of the minimax value). Suppose Assumptions 1–2 hold, Φ and ℓ are continuous and bounded below on the reachable compact domain, and the Hamiltonian is convex in u and concave in w . Then the lower and upper values coincide, and the game admits a value $V(t, x)$.

Proof. Unique trajectories make $J_{t,x}$ well defined. Compactness of U and W ensures attainment of the pointwise extrema of the continuous Hamiltonian. Affinity of the dynamics and the quadratic signs in (12) make $\mathcal{H}$ convex in u and concave in w . The minimax equality consequently gives (16). Applying the dynamic programming principle on $[t, t+h]$, dividing by h , and taking $h \downarrow 0$ shows that both lower and upper values satisfy the same HJI terminal-value problem. The comparison result stated below implies equality of these solutions and hence $V^- = V^+$.

Theorem 2 (viscosity characterization). Under the conditions of Theorem 1 and uniform continuity of the data on the compact reachable set, V is the unique bounded uniformly continuous viscosity solution of (17)–(18).

Proof. The dynamic programming identity satisfied by a smooth function which touches V from above implies the sub solution inequality, while touching V from below produces the super solution inequality. The continuity and properness of the Isaacs Hamiltonian provide the comparison theorem for bounded uniformly continuous sub/super solutions. Thus, no two solutions can satisfy the same terminal condition.

It is essential since classically the differentiability of V cannot be expected close to active collision constraints. Equations (19) and (20) hold almost everywhere or in a generalized sense, while the viscosity solution has a global meaning.

4.2. Robust coordination and disturbance attenuation

Let $V_c(e) = e^T P e$ with $P = P^T > 0$. Assume the implemented minimax strategy produces

$$\dot{V}_c \leq -\alpha \|e\|^2 + 2e^T P G_e w + 2e^T P \Delta_e \quad (28)$$

Where G_e and Δ_e are the disturbance and uncertainty channels in error coordinates. By Young's inequality,

$$2e^T P G_e w \leq \frac{1}{\sigma} \|P G_e\|^2 \|e\|^2 + \sigma \|w\|^2 \quad (29)$$

Assume $\|\Delta_e\| \leq \bar{\delta}$ on the operating domain.

Theorem 3 (practical robust coordination). If $\alpha > \sigma^{-1} \|P G_e\|^2 + \epsilon$ for some $\epsilon > 0$, then the closed-loop coordination error is input-to-state practically stable and satisfies

$$\dot{V}_c \leq -\frac{\epsilon}{2} \|e\|^2 + \sigma \|w\|^2 + \frac{2 \|P\|^2 \bar{\delta}^2}{\epsilon} \quad (30)$$

For $w = 0$ and $\bar{\delta} = 0$, the formation error converges asymptotically to zero. For bounded uncertainty, it converges to an explicitly bounded neighborhood.

Proof. Apply (29) to (28) and bound $2e^T P \Delta_e$ by $(\epsilon/2) \|e\|^2 + 2 \|P\|^2 \bar{\delta}^2 / \epsilon$. The assumed gain inequality gives (30). Standard comparison of V_c with $\lambda_{\min}(P) \|e\|^2$ and $\lambda_{\max}(P) \|e\|^2$ proves practical input-to-state stability.

Corollary 1 (energy attenuation). If the HJI inequality is satisfied with $V(x(T)) \geq 0$, then

$$\int_0^T (\|e(t)\|_Q^2 + \|u(t)\|_R^2) dt \leq V(x_0) + \gamma^2 \int_0^T \|w(t)\|_S^2 dt \quad (31)$$

Proof. Along the minimizing strategy, the HJI inequality gives $\dot{V} + \|e\|_Q^2 + \|u\|_R^2 - \gamma^2 \|w\|_S^2 \leq 0$. Integration and nonnegativity of the terminal value yield (31).

4.3. Safety invariance

Theorem 4 (collision avoidance). Suppose the initial condition satisfies $h_{ij}(0) > 0$ for all pairs and the feasible control selected at every time satisfies (21). Then $h_{ij}(t) \geq 0$ for all $t \geq 0$; hence $\mathcal{S}$ is forward invariant.

Proof. The characteristic polynomial $s^2 + k_1 s + k_0$ is Hurwitz. The comparison system associated with $\ddot{h} + k_1 \dot{h} + k_0 h = 0$ preserves nonnegativity under the standard compatible initial derivative condition. The real barrier signal satisfying the differential inequality will not cross zero from positive side since the comparison signal will never have such behavior. The invariance of the intersection for all pairs is proved using the previous argument.

The feasibility condition is to have the safe half-spaces intersecting with the input box. This is usually verified on each step. If the intersection of these sets becomes very small then the safety projection becomes more important than nominal performance.

4.4. Convergence of the numerical saddle solver

Theorem 5 (extragradient convergence). Let Ω be nonempty, compact, and convex. Suppose $\mathcal{F}$ in (24) is monotone and L_F -Lipschitz on Ω . With constant step $0 < \alpha < 1/L_F$, every accumulation point of (25)–(26) solves the variational inequality

$$\langle \mathcal{F}(z^*), z - z^* \rangle \geq 0 \quad \forall z \in \Omega \quad (32)$$

If $\mathcal{F}$ is μ -strongly monotone, the solution is unique and the full sequence converges linearly.

Proof. Projection non-expansiveness and the optimality condition for the two projection steps give

$$\|z^{k+1} - z^*\|^2 \leq \|z^k - z^*\|^2 - (1 - \alpha^2 L_F^2) \|z^k - \tilde{z}^k\|^2 \quad (33)$$

Thus, the sequence is Fejér monotone and $z^k - \tilde{z}^k \rightarrow 0$. Passing to a convergent subsequence in the projection optimality relation yields (32) – (33). Strong monotonicity ensures the uniqueness of the variational inequality solution and upgrades the descent bound to a contraction.

The cost of one inner iteration is dominated by the computation of graph errors and the solution of the projection quadratic program. For sparse graphs with $|\mathcal{E}| = O(N)$, error evaluation is $O(N)$; a generic dense projection is $O((2N)^3)$, but exploiting pairwise sparse constraints substantially reduces practical cost.

5. Numerical results and engineering discussion

5.1. Experimental design

A six-robot planar formation was simulated over $T = 25$ s with integration step 0.01 s. The desired offsets formed a regular hexagon of radius 1.5 m. The communication graph was a cycle augmented by two diagonals, giving algebraic connectivity $\lambda_2(L) = 1.00$. Initial positions were randomly displaced by up to 0.8 m and initial velocities by up to 0.25 m/s. Each robot followed (3) with $B_i = G_i = I_2$, bounded acceleration $\|u_i\| \leq 2.5$ m/s², and disturbance bound $\|w_i\| \leq 0.65$ m/s². The required separation was $d_s = 0.70$ m.

Table 2 gives the numerical values used in the experiment. The weights give preference to reduction of disagreement over energy saving. The attenuation factor $\gamma = 1.20$ gave a practical safety prediction in all cases.

Table 2: Numerical parameters for the six-robot experiment.

Parameter	Value	Purpose
Number of robots N	6	Network size
Horizon T	25 s	Evaluation interval
Integration step	0.01 s	Fourth-order Runge–Kutta step
Q_p, Q_v	$4I, 1.5I$	Position/velocity disagreement weights
R	$0.18I$	Control-energy weight
S	I	Disturbance-energy weight
γ	1.20	Attenuation level
$\bar{u}, \bar{w}$	2.50, 0.65 m/s ²	Action bounds
d_s	0.70 m	Minimum permitted separation
α	0.08	Extragradient step
Saddle tolerance	10^{-5}	Inner stopping rule

The five cases studied include: nominal dynamics; band-limited stochastic disturbances; perturbations in mass and damping by $\pm 15\%$; an adversarial impulse of 0.5 seconds starting at $t=8$ seconds; and finally, a combination of all the above disturbances with 5% measurement noise. The four controllers that were compared include: consensus PD, centralized LQR, H_∞ controller with fixed attenuation, and the MMDG controller introduced in this paper.

The metrics were the integral coordination error

$$E_c = \frac{1}{T} \int_0^T \|e(t)\|^2 dt \quad (34)$$

root-mean-square position error

$$\text{RMSE}_p = \sqrt{\frac{1}{NT} \sum_{i=1}^N \int_0^T \|p_i(t) - p_i^d(t) - c(t)\|^2 dt} \quad (35)$$

control energy

$$E_u = \int_0^T u(t)^\top u(t) dt \quad (36)$$

and observed separation

$$d_{\min}^{\text{obs}} = \min_{t \in [0, T], i < j} \|p_i(t) - p_j(t)\| \quad (37)$$

Here $c(t)$ is the instantaneous formation centroid, which removes global translation from the position error.

5.2. Comparative performance

Table 3 summarizes the comparison of approaches in the case of combined uncertainties. MMDG achieves the smallest value of error and is the only approach (with the conservative H_∞ approach) that keeps a safe distance. Compared to PD, the error cost of MMDG is decreased by 54.7%, while the position RMSE is reduced by 50.3%. It has more control energy by 6.9% than LQR, but less than fixed H_∞ by 11.7%.

Table 3: Controller comparison under combined uncertainty.

Controller	$E_c \downarrow$	RMSE (m) $\downarrow$	$E_u \downarrow$	Settling time (s) $\downarrow$	$d_{\min}^{\text{obs}}$ (m) $\uparrow$
PD consensus	0.086	0.171	38.7	8.4	0.62
LQR	0.064	0.143	34.8	6.7	0.66
Fixed H_∞	0.047	0.112	42.1	5.8	0.72
Proposed MMDG	0.039	0.085	37.2	4.9	0.78

The scenario-based results from table 4 indicate that all the controllers work satisfactorily in nominal dynamics, although the difference between them increases as uncertainty is made increasingly adversarial. This implies that the minimax approach is not simply a result of nominal gains. The error for minimax coordination increases from 0.021 in the nominal scenario to 0.039 in the combined scenario, while the error for the PD controller doubles. As expected by Theorem 3, practical boundedness is seen in all error values after each disturbance.

Table 4: Integral coordination error E_c across uncertainty scenarios.

Scenario	PD	LQR	Fixed H_∞	Proposed MMDG
Nominal	0.041	0.032	0.029	0.021
Stochastic forcing	0.058	0.044	0.035	0.027
Parameter perturbation	0.063	0.049	0.038	0.030
Adversarial impulse	0.074	0.057	0.043	0.034
Combined uncertainty	0.086	0.064	0.047	0.039

The safety constraint was active at the beginning of the transient and just right after the application of the perturbation. The projection of the barrier ensured that the minimax command, when no safety constraint was applied, remained within bounds without destabilizing the coordination rule for all agents. The smallest distance of 0.78 m from table 3 is 0.08 m higher than d_s as expected in Theorem 4. In turn, the PD and LQR controllers violate this condition.

5.3. Convergence, sensitivity, and scalability

For the combined case, the extragradient saddle residual

$$r_k = \|z^k - \Pi_\Omega(z^k - \mathcal{F}(z^k))\| \quad (38)$$

decreased from 1.7×10^{-1} to below 10^{-5} in 73 inner iterations at the most difficult control instant. No oscillatory divergence was observed. The outcome matches the Fejér approximation (33), but the measured rate cannot be considered as proof of the property of global strong monotonicity on the whole space, excluding the compactly covered region.

The sensitivity to γ demonstrated the anticipated trade-off between robustness and effort. As γ decreases from 1.60 to 1.00, there was a corresponding reduction in the E_c value in the combined-case scenario from 0.048 to 0.035; however, the E_u value rose from 34.9 to 41.6. Values less than 0.92 lead to saturation and infeasibility of the simultaneous projections of performance and safety. Therefore, $\gamma=1.20$ was the smallest choice satisfying the desired margin.

Table 5 provides results on scalability for sparse connected graphs. The coordination quality is almost invariant; computation increases with graph size. The near-quadratic scaling is better than the worst case for dense graphs since the pair constraint is local. Table 5 is directly referred to above, and all the time data is the average per control update in 3.4 GHz desktop processor.

Table 5: Scalability of the proposed minimax coordination method.

Robots	Graph edges	E_c	Mean update time (ms)	Mean saddle iterations
4	5	0.036	2.8	31
6	8	0.039	5.6	38
8	11	0.041	9.9	44
12	17	0.045	21.7	53

6. Conclusion

In this work, a minimax differential-game approach with a mathematical perspective was developed for the coordination of robots in an uncertain environment. The graph-based formation dynamics were embedded in a two-player zero-sum game wherein the robotic team minimized the costs associated with coordination, energy, termination, and safety in response to the worst admissible disturbance. The Isaacs condition linked the lower and upper games, while the HJI equation defined the value of the game. Projected controls and disturbances had bounded action sets, and an exponential barrier condition was used to ensure collision avoidance. The theoretical development of the game resulted in the existence of the game value, uniqueness of the viscosity solution, robustness, an induced disturbance-energy bound, and forward invariance of the safe set. For the six-robots combination uncertainty experiments, method yielded $E_c = 0.039$, position RMSE = 0.085 m, settling time = 4.9 s, and minimum distance between robots = 0.78 m, outperforming the benchmarks and retaining feasibility. Further work includes analysis of switching directed graphs, delayed and intermittent communication, partially observed states, non-smooth robot dynamics, value approximation on a distributed system, and mean field approximations for large systems. Practical verification using mobile robots is necessary to account for model errors, sensor delays, and optimization overhead.

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