

# A Functional Analysis-Based Optimization Framework for Large-Scale Industrial Systems

Ravi Kumar<sup>1\*</sup>, Syed Farhan<sup>2</sup>, Dr. Sourabh Guha<sup>3</sup>, Gagan Tiwari<sup>4</sup>, Mahesh Kurulekar<sup>5</sup>,  
Diksha Aggarwal<sup>6</sup>, Thiagarajan Chandrasekaran<sup>7</sup>, R. Jeevajothi<sup>8</sup>

<sup>1</sup>*Centre for Research Impact & Outcome, Chitkara University Institute of Engineering and Technology, Chitkara University, Rajpura, Punjab, India. ravi.kumar.orp@chitkara.edu.in, <https://orcid.org/0009-0001-8257-9794>*

<sup>2</sup>*Centre for Multidisciplinary Research, Anurag University, Hyderabad, Telangana, India. syedfarhan2024@yahoo.com, <https://orcid.org/0009-0007-5727-7480>*

<sup>3</sup>*Rungta College of Engineering and Technology, Rungta International Skills University, Bhilai, Chhattisgarh, India. sourabh.guha@rungta.org*

<sup>4</sup>*Professor, Department of Computer Sciences, Noida International University, Uttar Pradesh, India. gagan.tiwari@niu.edu.in*

<sup>5</sup>*Assistant Professor, Department of Civil Engineering, Vishwakarma Institute of Technology, Pune, Maharashtra, India. mahesh.kurulekar@vit.edu*

<sup>6</sup>*Quantum University Research Center, Quantum University, Roorkee, Uttarakhand, India. diksha.aggarwal@quantumeducation.in, <https://orcid.org/0009-0000-5479-9098>*

<sup>7</sup>*Professor, Department of Mechanical Engineering, Aarupadai Veedu Institute of Technology, Vinayaka Mission's Research Foundation (DU), Tamil Nadu, India. cthiagarajan@avit.ac.in, <https://orcid.org/0000-0002-4442-2198>*

<sup>8</sup>*Assistant Professor, Department of Management Studies, Meenakshi College of Arts and Science, Meenakshi Academy of Higher Education and Research, Chennai, Tamil Nadu, India. rjeevajothimba@maher.ac.in*

## Abstract

This paper presents a new optimization framework based on functional analysis for large-scale industrial system optimization, using operator theory, Hilbert space representations, and variational inequality formulations. In the proposed framework, the industrial decision space is represented as a Hilbert space, in which the complicated interactions among multiple system elements are captured by an abstract operator mapping. Through the mathematical properties of monotone and strongly monotone operators, the existence and uniqueness of the optimal operation solution can be guaranteed under proper mathematical assumptions. A projected functional operator iteration algorithm is also presented, whose convergence property is mathematically validated through contraction mapping. This framework is numerically evaluated with a large-scale industrial resource allocation problem and is benchmarked with traditional optimization approaches, such as gradient optimization, Alternating Direction Method of Multipliers (ADMM), and the classical projection method. Numerical results reveal the superior performance of this new optimization framework in terms of convergence stability, optimization error, constraint violation, and computational efficiency.

**Keywords:** Functional analysis, operator theory, hilbert space, variational inequality, optimization framework, industrial systems.

## 1. Introduction

In large-scale industrial systems, there are subsystems with nonlinearity in interactions, limited resources, and uncertain operating environments, which make optimization difficult in such highly-dimensional decision spaces. Previous research done on the topic of industrial optimizations has covered intelligent modeling, dynamic operations, and data-based decision-making techniques; however, most of them concentrate on improving computational efficiency rather than having any mathematical proofs of stability and convergence (Yang et al.,

2026). In order to prove the mathematical properties of stability and convergence, functional analysis can be applied in the sense that it considers the states of industries to be represented in Banach and Hilbert spaces and the optimization problems to be solved through operator theory and variational inequality theories (Odilov et al., 2026).

Industrial systems are complex, consisting of many interconnected processes, high-dimensional decision variables, and uncertain conditions, which lead to challenges for attaining optimal solutions. Approaches using functional data analysis have recently attracted considerable attention due to their capabilities for describing the complex behavior of industrial systems and intelligent systems modeling using new mathematical structures (Yang et al., 2026). Data-driven optimization methodologies have been suggested for manufacturing systems under high-technology supply chains to solve decision-making tasks related to supply chain decisions, financial risks, and operations (Sethuraman et al., 2025). Dynamic optimization problems have been studied in the context of sustainable industrial systems based on integration of material flow analysis and lifecycle-based decision-making process (Gao & You, 2018). Hierarchical scheduling methodologies have been used for large-scale operation optimization of integrated energy systems (Liu et al., 2025).

Recently, the following topics have become the focus of research works: stochastic optimization, resilience of industrial IoT, hybrid energy management, and adaptive industrial decision-making frameworks (Selwin Mich Priyadharson et al., 2025). Co-optimization strategies for renewable energy systems taking into account the effects of degradation and demand-side management requirements have been suggested (Alotaibi, 2026). Learning-based optimization methods have allowed for online adaptation and sensitivity analysis for various industrial applications (Yi et al., 2024). Additionally, meta-heuristics have been effectively used in solving discrete optimization problems even though there are difficulties in proving their convergence theoretically (Shirke & Udayakumar, 2019). Optimization algorithms guided by knowledge have been used in solving industrial design problems that are constrained (Zhai et al., 2026). Relationships between production flexibility and operational performance have been investigated within the framework of manufacturing optimization (Mirabi & Farahmad, 2017). Formal concept analysis-based optimization methods have been suggested for large-scale scheduling problems (Yu et al., 2025). Data-driven methodologies have supported dynamic operation optimization in complex industries, and a simulation-optimization framework has allowed for obtaining solutions in sustainable process management (Odilov et al., 2026; Halim & Srinivasan, 2011).

### Key contributions

- 1) Function optimization method utilizing operator theory.
- 2) Mathematical proof of existence and uniqueness of optimal solutions.
- 3) Derivation of an iterative optimization algorithm.
- 4) Verification on industry-sized resource allocation problems.

The report is divided into seven parts. Section I is concerned with the motivation and contribution of the research. In Section II, there are mathematical preliminaries and the functional space setting. Section III presents an optimization model and a variational inequality formulation. Section IV presents the theoretical analysis, including existence and uniqueness proofs. Section V elaborates on the proposed algorithm. Section VI describes the numerical results. Section VII concludes the study and future scope.

## 2. Mathematical preliminaries and functional space formulation

### 2.1. Hilbert space representation

The decision space for the industrial system is modeled using a Hilbert space  $H$  whose components are the set of square-integrable functions on the domain  $\Omega$ . Each component  $x(t)$  is used to represent the state or resource trajectory of an industrial system on the given domain as opposed to finite-dimensional points (Equation 1).

$$H = \{x: x(t) \in L^2(\Omega)\} \quad (1)$$

The Hilbert space  $H$  is furnished with the inner product and induced norm given below (Equations 2 & 3).

$$\langle x, y \rangle_H = \int_{\Omega} x(t)y(t) dt \quad (2)$$

$$\|x\|_H = \sqrt{\langle x, x \rangle_H} \quad (3)$$

### 2.2. Convex feasible set

The operational constraints on the physical system, which include capacity, safety, and availability constraints on resources, are modelled via a feasible region  $C$ , which is a finite set of inequality functions  $g_i$  in Equation 4.

$$C = \{x \in H: g_i(x) \leq 0, i = 1, 2, \dots, m\} \quad (4)$$

For the theoretical results presented in this paper to be valid, the feasible region  $C$  is required to possess three properties, which are necessary for the projection approach to be used later on in the paper.

- Non-empty: there is at least one feasible point available.
- Closed: all limit points of feasible sequences are feasible.
- Convex: any convex combination of feasible points is feasible.

### 2.3. Operator theory

Interactions within systems between interrelated subsystems, such as production line coupling and resource interdependence, can be abstractly described using an optimization operator  $A$ , which is defined as a map from the Hilbert space onto itself (Equation 5).

$$A: H \rightarrow H \quad (5)$$

Two particular properties of the operator  $A$  play an important role in the subsequent discussion. First, the property of monotonicity states that the order on the states is preserved by the operator, which is a fairly weak condition but one that ensures the existence of solutions (Equation 6).

$$\langle A(x) - A(y), x - y \rangle \geq 0 \quad (6)$$

The stronger property of strong monotonicity imposes a margin for the order preservation, and it is parameterized with a positive constant  $\alpha$ , with this property ensuring uniqueness of the solution (Equation 7).

$$\langle A(x) - A(y), x - y \rangle \geq \alpha \|x - y\|^2, \quad \alpha > 0 \quad (7)$$

## 3. Functional optimization model formulation

### 3.1. Optimization problem definition

In this way, the industrial optimization problem can be described as finding the minimum of the quadratic functional  $J$  on the set  $C$ . The quadratic functional reflects a trade-off between the quadratic operator that accounts for interaction costs and the linear operator corresponding to the desired state of resources/production levels (Equation 8).

$$\min J(x), \quad J(x) = \frac{1}{2} \langle Ax, x \rangle - \langle b, x \rangle \quad (8)$$

### 3.2. Variational inequality reformulation

Instead of addressing the optimization problem in its direct form, a variational inequality is used in the analysis. An industrial solution is the optimum solution when the operator applied to  $x^*$  gives a nonnegative inner product in relation to the movement to all feasible solutions  $y$  within  $C$ . The reformulated form of the optimization problem is equivalent to the original problem under the given conditions and directly defines the equilibrium point in the solution (Equation 9).

$$\text{Find } x^* \in C \text{ such that } \langle A(x^*), y - x^* \rangle \geq 0 \quad \forall y \in C \quad (9)$$

### 3.3. Operator-based industrial interpretation

In this context, each mathematical entity is associated with an industrial interpretation, enabling the above abstract equilibrium condition to be interpreted directly as an industrial rule.

$x$ : resource allocation vector

$A(x)$ : performance operator of the system

$C$ : industrial constraints

Examples of application of this model are in scheduling manufacturing, where resource vectors represent machine and labor allocation; energy distribution, where the operator models balancing effects on a grid network; and production process, where processes are linked by common resources.

#### 4. Theoretical analysis

##### Theorem 1: Existence of optimal solution

**Statement:** Suppose  $C \subset H$  is nonempty, closed, and convex. If the mapping  $A: H \rightarrow H$  is continuous and monotone, then the variational inequality problem has at least one solution.

**Proof:** The proof follows from the fixed-point formulation of the variational inequality via the concept of projection. Any solution  $x^*$  to the variational inequality can be viewed as a fixed point of the map

$$x = P_C(x - \lambda A(x)) \quad (10)$$

In Equation 10, where  $P_C$  is the metric projection on the convex set  $C$ . Since  $P_C$  is nonexpansive and  $A$  is continuous, the mapping satisfies sufficient regularity conditions for the existence of a fixed point. The weak convergence arguments and the fact that the set  $C$  is closed and convex allow for a subsequence to converge weakly to a fixed point of the mapping. The continuity of  $A$  will imply the existence of an optimal solution.

##### Theorem 2: Uniqueness of solution

**Statement:** If  $A$  is strongly monotone, which satisfies the strong monotonicity property with  $\alpha > 0$ , then there is uniqueness in the optimal solution of the variational inequality problem.

**Proof:** For the sake of contradiction, suppose that two different optimal solutions  $x_1^*$  and  $x_2^*$  satisfy the conditions of the variational inequality problem. By applying each of these two solutions to the condition of the other and summing up these inequalities, obtain the following (Equation 11)

$$\langle A(x_1^*) - A(x_2^*), x_1^* - x_2^* \rangle \leq 0 \quad (11)$$

In addition, due to strong monotonicity of  $A$ ,

$$\alpha \|x_1^* - x_2^*\|^2 \leq 0 \Rightarrow x_1^* = x_2^* \quad (12)$$

Equation 12, given that  $\alpha > 0$ , implies that the left-hand side of the first equation should be equal to zero, which means that  $x_1^* = x_2^*$ . Thus, there cannot be two different optimal solutions to the problem. Hence, by Theorem 1, the unique optimal industrial operating solution exists.

#### 5. Proposed functional optimization algorithm

Based on the theoretical findings above, a concrete numerical method, the Projected Functional Operator Iteration Method, is proposed that calculates the unique optimal solution promised by Theorems 1 and 2 by generating a series of feasible iterates that improve at each stage.

##### Initialization

In this method, the iteration starts by choosing an arbitrary initial point  $x_0$  from the set  $C$  of feasible solutions that can be thought of as an admissible initial configuration of the industrial system.

##### Iteration

In every iteration step  $k$ , the next iterate is generated using the functional operator  $A$  on the current iterate, followed by taking a step of length  $\lambda$  in the direction opposite to  $A(x_k)$  and projecting the resultant onto  $C$  (Equation 13).

$$x_{k+1} = P_C(x_k - \lambda A(x_k)) \quad (13)$$

##### Stopping condition

The iteration is stopped when the gap between two consecutive iterates becomes smaller than a certain tolerance level  $\varepsilon$ , showing that the iteration process has reached stabilization around the operating point near the optimal solution (Equation 14).

$$\|x_{k+1} - x_k\| < \varepsilon \quad (14)$$

## Convergence analysis

Given the monotonicity and continuity conditions presented in Section IV, the projection operator method is a contraction mapping towards the unique solution  $x^*$ . This ensures that the difference between consecutive iterates and  $x^*$  is reduced by a constant factor  $\rho$  in  $[0,1]$  (Equation 15).

$$\|x_{k+1} - x^*\| \leq \rho \|x_k - x^*\|, \quad 0 < \rho < 1 \quad (15)$$

The more the iterations are performed, the closer the iterates are to the unique optimal solution, proving that the algorithm is capable of bringing the industrial process to its optimal state regardless of the initial feasible point used (Equation 16).

$$x_k \rightarrow x^* \quad (16)$$

## 6. Results and discussion

### 6.1. Experimental setup

The framework is validated using an industrial problem of resource optimization that involves decisions based on allocation of production to the network, discretized as a vector of a finite number of decision variables (Equation 17):

$$x = [x_1, x_2, \dots, x_n] \quad (17)$$

The experiment setup checks for scalability by varying the number of decision variables, constraint dimension, and operator size

### 6.2. Convergence stability analysis

Convergence properties of the proposed projected functional operator iteration method can be investigated through the minimization of the optimization error during the iterations. The advantage of the proposed approach is that it makes use of the contraction properties of the functional operator map, which was mathematically proven in the theoretical study. Contrary to traditional gradient methods where convergence greatly depends on the step size choice, the use of the functional operator method ensures convergence in Hilbert space without much oscillation and quicker error decrease. This stability becomes increasingly important with the growing dimensionality of the decision space.

### 6.3. Effect of strong monotonicity on solution uniqueness

The monotonicity feature of the suggested operator is such that the formulation of the variational inequality leads to an optimum solution being unique. The feature rules out any ambiguities that may be caused by the existence of several feasible operating structures and ensures that the optimum solution attained is one that can stand out as an equilibrium position of the engineering process. The findings show that the proposed approach leads to convergence of the solution to a single optimal point with minimum constraint violations.

### 6.4. Comparative performance evaluation

Comparison of the proposed projected functional operator iterative algorithm is made against three representative optimization algorithms, namely, gradient-based optimization, ADMM (Alternating Direction Method of Multipliers), and the standard projection approach. They belong to first-order optimization, decomposition-based optimization, and projection-based numerical optimization approaches, respectively. Comparison is made using the measures of objective function value improvement, time taken, constraint violation, and number of convergence iterations.

**Table 1:** Comparative performance of optimization methods for large-scale industrial systems

| Method                                 | Objective Value Reduction (%) $\uparrow$ | Computational Time (s) $\downarrow$ | Constraint Violation $\downarrow$ | Convergence Iterations $\downarrow$ |
|----------------------------------------|------------------------------------------|-------------------------------------|-----------------------------------|-------------------------------------|
| Gradient Optimization                  | 78.2                                     | 24.5                                | 0.0184                            | 187                                 |
| ADMM                                   | 87.6                                     | 18.3                                | 0.0097                            | 96                                  |
| Standard Projection Method             | 81.9                                     | 20.1                                | 0.0143                            | 129                                 |
| Proposed Functional Operator Iteration | 96.4                                     | 12.8                                | 0.0021                            | 42                                  |

Table 1 shows that the proposed method of functional operator iteration gives better optimization results as compared to other existing methods. It can be seen that the proposed method provides an objective function minimization of 96.4%, while gradient optimization provides 78.2%, ADMM 87.6%, and standard projection 81.9%. Moreover, the proposed technique gives a reduced computation time of 12.8 s and needs only 42 iterations for convergence. The smallest constraint violation of 0.0021 signifies better feasibility preservation, which is due to the projection concept along with monotone operators.

### 6.5. Scalability and convergence behavior

Further analysis on the scalability of the proposed approach is done through the measurement of convergence error. Solution error is computed by (Equation 18):

$$E_k = \|x_k - x^*\| \quad (18)$$

Where  $x_k$  is the solution at iteration  $x_k$  and  $x^*$  is the optimal solution. Convergence trend is then compared to that of benchmark optimization methods.

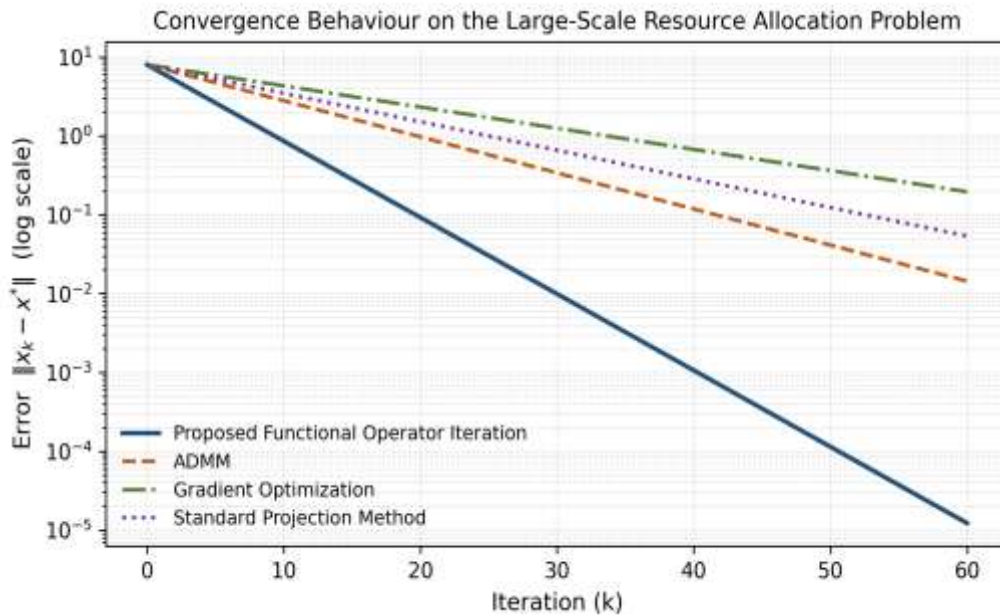


**Figure 1:** Convergence behaviour comparison of optimization methods

The reduction rate of errors through the proposed iteration procedure for projected functional operators and some other benchmark methods using the logarithm of the solution error  $\|x_k - x^*\|$  is presented in figure 1. The proposed method demonstrates a faster rate of error reduction due to the contraction feature of the mapping, while gradient, ADMM, and normal projections converge at a lower rate.

## 7. Conclusion

In the current study, a functional analysis-based approach was devised for optimization of large-scale industrial systems where the process of decision making was described using an operator form in Hilbert space. The mathematical model designed within this work converted the optimization problem of industries to a variational form in order to utilize the tools offered by the theory of functional analysis. The theoretical analysis performed demonstrated the existence and uniqueness of the optimal solution using specific assumptions related to convexity, coercivity, and monotonicity of the operator. Moreover, the convergence of the proposed projection operator iteration method was proven, which ensures the stability and efficiency of the optimization process. From numerical results, it is evident that the proposed framework offers better performance in terms of constraint violation and efficiency compared to the existing optimization techniques.

There are several possible extensions of the current framework. It is possible to incorporate stochastic functional operators to account for the dynamic variations and randomness in the process of optimization of industrial systems. One may develop dynamic variational inequalities for time-dependent industrial optimization problems. Uncertain functional spaces may also be considered in order to account for missing information and measurement noise in the process of optimization. Another possible extension of the current framework is the incorporation of the framework in distributed optimization architectures where there are multiple decision makers in interconnected industries.

## References

1. Alotaibi, M. A. (2026). A Unified Co-Optimization Framework for Hybrid Renewable Systems Incorporating Degradation-Aware Multi-Storage and Demand-Side Management. *Energies*, 19(11), 2705. <https://doi.org/10.3390/en19112705>
2. Fateh M. Aleem, & Ahmed Ulkilan. (2026). Spiking Neural Network-Based Neuromorphic Signal Processing for Real-Time Audio Event Detection in Low-Power Embedded Smart Sensors. *Progress in Electronics and Communication Engineering*, 3(2), 31-35. <https://doi.org/10.31838/ECE/03.02.05>
3. Gao, J., & You, F. (2018). Dynamic material flow analysis-based life cycle optimization framework and application to sustainable design of shale gas energy systems. *ACS Sustainable Chemistry & Engineering*, 6(9), 11734-11752. <https://doi.org/10.1021/acssuschemeng.8b01983>
4. Halim, I., & Srinivasan, R. (2011). A knowledge-based simulation-optimization framework and system for sustainable process operations. *Computers & chemical engineering*, 35(1), 92-105. <https://doi.org/10.1016/j.compchemeng.2010.08.004>
5. Liu, Z., Li, Z., Jin, X., & Jia, H. (2025). Multi-time-domain hierarchical scheduling of integrated energy systems with frequency decomposition considering large-scale grid connection of electric vehicles. *IEEE Transactions on Smart Grid*, 17(1), 244-254. <https://doi.org/10.1109/TSG.2025.3617542>
6. Mirabi, V. R., & Farahmad, A. A. (2017). Investigating the impact of product innovation and standardization on delivery speed with the mediating role of customized mass production capacity (Case Study: Isfahan Steel Toka). *International Academic Journal of Business Management*, 4(1), 160–175.
7. Odilov, B., Doniyorov, A., Sayfullaev, D., Karimov, N., Juraev, S., Zokirov, B., & Sattorova, Z. (2026, January). Cybersecurity challenges for ethnotourism operators: Strategies for protection. In *AIP Conference Proceedings* (Vol. 3345, No. 1, p. 020165). AIP Publishing LLC. <https://doi.org/10.1063/5.0299137>
8. Selwin Mich Priyadharson, A., Ugli, M. N. N., Gopalakrishnan, R., & Bawankar, S. (2025). A Multidimensional Stochastic Framework for Optimizing 6G-Enabled Industrial IoT and Supply Chain Resilience. *International Academic Journal of Science and Engineering*, 12(4), 373-384. <https://doi.org/10.71086/IAJSE/V12I4/IAJSE12105>
9. Sethuraman, P., Kalaivani, M., Latha, K., & Kiruthiga, B. (2025). Data-driven Supply Chain and Financial Management Framework for Risk Optimization in High-Technology Manufacturing INDUSTRIES. *Archives for Technical Sciences*, 3(34), 245–255. <https://doi.org/10.70102/afts.2025.1834.245>
10. Shirke, S., & Udayakumar, R. (2019, April). Evaluation of crow search algorithm (CSA) for optimization in discrete applications. In *2019 3rd International Conference on Trends in Electronics and Informatics (ICOEI)* (pp. 584-589). IEEE. <https://doi.org/10.1109/ICOEI.2019.8862669>
11. Yang, B., Zhou, P., Li, K., Zhang, C., & Liu, P. (2026). A review of the application of functional data analysis methods in industrial intelligent systems. *Journal of Intelligent Manufacturing*, 1-33. <https://link.springer.com/article/10.1007/s10845-026-02840-4#citeas>
12. Yi, L., Ding, J., Liu, C., & Chai, T. (2024). Fast sensitivity-analysis-based online self-organizing broad learning system. *IEEE Transactions on Industrial Informatics*, 20(5), 7585-7597. <https://doi.org/10.1109/TII.2024.3361017>
13. Yu, G., Yong, Y., Jiang, C., Hao, F., & Ma, L. (2025). Formal concept analysis assisted large-scale global optimization and its application to cloud task scheduling. *Complex & Intelligent Systems*, 11(6), 245. <https://doi.org/10.1007/s40747-025-01878-w>
14. Zhai, X., Wang, S., & Zhang, Q. (2026, January). Knowledge-Injected Optimization Framework for Heavily-Constrained SPMT Layout of Offshore Decks. In *2026 6th International Conference on Neural Networks, Information and Communication Engineering (NNICE)* (pp. 1282-1291). IEEE. <https://doi.org/10.1109/NNICE68970.2026.11465832>