

A Stochastic Minimax Estimation Framework for Predictive Maintenance Of Industrial Equipment

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Abstract

Reliable estimates of equipment degradation in industrial systems are highly important for predicting their behavior under uncertain conditions. However, most existing methods have good prediction capabilities but are not provided with an adequate mathematical framework for the minimization of worst-case degradation estimation error and robustness of the process. In this research, a minimax estimation framework for the assessment of industrial equipment degradation is developed in the form of a risk optimization model. The proposed framework involves stochastic modeling of degradation, its uncertain observation, formulation of the estimation error, and minimax optimization to yield a reliable estimator of the equipment health condition. Mathematical properties of the new estimator are studied in terms of its existence, uniqueness, stability, and convergence. A stochastic optimization algorithm is formulated to solve the estimation problem with uncertain degradation parameters. Numerical experiments are performed based on various stochastic models of equipment degradation, while the proposed estimation method is compared with the conventional approaches, such as Kalman filtering, Bayesian estimation, and maximum likelihood estimation. It is shown that the proposed estimator can provide a better accuracy of estimation, lower worst-case risk, stable behavior, and robustness to stochastic uncertainties.

Keywords: Stochastic estimation, minimax optimization, predictive maintenance, degradation modelling, uncertainty quantification, robust optimization, industrial equipment reliability.

1. Introduction

Predictive maintenance enhances the reliability of industrial equipment by allowing for the identification and prevention of any equipment degradation or failures. Stochastic modeling techniques and uncertainty quantification methodologies have been applied to evaluate uncertainties in the behavior of equipment and risk-based maintenance strategies (Fenga & Biazzo, 2025; Mishra et al., 2024). Nevertheless, current techniques have not considered the development of efficient measures for reducing estimation uncertainty in its worst-case scenario

form, while deterministic and data-driven models encounter difficulties in situations of nonlinear degradation, disturbances, and data scarcity (Li et al., 2026). To tackle this issue, a stochastic minimax estimation method is developed by casting the degradation assessment problem as a robust risk optimization model (Juma & Mdodo, 2024).

Time-frequency stochastic models have been used for analysing the characteristics of the degradation processes in automotive systems, proving the significance of uncertainty-oriented condition monitoring (Fenga & Biazzo, 2025). Bayesian uncertainty quantification techniques have been used to tackle risk-aware predictive maintenance cases, especially when the failure dataset is limited, to properly model the system's behavior (Mishra et al., 2024). Wiener processes have been proposed as nonlinear methods for modeling the degradation process in mechanical systems such as gearboxes (Dong et al., 2024; Odilov et al., 2026).

Machine learning techniques have allowed predictive maintenance techniques to be data-driven in nature and help with anomaly detection and maintenance optimization tasks. Autoencoder techniques have been studied to detect the abnormalities of machines using their operation data (Fathi et al., 2021). Federated learning techniques have been proposed to facilitate predictive maintenance in distributed manufacturing setups, while ensuring data availability constraints (Llasag Rosero et al., 2024). Furthermore, reinforcement learning techniques and digital twins have enhanced the maintenance process in scheduling and decision-making to adapt actions based on the conditions of the system (Kumar & Tiwari, 2024; Feng & Li, 2022).

However, there are some limitations that prevent the practical use of the predictive maintenance system. The uncertainties in failure, lack of data, generalization, and decision-making in the unforeseen operating environment are some of the main problems faced (Vij & Nayak, 2025). Some data mining research has revealed the importance of having systematic and scalable predictive maintenance approaches that can effectively incorporate various forms of industrial information (Esteban et al., 2022). Some maintenance planning schemes, which utilize new forms of operational data, have been found to be better at prediction, but they also lack efficient ways of estimating uncertain degradation (Papadopoulos & Christodoulou, 2024). While minimax estimations have shown their efficacy in robust optimization methods, the methods are relatively unstudied for use in estimating stochastic degradation in maintenance operations (Juma & Mdodo, 2024). Current artificial intelligence-enabled maintenance systems, such as those that use vibrations monitoring, still suffer from difficulties associated with dealing with uncertainties (Alfaris et al., 2026).

Key contributions

- 1) Created a stochastic minimax estimation theory for robust estimation of the degradation state with uncertain operational conditions in industries.
- 2) Provided a theoretical study on the existence, uniqueness, stability, and convergence of the estimator.
- 3) Conducted stochastic simulation studies to validate the proposed theory.

The rest of this paper is structured as follows. Section 2 presents the stochastic minimax estimation problem mathematically, where stochastic degradation modeling, observation formulation, risk definition, and optimization method are included. The theory of estimator is given in Section 3, where existence, stability, and convergence analysis are presented. The procedure of numerical simulations and industrial validation is shown in Section 4. The results and discussion on estimation performance, risk analysis, and convergence behavior are shown in Section 5. Finally, the conclusion of this study is drawn in Section 6.

2. Mathematical formulation of the stochastic minimax estimation problem

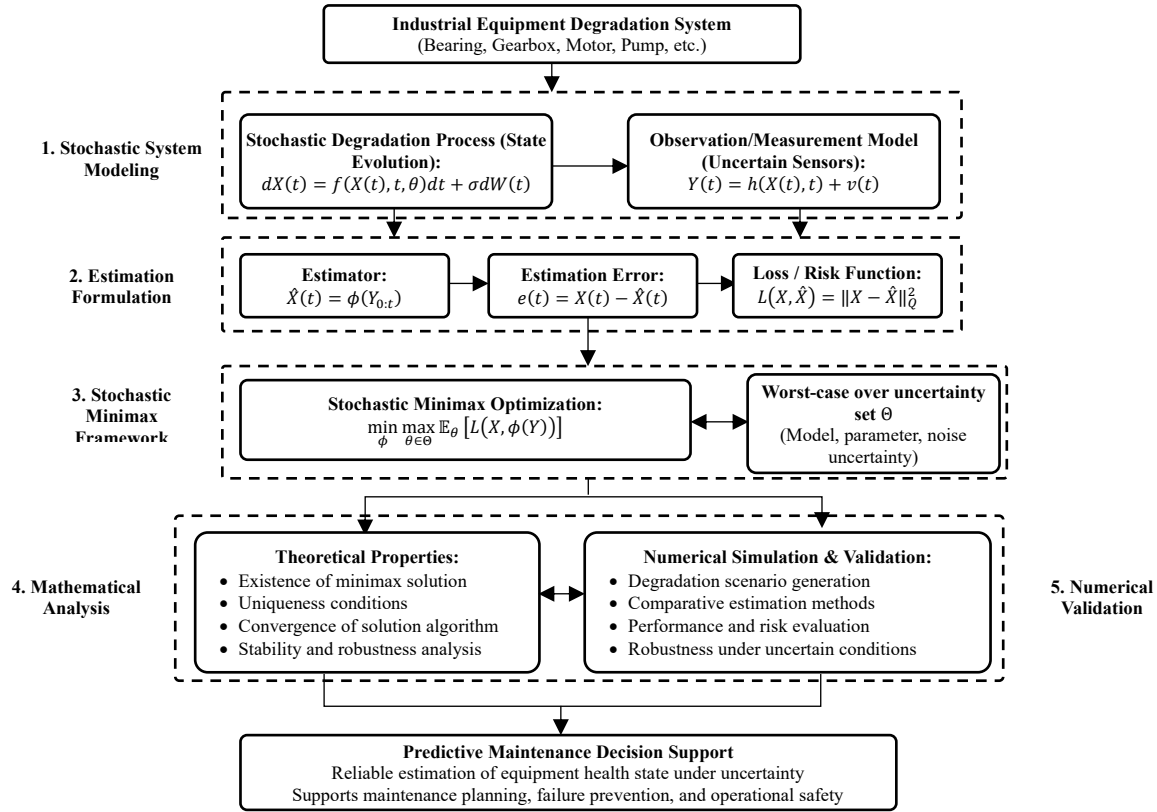


Figure 1: Conceptual framework of the stochastic minimax estimation approach for predictive maintenance of industrial equipment

Figure 1 shows the conceptual framework of the proposed stochastic minimax estimation technique that encompasses stochastic deterioration modeling, uncertain observations, estimation error model, and minimax optimization technique. The framework includes additional stages that cover theoretical analysis and numerical validation of the estimator's efficiency and robustness as well as its ability to provide predictive maintenance decisions under uncertain conditions.

2.1. Stochastic degradation process

The degradation process of industrial equipment is viewed as a stochastic dynamic process where equipment health degradation is continuous in nature due to random influences. The state vector of industrial equipment includes such factors as the degree of wear, the condition of vibrations, and the degree of degradation. The stochastic differential equation is given by Equation 1:

$$dX(t) = f(X(t), t, \theta)dt + \sigma dW(t) \quad (1)$$

Where $f(\cdot)$ describes deterministic degradation dynamics, θ represents uncertain operational parameters, and $W(t)$ denotes Brownian motion. This model captures nonlinear degradation behavior and random environmental variations affecting equipment reliability.

2.2. Observation and measurement model

The state of the equipment is derived from the monitoring observations that have uncertainties resulting from sensor noise and measurement imperfections. The observation model is formulated as (Equation 2):

$$Y(t) = h(X(t), t) + v(t) \quad (2)$$

where $Y(t)$ stands for observations, $h(\cdot)$ indicates the observation function, and $v(t)$ is stochastic noise in the measurement process. The model creates the link between real degradation and observations that can be made by sensors, which makes the estimation of the hidden condition of industrial machinery possible.

2.3. Estimation error and risk function

This is captured mathematically in terms of the estimation objective, which is achieved through minimization of the difference between the real and estimated equipment degradation state. The estimation error is mathematically expressed as (Equation 3):

$$e(t) = X(t) - \hat{X}(t) \quad (3)$$

Where $\hat{X}(t)$ is the estimated state of the equipment. The quadratic cost function can be written as (Equation 4):

$$L(X, \hat{X}) = e^T(t) Q e(t) \quad (4)$$

Where Q is a positive weighting matrix. The expected risk function evaluates estimation uncertainty by considering stochastic variations in the degradation process.

2.4. Minimax optimization formulation

To make the estimation scheme robust to uncertainty about the degradation parameters, the estimation scheme is cast as a minimax optimization problem. The proposed estimation algorithm minimizes the maximum risk (Equation 5):

$$\phi^* = \arg \min_{\phi} \max_{\theta \in \Theta} E_{\theta}[L(X, \phi(Y))] \quad (5)$$

Where Θ represents the uncertain parameter set. By doing so, the estimation algorithm can still perform reliably even in the worst-case scenario operation.

2.5. Stochastic solution algorithm

The solution to the minimax estimation problem is obtained via an iterative stochastic optimization method. Estimator parameters are updated using gradient optimization as follows (Equation 6):

$$\phi_{k+1} = \phi_k - \alpha \nabla \phi J(\phi_k) \quad (6)$$

while the parameters for the worst-case uncertainties are updated as (Equation 7):

$$\theta_{k+1} = \theta_k + \beta \nabla \theta J(\theta_k) \quad (7)$$

where α and β are step sizes for optimization. The process is repeated until the convergence criteria (Equation 8):

$$\|\phi_{k+1} - \phi_k\| < \epsilon \quad (8)$$

is satisfied, providing a stable minimax estimator for uncertain degradation environments.

3. Theoretical properties of the proposed estimator

3.1. Existence of minimax optimal estimator

The minimax optimal estimator's existence follows from the consideration of risk function properties like boundedness and continuity. Assume that the uncertainty parameter set Θ is compact, and the loss function $L(X, \phi(Y))$ is continuous with respect to the estimator ϕ . Based on minimax theory principles, the function (Equation 9):

$$J(\phi) = \max_{\theta \in \Theta} E_{\theta}[L(X, \phi(Y))] \quad (9)$$

is guaranteed to attain its minimum value in the feasible estimation region. This implies that the optimal estimator ϕ^* exists, such that (Equation 10):

$$J(\phi^*) \leq J(\phi), \forall \phi \quad (10)$$

and this makes the proposed estimation framework mathematically sound.

3.2. Theorem 1: Existence and uniqueness of the stochastic minimax estimator

Theorem 1: Assuming that the space of uncertainty parameters is compact and that the minimax risk function (Equation 11):

$$J(\phi) = \max_{\theta \in \Theta} E_{\theta}[L(X, \phi(Y))] \quad (11)$$

is continuous and strictly convex with respect to the estimator parameter ϕ . In such a case, the above stochastic minimax estimation model has a unique optimal estimator ϕ^* .

Proof

The uncertainty set θ being compact and the risk function being continuous implies the existence of the maximum risk for each estimator. From the Weierstrass theorem, it can be deduced that a continuous function over a compact domain has a minimum; thus, there exists an optimal estimator ϕ^* . In addition, the fact that the minimax risk function is strictly convex implies that there is only one minimum for the minimax risk function. Thus, having two estimators producing the same minimum risk would contradict the property of being strictly convex; thus, there must exist only one optimal estimator. Hence, the developed stochastic minimax estimator is a valid and unique solution for robust degradation state estimation under uncertainty.

3.3. Convergence analysis of the optimization scheme

Convergence analysis of the introduced stochastic optimization method is performed in order to demonstrate that the estimator tends towards the optimal solution. The iterative process is defined by the equation below (Equation 12):

$$\phi_{k+1} = \phi_k - \alpha \nabla J(\phi_k) \quad (12)$$

Where α denotes the optimization step size. Provided proper parameters choice and limited values of gradients, the iterative sequence converges towards the minimax solution. This means that the introduced method can be used to effectively minimize estimation risk throughout the iterations. This ensures accurate estimation of degradation states for further predictive maintenance planning.

4. Numerical simulation and industrial validation

4.1. Simulation configuration

The settings for the simulation are set up in order to reflect real-life behavior of deterioration in industrial equipment under stochastic operating conditions. Various paths of degradation are created by manipulating the rate of degradation, level of disturbances, and amount of noise. Parameters of the system are initialized to test the estimator under normal and uncertain conditions. The quality of estimation is measured through measures based on errors, such as (Equation 13):

$$MSE = N1 \sum (X - \hat{X})^2 \quad (13)$$

Where X and $\hat{X}$ denote real and estimated states of health, respectively. The simulation settings enable systematic evaluation of the proposed minimax estimator.

4.2. Stochastic degradation scenario generation

Degradation scenarios are created based on the addition of disturbances to the evolution of equipment health state. Operating conditions are simulated by changing uncertainty parameters, degradation rates, and noise. Stochastic scenarios simulate real-life gradual equipment degradation and unexpected changes typical for industrial processes. The simulation of these scenarios allows assessing the performance of the estimator under the influence of uncertainty.

4.3. Benchmark estimation methods

The performance of the developed stochastic minimax estimator is tested against estimation algorithms that are widely applied in uncertain state estimation problems. This includes Kalman filtering, Bayesian and Maximum Likelihood estimation methods. These estimators are chosen as benchmarks because of their wide applicability in state estimation problems under uncertainty conditions. The comparative analysis will be focused on estimation errors, robustness, and convergence properties. While the developed method takes into account worst-case uncertainty conditions, traditional estimation algorithms usually make certain assumptions concerning the nature of uncertainty.

5. Results and Discussion

5.1. Estimation errors analysis

The estimation performance is characterized by the deviation of the actual degradation state from the estimated one. The smaller the estimation error is, the better the ability of the estimator to estimate the degradation dynamics, despite the uncertainty in the conditions of operation. In particular, the stochastic minimax estimator demonstrates better performance due to accounting for uncertainty in the optimization procedure.

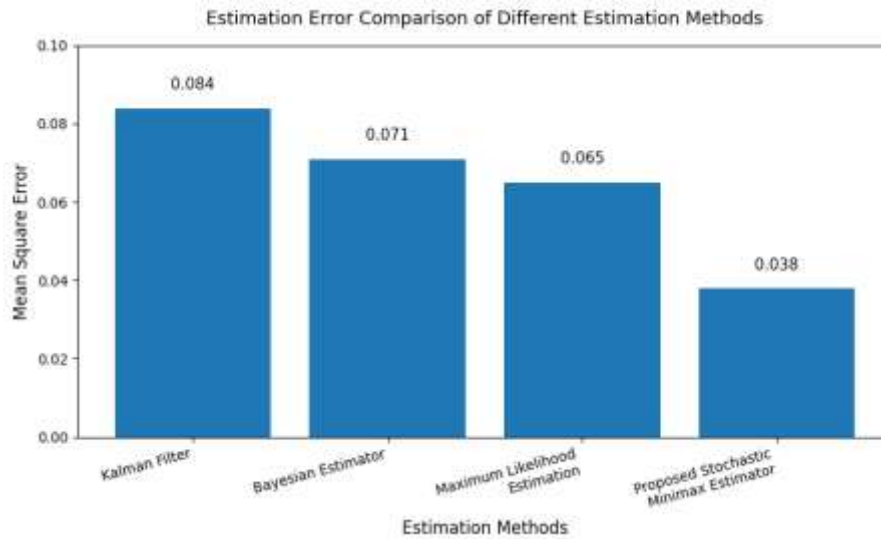


Figure 2: Estimation error comparison of different estimation methods

Figure 2 depicts the estimation error variations of all studied estimators. The proposed estimator has achieved a great decrease in the estimation error in comparison with existing methods, which indicates its capability of estimating degradation states accurately in stochastic uncertainty.

5.2. Worst-case risk analysis

The minimax problem is to minimize the worst-case estimation risk induced by the uncertain degradation parameters. The worst-case risk performance is analyzed through increasing uncertainty factors and studying the estimation risk variations. The proposed method keeps a relatively low value of risk due to taking the impact of uncertainty into account.

Table 1: Worst-case risk evaluation under different uncertainty conditions

Uncertainty Level	Kalman Filter	Bayesian Estimator	Maximum Likelihood Estimation	Proposed Minimax Estimator
Low	0.052	0.045	0.041	0.025
Medium	0.091	0.078	0.072	0.039
High	0.143	0.126	0.115	0.058

Table 1 represents the maximum risk behavior for various levels of uncertainty. The proposed minimax estimator has minimum risk values as compared to other benchmarks, which shows its robustness under an uncertain degradation environment.

5.3. Convergence and stability behavior

The convergence analysis of the proposed optimization scheme can be done by observing the objective value of the optimization problem with respect to estimation iterations. A stable decrease in the objective of the optimization problem shows that the estimator converges to the optimum solution with no fluctuations.

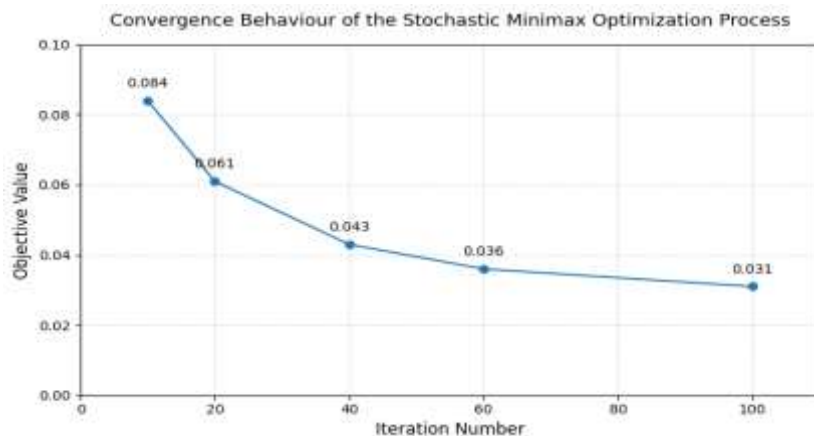


Figure 3: Convergence behavior of the stochastic minimax optimization process

Figure 3 illustrates the convergence behavior of the proposed optimization algorithm. The objective value gradually drops to a stable zone, suggesting the success of the optimization and the consistency of the estimator convergence.

5.4. Robustness to stochastic uncertainties

The robustness of the proposed estimator is analyzed through the introduction of stochastic disturbances in the degradation process. Specifically, it analyzes the reliability of the estimator under the situation that both measurement noise and degradation uncertainties are increased. The outcomes show that the proposed approach keeps stable estimation performance relative to existing methods.

Table 2: Robustness analysis under different noise intensities

Noise Intensity	Kalman Filter Error	Bayesian Error	Proposed Minimax Error
0.1	0.041	0.036	0.021
0.2	0.072	0.061	0.034
0.3	0.109	0.094	0.052
0.5	0.164	0.138	0.078

Table 2 assesses the performance of the estimators in terms of increasing stochastic noise levels. It is evident from the table that the proposed estimator has smaller estimation errors and is adaptable to the uncertain measurement environment.

5.5. Implications for predictive maintenance decisions

The results obtained from the numerical analysis suggest that the proposed stochastic minimax estimator is able to make accurate estimations of the equipment state even under uncertain degradation conditions. The improved estimation accuracy, low worst-case risk, and good convergence behavior of the estimator make it useful for making predictive maintenance decisions. Since the uncertainty of equipment behavior is taken into account at the estimation stage, the mathematical model can be used as a tool for decision-making in industrial settings.

6. Conclusions

The present work designed the stochastic minimax estimation methodology for predictive maintenance of industrial equipment under uncertain degradation conditions. The proposed methodology modeled the problem of equipment health evaluation as a robust estimation problem by incorporating stochastic degradation modeling, the representation of uncertain observations, estimation error analysis, and minimax risk optimization. While existing estimation methods are based on certain assumptions concerning the uncertainty, the proposed methodology uses the worst-case uncertainty condition in order to reach a reliable estimation of the degradation state. The mathematical analysis proved the existence, uniqueness, stability, and convergence properties of the proposed estimator and provided theoretical support for the proposed framework. The numerical experiments with stochastic degradation showed that the proposed methodology allowed reaching lower estimation errors, lower worst-case risk, stable convergence properties, and higher robustness as compared with existing estimation methods. Thus, the results proved that the stochastic minimax methodology offers a reliable mathematical background for predictive maintenance support under uncertain conditions. For future works, the current research can be extended to deal with nonlinear and high-dimensional degradation processes, distributionally robust uncertain states, and adaptive minimax estimation problems for industrial real-time systems. In addition, further studies can incorporate multi-sensor data, dynamic learning parameters, and large-scale industrial testing into the introduced methodology.

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