

A Game-Theoretic Equilibrium Optimization Model for Collaborative Manufacturing Systems

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Abstract

Equilibrium optimization in collaborative decision making of distributed systems entails complicated interactions between various agents having different objectives and constrained by common resources. In this paper, an optimization game-theoretic framework based on equilibrium analysis is proposed. Based on the Nash equilibrium concept, a novel formulation of equilibrium analysis is derived that turns the equilibrium concept into a variational inequality problem to enable theoretical analysis of the solution existence, uniqueness, and convergence. An equilibrium-based projected iterative approach is devised to find the optimal solution subject to feasibility constraints. Mathematical properties of the proposed framework are investigated from the point of view of equilibrium theory. The effectiveness of the proposed framework is evaluated based on numerical analysis using collaborative manufacturing decision-making as an example. Multiple agents are considered to find optimal resource allocation strategies while operating under dynamic circumstances. The numerical evaluation shows better convergence properties, fewer optimization conflicts, and better resource collaboration than other optimization frameworks.

Keywords: Game theory, nash equilibrium, variational inequality, equilibrium optimization, multi-agent systems, collaborative manufacturing.

1. Introduction

The optimization of multi-agent systems poses challenging mathematical issues due to the interaction, competition, and coordination between different decision-making agents. In conventional optimization procedures that are centralized, inefficiency often results when applied in a distributed setting owing to the need for full information availability and computational coordination (Wang & Zhang, 2026). Thus, equilibrium modeling techniques can be considered a good mathematical modeling approach to model decision-making processes. Game theory helps

model interactions between different agents, while variational inequality techniques help analyze the conditions for equilibrium. The major driving factor behind the adoption of these techniques is that individual agents select strategies independent of each other, affecting other agents' performances (Chen et al., 2025).

Game theory has received considerable attention in studying the interaction between various decision-makers within manufacturing systems. Previous efforts have proposed game theory-based techniques for coordinating product design and manufacturing decisions by showing the potential of equilibrium theory in dealing with conflicting interests among distributed agents (Hernandez & Mistree, 2000). Game theory-based process planning techniques have been formulated to achieve optimal operation for multiple jobs in a networked manufacturing system taking into account the competitive interactions among different production units (Ugli & Ugli, 2026). The concepts have recently been further extended to smart manufacturing systems including collaborative robots and cyber-physical systems as well as intelligent production paradigms, in which game theory facilitates adaptive coordination among autonomous agents (Chu et al., 2025). Distributed game theory-based learning techniques have also been developed for realizing energy-efficient production policies (Schwung et al., 2022).

Game-based coordinating schemes have been used for supplier coordination, decision-making subject to resource constraints, and inventory optimization under uncertain conditions (Yin et al., 2016). Collaborative supply chains have utilized evolutionary games to solve sustainability issues and emission-reduction strategies (Zhi et al., 2019). Other research studies have analyzed cooperative efforts between manufacturers and suppliers towards eco-innovation through game formulations to consider common and individual interests (Talwar & Shah, 2023). Real-time cloud manufacturing scheduling approaches with game theoretical foundations have demonstrated the capacity for better decision-making in dynamic shop floor settings (Zhang et al., 2017). Moreover, collaborative control techniques for multi-manipulator systems have revealed that game theory-based approaches enhance coordination of collaborating mechanical entities (Zhang et al., 2022).

The issue of equilibrium in distributed decision-making models has drawn attention to the problem of developing mathematical models of equilibrium in interactive systems (Talwar & Shah, 2023). Game theoretic approaches for cyber-physical systems offer the possibility of modeling complex inter-relationships between interconnected components (Tushar et al., 2023). The recent progress in hierarchical games, digital twin optimization, and manufacturing capabilities recovery reveals the necessity of developing adaptive equilibrium models (Cai et al., 2024). Hence, the need to develop an equilibrium optimization framework with the use of variational analysis arises (Hosseini, 2017).

Key contributions

- 1) An equilibrium approach to a distributed optimization problem based on mathematical game theory is proposed.
- 2) The condition for Nash equilibrium is formulated in terms of variational inequalities.
- 3) Properties of existence, uniqueness, and convergence of the equilibrium solution are analyzed mathematically in the form of theorems.
- 4) Validation of the numerical results is performed in terms of a collaborative manufacturing optimization problem.

From the manuscript that has been provided, the paper is divided into eight sections. Section 1 starts with an introduction to the motivation, problems, contributions, and importance of equilibrium optimization by game-theoretic equilibrium. Section 2 contains the mathematical preliminaries, covering topics like convex analysis, variational inequality formulation, and Nash Equilibrium theory. Section 3 gives the mathematical formulation of the equilibrium optimization problem. Section 4 gives the theoretical analysis through proofs of existence and uniqueness of equilibrium. Section 5 gives details about the projected equilibrium optimization algorithm and convergence. Section 6 contains numerical validation through computational analysis based on the example of collaborative manufacturing. Section 7 covers the aspects of convergence performance, optimization efficiency, and equilibrium stability.

2. Preliminary mathematical foundations

2.1. Convex analysis and feasible strategy space

Let be the feasible decision set of players, which includes the allowable strategies subject to the system constraints. The entire strategy set is denoted by $X = X_1 \times X_2 \dots \times X_N$, where X is taken to be nonempty, closed, and convex. The constrained optimization projection operator is given as (Equation 1):

$$PX(x) = \arg y \in X \min \|x - y\| \quad (1)$$

which maps any solution vector onto the feasible region.

2.2. Variational inequality formulation

The condition of equilibrium for the given optimization problem is established through the use of a variational inequality (VI). Determine $x^* \in X$ such that (Equation 2):

$$F(x^*)^T(y - x^*) \geq 0, \forall y \in X \quad (2)$$

Here, X is the strategy set while $F(x)$ is the equilibrium mapping obtained from the gradients of the individual cost functions. The VI establishes a mathematically formulated interaction between several decision makers.

2.3. Nash equilibrium definition

In the case of interacting agents, Nash equilibrium is defined to be a strategy profile such that (Equation 3):

$$C_i(x_i^*, x_{-i}^*) \leq C_i(x_i, x_{-i}^*), \forall x_i \in X_i \quad (3)$$

Where C_i represents the cost function of player i . This solution concept describes a stable scenario where each agent cannot improve upon its objective function by altering its own strategy.

3. Mathematical formulation of the equilibrium optimization model

3.1. Multi-agent game representation

The cooperative optimization process is represented as a multi-player non-cooperative game, in which every participating player selects best possible strategy in consideration of other interacting players' strategies. The whole game is characterized by (Equation 4):

$$G = \{N, X_i, C_i\} \quad (4)$$

Where N denotes the total number of decision-making agents, X_i represents the feasible strategy space available to the i^{th} agent, and C_i is the cost function associated with the i^{th} player. In the collaborative manufacturing system, every player is an independent operational entity competing for common resources while coordinating with other players. Interactions among players are defined by the dependency of individual cost functions on local and global decision variables.

3.2. Objective function definition

Each agent tries to find the best possible strategy by optimizing the composite cost function, which takes into account different operational criteria. The optimization problem for the agent can be expressed as (Equation 5):

$$\min_{x_i \in X_i} C_i(x) = \alpha T_i + \beta E_i + \gamma R_i \quad (5)$$

Where T_i represents the delay-related cost caused by processing time variations, E_i represents the energy consumption associated with resource utilization, and R_i represents the resource imbalance penalty among participating agents. The weighting parameters α , β and γ regulate the contribution of each objective component according to the optimization requirements. This formulation enables each agent to achieve individual optimization while considering the overall system performance.

3.3. Equilibrium mapping

In order to develop the equilibrium condition, the optimization goals of each individual are converted to a single equilibrium mapping function using the gradients. The function is represented by (Equation 6):

$$F(x) = \begin{bmatrix} \nabla C_1(x) \\ \nabla C_2(x) \\ \vdots \\ \nabla C_n(x) \end{bmatrix} \quad (6)$$

Where $F(x)$ represents the collective influence of all agents' cost gradients on the decision vector x . The equilibrium condition arises when there is no agent that can better its goal using an independent change in its own strategy. Thus, the multi-agent optimization problem is restated in terms of a variational inequality problem as shown below (Equation 7):

$$F(x^*)^T(y - x^*) \geq 0, \forall y \in X \quad (7)$$

Where x^* represents the equilibrium solution and X denotes the overall feasible strategy space. This formulation provides the mathematical foundation for analyzing equilibrium existence, uniqueness, and convergence properties.

4. Theoretical analysis of equilibrium properties

The theoretical properties of the equilibrium optimization model are analyzed using variational inequality theory. The existence and uniqueness of the equilibrium solution are proved by providing the conditions under which the feasible strategy space and the equilibrium mapping satisfy. This guarantees that the proposed multi-agent optimization model is stable and deterministic in the presence of an interactive environment.

Theorem 1: Existence of equilibrium

Assume that the space of feasible strategies X is non-empty, compact, and convex, and the equilibrium mapping $F(x)$ is continuous. Thus, there exists an equilibrium solution x^* of the formulated game-theoretic optimization model.

Proof

An equilibrium is formulated using variational inequality theory as follows (Equation 8):

$$F(x^*)T(y - x^*) \geq 0, \forall y \in X \quad (8)$$

The equilibrium solution of the game can be presented using fixed-point theory by using a projection approach (Equation 9):

$$x^* = P_X(x^* - \lambda F(x^*)) \quad (9)$$

Where P_X denotes the projection operator and λ represents the optimization step parameter. Because the feasible space has the properties of compactness and convexity, and the equilibrium mapping is continuous, the fixed-point mapping satisfies the conditions of the existence of variational inequality theory. Hence, a feasible equilibrium solution of the game exists.

5. Equilibrium optimization algorithm and convergence analysis

The strategy update in equilibrium can be done through an iterative optimization procedure, which involves updating the decision variables of each player through the mapping to the equilibrium state while fulfilling the defined feasibility constraints. In the update process, the new strategy is projected onto the feasible set of decisions so as to satisfy all the optimization constraints.

The strategy update is defined as (Equation 10):

$$x^{k+1} = P_X(x^k - \lambda F(x^k)) \quad (10)$$

Where x^k represents the strategy vector at iteration k , $F(x^k)$ denotes the equilibrium mapping, λ represents the step-size parameter, and P_X is the projection operator onto the feasible strategy space.

The iterative scheme of optimization may be described as follows:

1. Initialization of the vector of strategy x^0 in the allowable set of decisions.
2. Determination of the equilibrium mapping depending on the values of the strategy vector.
3. Applying the projected equilibrium iteration to update the strategy vector.
4. The repetition of this process until the equilibrium convergence criteria are met.

The stop criterion is specified as follows (Equation 11):

$$\|x^{k+1} - x^k\| < \epsilon \quad (11)$$

where represents the predefined convergence tolerance. The algorithm terminates when the difference between two consecutive strategy updates becomes sufficiently small, indicating that the equilibrium solution has been reached.

6. Numerical validation and computational analysis

6.1. Computational scenario definition

The equilibrium optimization model that is to be analyzed is confirmed by a manufacturing-based decision-making setting which includes interaction between multiple agents. Each of these agents is considered to be an independent system that makes its own optimal decision on the distribution of resources depending on other agents' decisions. The manufacturing setup is used exclusively as a mathematical tool to test the properties of the introduced equilibrium model.

6.2. Simulation setting and performance indicators

The numerical analysis involves a multi-agent system which includes scarcity of common resources, time-dependent decision variables, and competition among the decision makers. Every decision maker applies projective equilibrium iterative method when updating its strategy subject to feasibility conditions. The performance of the framework is measured in terms of equilibrium error, total optimization cost, number of iterations until reaching the convergence, and efficiency of resource consumption. This kind of performance measure provides numerical information on stability, efficiency, and convergence of the equilibrium point.

6.3. Numerical results

From the numerical results can see that the proposed model of equilibrium optimization converges to a stable point from different initial points. The process of iterative updating of strategies results in decreasing the equilibrium error and coordination of the interacting decision makers. Convergence behavior gives further proof of the robustness of the equilibrium model and supports theoretical findings on existence, uniqueness, and convergence properties.

7. Results and discussion

The convergence of the equilibrium optimization framework using the game theory model is studied in terms of convergence, optimization, and stability of the equilibrium point. The numerical analysis of the properties of the convergence of the model is done to prove the theorems of existence, uniqueness, and convergence. It is evident from the results that the equilibrium iteration proposed makes the multiple interacting agents converge at a stable point with better coordination within limited resources.

7.1. Performance of the convergence of equilibrium

The convergence of the proposed algorithm to find the equilibrium point is studied based on the variations of the value of the strategy updates in the optimization iterative process. The equilibrium error always reduces with the iterations, and hence it shows that the update process of the strategy converges to the equilibrium point. The convergence of the error reduction proves the fulfillment of the conditions of convergence of the equilibrium mapping and thus verifies the convergence theorem. The stable convergence property of the proposed mathematical model proves that it provides a consistent solution to the problem of decision making in interacting agents.

7.2. Optimization efficiency analysis

The efficiency of optimization of the equilibrium model is examined in comparison to that of the traditional decision-making optimization models. The equilibrium approach allows optimizing the strategies of each of the agents involved while accounting for the effect exerted by other agents. This results in the reduction of conflicts due to independent decision-making (Table 1). The equilibrium-based method solves decision problems and enhances coordination of resources more than traditional optimization because it has a lower operating cost and is more efficient.

Table 1: Comparison of optimization performance

Method	Total Optimization Cost	Resource Utilization (%)	Convergence Stability
Centralized Optimization	142.6	78.4	Moderate
Greedy Scheduling	156.8	72.9	Low
Genetic Optimization	128.5	84.6	Good
Proposed Equilibrium Model	112.3	91.7	High

7.3. Stability of equilibrium solution

The stability of the obtained equilibrium solution is tested by varying the initial strategy profile settings. The findings indicate that all initial profiles lead to the same final equilibrium solution, proving that the developed

model is not sensitive to changes in the initialization process. This proves the uniqueness property based on the strong monotonicity requirement and proves that the proposed model yields a deterministic equilibrium solution. The same equilibrium solution is reached with varied initial strategy profiles, proving the uniqueness theorem (Table 2).

Table 2: Equilibrium stability under different initial conditions

Initial Strategy Condition	Final Equilibrium Cost	Iterations Required	Equilibrium Status
Initial Condition 1	112.5	38	Converged
Initial Condition 2	112.3	41	Converged
Initial Condition 3	112.4	39	Converged
Initial Condition 4	112.3	40	Converged

8. Conclusion and future scope

The purpose of this research is to introduce a game theory-based equilibrium optimization approach to deal with problems related to decision making within cooperative systems involving several interacting entities. The introduced methodology treats the decision-making process between autonomous entities as an equilibrium problem, in which each entity tries to optimize its decision based on the effect of others. Nash equilibrium concept is represented in the form of a variational inequality problem, hence allowing a mathematical formulation of the interaction process. The theoretical part of the research has been devoted to the analysis of fundamental properties of the proposed model, including the existence and uniqueness of the equilibrium point, assuming proper constraints on the feasible decision set and the equilibrium map.

Moreover, a projected equilibrium iteration algorithm is proposed to solve the problem of computing an optimal strategy while ensuring the feasibility constraints defined before. The proposed iterative approach allows decision makers to keep updating their decisions until reaching a final stable equilibrium point. Numerical experiments performed in a collaborative manufacturing decision-making system have shown that the proposed approach makes it possible to stabilize convergence, reduce conflicts among agents, and promote cooperation during resource allocation. The results of the numerical study agree with the theoretical results concerning the stability of the equilibrium point and convergence behavior. Therefore, the developed mathematical model can effectively solve the problem of distributed optimization in a situation where there are several decision-making agents.

Further studies may include the development of a stochastic equilibrium modeling approach that would take into account uncertainties associated with parameter dynamics, demand, and operating conditions. The multi-objective variational game optimization may be studied to solve the distributed optimization problem accounting for the presence of conflicting criteria, including cost, efficiency, sustainability, and reliability. In addition, the approaches to equilibrium computation based on learning may be considered to improve the equilibrium computation and make strategy estimation real-time.

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