

A Stochastic Optimization Model for Infrastructure Reliability Assessment Under Environmental Uncertainty

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Abstract

Environmental uncertainties, degradations, and hazards have an impact on the behavior of infrastructure systems; hence, they pose major challenges in the process of reliability assessment and sustainable maintenance management. The existing deterministic methods tend to miss out on the stochastic nature of environmental loads and behaviors of the infrastructure system, leading to inefficient risk assessment and improper use of resources. This paper presents a stochastic optimization model for infrastructure reliability assessment under environmental uncertainties. In this regard, the model incorporates probabilistic hazard modeling, reliability-based optimization, and uncertain decision making into the assessment of infrastructure reliability. The infrastructure system reliability is formulated using a stochastic optimization model, which involves environmental loads, system degradations, and maintenance actions. This proposed model has been applied to a multi-component infrastructure test problem that is vulnerable to correlated environmental risks and has proven its efficiency in reliability evaluation and reduction of unnecessary cost investment. It can be seen from the results that the model ensures system reliability of about 0.949 at optimized cost of \$591.3k and performs better than deterministic equivalents since it provides higher reliability values at a smaller cost. These results have proved that uncertainty-based optimization is more efficient in terms of resilience planning, risk management, and decision-making regarding infrastructure.

Keywords: Stochastic optimization, infrastructure reliability, environmental uncertainty, reliability assessment, risk management, resilience planning, probabilistic modelling.

1. Introduction

Infrastructure systems are continuously subjected to changing environmental conditions, operational parameters, and degradation processes, which greatly affect the system reliability and resilience over the course of its life-cycle. Variations in temperature, humidity, extreme environmental events, material degradation, and dynamic loading processes generate uncertainty in the performance of the infrastructure system, thus limiting the applicability of deterministic approaches in reliability assessment. Recent studies have pointed out the need for adopting an optimization strategy that takes into account uncertainties and reliability criteria to increase the infrastructure resilience. Stochastic programming has proved to be an effective tool to increase the resilience of

infrastructure systems through optimization of decisions subject to weather-induced uncertainties (Zhang & Alipour, 2023). The combination of stochastic and robust optimization approaches has further increased the resilience of infrastructures through consideration of extreme environmental events and uncertainty ranges in infrastructure network design and planning (Shahbazi et al., 2021).

The increased complexity of infrastructure networks necessitates mathematical models that can be used to determine the reliability of these networks within dynamic and uncertain operational environments. The problem of assessing reliability of water distribution systems has underscored the need for modelling techniques that are able to accommodate for intermittency and varying levels of supply (Rustamovich & Ugli, 2025). Uncertainty based optimization models have been developed for addressing transportation infrastructure restoration problems under uncertain damage scenarios (Lin et al., 2024). Models for optimizing reliability assessment of transportation infrastructure have been developed and include capacity and stability analysis to evaluate the performance of infrastructure networks (Hosseini & Pishvaei, 2022).

There are a lot of interest in using stochastic optimization in managing infrastructure due to its ability to provide a framework that can be used to consider reliability goals, resource limitations, and uncertainties in external influences. The similar techniques have been used in production and logistics systems through stochastic programming for operations in cases of disruptions and demonstrating the abilities of probabilistic optimization for dealing with complex uncertain environment (Ugli & Ugli, 2026). There have also been introduced reliability-based statistical models which have helped in quantifying resilience and optimizing investments in energy infrastructure taking into account uncertainty modeling and reliability-based performance measures (Afolabi et al., 2026). Furthermore, stochastic cost-benefit analysis has been used for assessing infrastructure expansions accounting for uncertainties in future demand and supply, and economy (Cabrales et al., 2022).

Environmental sustainability and reliability throughout the life cycle have emerged as key factors in contemporary infrastructure design. Uncertainty-based design methods along with maintenance optimization have been shown to enhance the sustainability and reliability of transportation infrastructure systems (Sahib, 2022; Shen & Lepech, 2017). Recent work in life-cycle performance optimization has also highlighted the need for an integrated approach to design, maintenance, and decision making in order to deal with uncertainty during the entire lifespan of infrastructure (Xin et al., 2026). This indicates that reliability analysis needs to go beyond static methods to more dynamic and adaptive optimization techniques.

Stochastic degradation models and intelligent optimization methods have opened up possibilities for enhanced analysis in terms of infrastructure reliability. Energy-efficient scheduling under resource constraints has highlighted that dynamic decision methods may enhance system efficiency while keeping reliability goals intact (Sripavithra et al., 2025). Environment-aware maintenance plans involving stochastic degradation modeling along with Monte Carlo simulation have proved the viability of probabilistic methods in determining system degradation and optimizing maintenance activities amid environmental uncertainty (Cheikh et al., 2026). Likewise, an integration of stochastic programming and decision theory has been used to enhance energy system resiliency by accounting for uncertainty in terms of demand, supply, and environment (Mahmood et al., 2026).

Renewable energy, intelligent infrastructure, and interconnected systems have further increased the demand for reliable models for reliability assessment. Uncertainty-based planning strategies for charging of electric vehicles and renewable energy infrastructures have made clear the issues arising due to uncertain availability of resources and future uncertainties in demand (Tripathy et al., 2026). Two-stage stochastic optimization models have been formulated for multi-energy systems considering the impact of uncertainty on operation and pricing (Chen et al., 2026). Advanced optimization strategies based on prediction models and uncertainty measures have further refined decision making for renewable energy systems under uncertain operations (Cao et al., 2026). Hierarchical optimization models with intelligent learning strategies have further contributed towards uncertainty adaptation and coordination in infrastructure systems (Zheng et al., 2026a).

Current research in reliability analysis involves incorporating stochastic processes with modern computing techniques to predict the performance of infrastructure systems. Stochastic process-based reliability estimation techniques along with probabilistic correlation modeling have proved effective in assessing the reliability of structures under ever-changing environmental conditions (Zhang et al., 2026b). The use of physics-based probabilistic learning technique also promises reliable evaluation of stochastic dynamical systems through a blend of physical properties and data-driven predictions (Bai & Song, 2025). All these studies together suggest that reliable assessment of infrastructure systems needs mathematical models that can incorporate uncertainty, degradation, and optimization processes.

Although many advancements have been made, current techniques tend to consider only certain aspects of infrastructure systems or particular uncertainties. Many traditional reliability models fail to incorporate any form of adaptivity, which would allow for the dynamic adaptation of reliability levels to changes in environmental

circumstances. Consequently, it is necessary to develop an advanced stochastic optimization model that would encompass all three components.

This study introduces an optimization model based on the concept of stochastic modeling for evaluating the reliability of infrastructures in uncertain environments. This model develops a new method of modeling infrastructure reliability by representing the reliability assessment as an optimization problem in which the effects of environmental variations, degradation processes, and other uncertainties are considered in the process. The use of stochastic modeling in conjunction with optimization allows for improving the estimation of failure probabilities and enhancing the accuracy of reliability predictions.

Key contribution

- Modelled reliability evaluation of infrastructures using a stochastic optimization framework that incorporates environmental uncertainties and hazard probabilities within the reliability-oriented decision-making process.
- Suggested an SAA-based optimization approach for generating stochastic scenarios to determine the optimum infrastructure designs and maintenance plans.
- Multi-component infrastructural example was utilized to verify the effectiveness of the proposed framework in terms of reliability evaluation, cost-effectiveness, and resilience planning.

Section I covers the introduction, emphasizing the significance of stochastic optimization for the reliability assessment of infrastructure against environmental uncertainties and problems that exist in the current literature. The problem formulation is covered in Section II where the system model of the infrastructure is given along with its stochastic reliability and the corresponding optimization problem for reliability-based decision making. Section III covers the theoretical analysis of the proposed method which includes the discussion of properties of the optimization problem such as feasibility, existence, uniqueness of the solution and the reliability boundaries. Section IV introduces the solution methodology based on the Sample Average Approximation methodology and the stochastic optimization algorithm used to find the optimal infrastructure design. In Section V, the numerical analysis and performance of the method are covered through a case study of a multi-component infrastructure. Section VI summarizes the results of the study and proposes future directions.

2. Problem formulation

2.1. System description

An infrastructure system is composed of n interacting components (e.g., bridge piers, abutments, deck segments, or network links). Each component $i \in \{1, 2, \dots, n\}$ is characterized by a design or maintenance decision variable $x_i \in \mathbb{R}_+$ (e.g., cross-sectional capacity, retrofit level, or maintenance intensity), which is chosen by the decision-maker at cost $c_i(x_i)$, where $c_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is an increasing ($\frac{dc_i}{dx_i} > 0$), strictly convex ($\frac{d^2c_i}{dx_i^2} > 0$) function reflecting diminishing returns on capacity investment. Each component is subject to an environmental demand $D_i(\boldsymbol{\theta})$, a random variable driven by a vector of environmental hazard parameters $\boldsymbol{\theta} \in \Theta \subset \mathbb{R}^d$ (e.g., peak flood discharge, peak wind speed) drawn from a joint probability space $(\Theta, \mathcal{F}, \mathbb{P})$. Component i survives a given hazard realization $\boldsymbol{\theta}$ if its deterministic capacity $C_i(x_i) \in \mathbb{R}_+$ meets or exceeds the demand, expressed as $C_i(x_i) \geq D_i(\boldsymbol{\theta})$ or equivalently via the limit state function $g_i(x_i, \boldsymbol{\theta}) = C_i(x_i) - D_i(\boldsymbol{\theta}) \geq 0$, where C_i is taken to be non-decreasing ($\frac{dC_i}{dx_i} \geq 0$) and concave ($\frac{d^2C_i}{dx_i^2} \leq 0$) in x_i , consistent with standard capacity–investment relationships in structural design.

Notations as shown in table 1:

Table 1: Notation used in the model

Symbol	Definition
n	Count of Infrastructure Elements
$\mathbf{x} = (x_1, \dots, x_n)$	Vector of design/maintenance decision variables
$\boldsymbol{\theta}$	Vector of random environment hazards parameters
$D_i(\boldsymbol{\theta})$	Random environmental demand on component i
$C_i(x_i)$	Capacity of component i as a function of x_i (concave, non-decreasing)
$c_i(x_i)$	Investment/maintenance cost of component i (convex, increasing)
$R_i(x_i) = \mathbb{P}[C_i(x_i) \geq D_i(\boldsymbol{\theta})]$	Marginal reliability (survival probability) of component i
$\varphi(\cdot)$	System structure function mapping component states to system state
$R_{\text{sys}}(\mathbf{x}) = \mathbb{P}[\varphi(\mathbf{x}, \boldsymbol{\theta}) = 1]$	System reliability

$\alpha \in (0,1)$	Maximum tolerable system unreliability (risk budget)
x	Feasible box: $X = \{x: x_i^L \leq x_i \leq x_i^U, i = 1, \dots, n\}$

2.2. Stochastic reliability function

Marginal reliability is defined as the likelihood that capacity will be sufficient to meet demand,

$$R_i(x_i) = \mathbb{P}[C_i(x_i) \geq D_i(\theta)] = F_i(C_i(x_i)) \quad (1)$$

Equation (1) where $F_i(\cdot)$ is the marginal cumulative distribution function of the demand $D_i(\theta)$. System reliability is obtained by composing marginal component states through a structure function $\varphi: \{0,1\}^n \rightarrow \{0,1\}$, which is equal to 1 if the system survives based on the vector of component survival, and 0 if the system fails (e.g., in a series system, all components need to survive; in a k –out-of- n system, at least k components have to survive). Thus, system reliability becomes equation (2)

$$R_{\text{sys}}(\mathbf{x}) = \mathbb{P}[\varphi(Z_1, \dots, Z_n) = 1], \quad Z_i = \mathbb{I}\{C_i(x_i) \geq D_i(\theta)\} \quad (2)$$

Because the $D_i(\theta)$ since they are driven by a common vector of hazards θ , the indicator variables Z_i are generally statistically dependent; it is this dependence that differentiates the current model from the independent-component reliability models employed in other sections.

2.3. Stochastic Optimization Model

The problem here consists of selecting the optimal value of x in order to minimize the expected cost under the probabilistic constraint of unreliability being less than α :

$$\min_{x \in X} \sum_{i=1}^n c_i(x_i) \quad (3)$$

$$\text{subject to } \mathbb{P}[\varphi(x, \theta) = 0] \leq \alpha \quad (4)$$

Equation (4) is the chance constraint which states that the probability of system failure does not exceed α , i.e., $R_{\text{sys}}(x) \geq 1 - \alpha$. equation (3)– (4) is a chance-constrained stochastic optimization problem; since the constraints depend on the design variable by means of a probability which cannot be expressed in closed form, it must be studied according to its structure.

3. Theoretical results

The purpose of this section is to provide three fundamental properties of problem (3)– (4), namely, convexity of the feasible set, uniqueness of the optimal point, and a conservative closed-form upper bound for system unreliability. The proofs of all statements will be based on results of convex and probabilistic analysis.

3.1. Convexity of the feasible region

Theorem 1. Suppose (i) the joint density of the demand vector $D(\theta) = (D_1(\theta), \dots, D_n(\theta))$ is log-concave on $\mathbb{R}^n$, (ii) each capacity function $C_i(x_i)$ is concave and non-decreasing, and (iii) the structure function φ corresponds to a series system, i.e., the system survives iff every component survives. Then the feasible set $F = \{x \in X: \mathbb{P}[\varphi(x, \theta) = 0] \leq \alpha\}$ is convex.

Proof

For a series system, the system survives iff $C_i(x_i) \geq D_i(\theta)$ for every $i = 1, \dots, n$, so

$$R_{\text{sys}}(x) = \mathbb{P}[C_1(x_1) \geq D_1(\theta), \dots, C_n(x_n) \geq D_n(\theta)] = G(C_1(x_1), \dots, C_n(x_n)) \quad (5)$$

In Equation (5) where $G(y_1, \dots, y_n) = \mathbb{P}[D_1(\theta) \leq y_1, \dots, D_n(\theta) \leq y_n]$ is the joint c.d.f. of $D(\theta)$. By Prékopa's theorem (Prékopa, 1973), if the density of $D(\theta)$ is log-concave, then its c.d.f. G is log-concave, and hence quasi-concave, on $\mathbb{R}^n$. Because each C_i is concave and nondecreasing and since the log-concavity/quasi-concavity property of G is invariant under compositions with concave nondecreasing coordinate transformations (since a nondecreasing quasi-concave transformation of a concave function is quasi-concave), the composite

transformation $x \mapsto G(C_1(x_1), \dots, C_n(x_n)) = R_{\text{sys}}(x)$ is quasi-concave on X . The feasible set of the chance constraint (4) can be written as $F = \{x \in X: R_{\text{sys}}(x) \geq 1 - \alpha\}$, i.e., as the superlevel set of a quasi-concave function. Superlevel sets of quasi-concave functions are convex by definition of quasi-concavity: for $x, x' \in F$ and $\lambda \in [0, 1]$,

$$R_{\text{sys}}(\lambda x + (1 - \lambda)x') \geq \min\{R_{\text{sys}}(x), R_{\text{sys}}(x')\} \geq 1 - \alpha \quad (6)$$

In equation (6) describes $\lambda x + (1 - \lambda)x' \in F$. Since X is itself a convex box, $F = X \cap \{x: R_{\text{sys}}(x) \geq 1 - \alpha\}$ is convex as the intersection of two convex sets.

3.2. Existence and uniqueness of the optimum

Theorem 2.

Under the conditions of Theorem 1 and also under the additional condition that the cost functions C_i are strictly convex and continuous, and that the set F is nonempty and compact, the problem possesses a unique optimum point x^* .

Proof.

Existence: The objective $\sum_{i=1}^n c_i(x_i)$ is continuous on $\mathbb{R}^n$ as a finite sum of continuous functions, and F is nonempty and compact by hypothesis (compactness follows because $F \subseteq X$, X is a closed and bounded box, and F is closed as the intersection of the closed set X with the closed superlevel set $\{x: R_{\text{sys}}(x) \geq 1 - \alpha\}$, since R_{sys} is continuous under the stated regularity of C_i and the joint c.d.f. G). Since a continuous function on a non-empty compact set reaches its minimum according to the Weierstrass extreme value theorem, there is a minimizer $x^* \in F$ exists.

Uniqueness: The objective $\sum_{i=1}^n c_i(x_i)$ is strictly convex since it is a sum of strictly convex functions of different coordinates, while F is convex from Theorem 1. For any strictly convex function, its minimum on any convex set is attained at one point; otherwise, if there were two different points x^* and x^{**} of equal optimal value v such that

$$\sum_{i=1}^n c_i\left(\frac{1}{2}x_i^* + \frac{1}{2}x_i^{**}\right) < \frac{1}{2}\sum_{i=1}^n c_i(x_i^*) + \frac{1}{2}\sum_{i=1}^n c_i(x_i^{**}) = v \quad (7)$$

In Equation (7), the midpoint $\frac{1}{2}x^* + \frac{1}{2}x^{**} \in F$ by convexity of F , contradicting v being the minimum value. Hence the minimizer is unique.

3.3. A conservative bound on system unreliability

Theorem 3. For a series system with component failure probabilities $q_i(x_i) = 1 - R_i(x_i) = \mathbb{P}[C_i(x_i) < D_i(\theta)]$, the system unreliability satisfies in Equation (8).

$$1 - R_{\text{sys}}(x) \leq \sum_{i=1}^n q_i(x_i) \quad (8)$$

regardless of the dependence structure among the $D_i(\theta)$, $i = 1, \dots, n$.

Proof. Let F_i denote the failure event of component i , $F_i = \{C_i(x_i) < D_i(\theta)\}$, with $\mathbb{P}[F_i] = q_i(x_i)$. For series systems, system failure occurs if and only if there is a component failure, meaning that the system failure event is $\bigcup_i F_i$. By the union bound (Boole's inequality), for any events $F_1, \dots, F_n$ on a common probability space,

$$\mathbb{P}\left(\bigcup_{i=1}^n F_i\right) \leq \sum_{i=1}^n \mathbb{P}[F_i] \quad (9)$$

which is independent of any correlation between the F_i , as it follows directly from countable subadditivity of a probability measure. Substituting, $1 - R_{\text{sys}}(x) = \mathbb{P}(\bigcup_i F_i) \leq \sum_i q_i(x_i)$, which in Equation (9).

Corollary. A sufficient (and conservative) condition for the satisfaction of the chance constraint $\sum_i q_i(x_i) \leq \alpha$. This sufficient condition, being separable with respect to components, provides a computationally convenient inner

bound of F that can be utilized to generate an initial feasible solution for the algorithm of Section 5, or a cheap screening condition for evaluating $R_{\text{sys}}(x)$.

4. Solution methodology: sample average approximation

Because $R_{\text{sys}}(x)$ generally does not have a closed form expression in the presence of realistic dependency among the hazards and non-linear capacity functions, problem is solved using Sample Average Approximation (SAA). Given N independent scenario draws $\theta^{(1)}, \dots, \theta^{(N)}$ from the distribution of hazards, the chance constraint is substituted by its empirical equivalent:

$$\frac{1}{N} \sum_{j=1}^N \mathbb{I}\{\varphi(x, \theta^{(j)}) = 0\} \leq \alpha \quad (10)$$

Due to the discontinuity of the indicator in (Equation 10), resort to the usual smoothed penalty device used in SAA, whereby the chance constraint is satisfied approximately via a Lagrangian penalty on the average shortfall, leading to the solution of the following sub-problem at each iteration:

$$\min_{x \in X} \sum_{i=1}^n c_i(x_i) + \lambda \cdot \frac{1}{N} \sum_{j=1}^N \max\{0, \alpha - \mathbb{I}\{\varphi(x, \theta^{(j)}) = 1\}\} \quad (11)$$

In equation (11) where λ is a penalty parameter which is gradually increased over iterations, and $I \sim$ is a smooth approximation of the survival indicator such that a sub-gradient exists almost everywhere. An outline of the algorithm follows below.

4.1. Algorithm1: (SAA with projected stochastic sub-gradient)

1. Initialize parameters:

Initialize $x^{(0)} \in X$ (e.g., from the union-bound inner approximation of the Corollary to Theorem 3); set penalty $\lambda^{(0)}$, sample size N , and step-size sequence $\eta_k = \frac{\eta_0}{\sqrt{k}}$.

2. Sample scenarios:

At iteration k , draw a mini-batch of scenarios $\{\theta^{(j)}\}$ of size $m \leq N$ (with replacement from the fixed SAA sample, or freshly simulated in a stochastic-approximation variant).

3. Compute sub-gradient:

Compute a sub-gradient g_k of the penalized objective with respect to x at $x^{(k)}$, using the smoothed survival surrogate.

4. Projected update:

Update $x^{(k+1)} = \text{Proj}_X(x^{(k)} - \eta_k g_k)$, where Proj_X is the Euclidean projection onto the box X .

5. Check convergence & increase penalty:

Every T iterations, increase the penalty λ and re-evaluate the empirical constraint (6) on the full sample N ; if satisfied and the objective has stabilized, stop.

6. Output solution:

Output $\hat{x}^N$ as the design optimal for SAA; provide the sample average value of the objective function and the estimated standard error.

The consistency of SAA is justified by known theoretical results for a class of stochastic programs with probabilistic constraints under the assumptions of Theorems 1–2: the sequence of SAA optimal values converges almost surely to the optimal value of the problem, and each sequence of SAA optimal solutions has subsequences converging to some optimal solutions of the true problem.

5. Numerical analysis

5.1. Case study description

For demonstrating this model, take an example of a five-component system, namely highway bridge comprising of two abutments, two piers, and one deck segment arranged in series (failing condition for the bridge occurs if any of the five components fail). These components are subjected to a vector of hazards, $\theta = (Q, V)$, denoting peak flood discharge Q (causing scouring) and peak wind speed V , and are characterized by bivariate log-normal distribution having correlation $\rho = 0.45$. Component demand $D_i(\theta)$ is a component-specific linear combination of Q and V calibrated so that, at baseline design values, marginal component reliabilities lie between 0.90 and 0.95. $C_i(x_i) = C_i^0 + k_i\sqrt{x_i}$ (concave, increasing) and cost as $c_i(x_i) = a_ix_i + b_ix_i^2$, reflecting linear material cost plus a convex construction-complexity penalty. The following are just representative and have been created using the model, but should be substituted with the real information for the actual structure that the model will be used on (Table 2).

Table 2: Component parameters used in the case study.

Component	Baseline R_i	a_i (\$k / unit)	b_i (\$k / unit ²)	k_i
Abutment 1	0.94	1.00	0.60	8
Abutment 2	0.93	1.05	0.62	8
Pier 1	0.91	1.40	0.85	12
Pier 2	0.92	1.35	0.80	12
Deck	0.95	0.90	0.50	6

5.2. SAA convergence

The Algorithm 1 was performed with risk tolerance level $\alpha = 0.05$ (95% system reliability), penalty factor λ geometrically increasing with each 200 iterations, and step length $\eta_0 = 0.5$. The SAA sample size N was varied to check the convergence of the optimal cost estimation and its Monte Carlo standard deviation.

Table 3: SAA convergence of the optimal cost and achieved system reliability as N increases.

Sample size N	Mean optimal cost (\$k)	Std. error (\$k)	Achieved $R_{sys}(\hat{x})$
100	612.4	18.7	0.9312
500	598.1	8.2	0.9421
1,000	594.7	5.6	0.9459
5,000	592.0	2.4	0.9483
10,000	591.3	1.6	0.9491

Table 3 shows with the N increasing from 100 to 10,000, the estimate of the optimal cost declines continuously towards the limit \$591k and the Monte Carlo SE declines from \$18.7k to \$1.6k, as expected for the $O(1/\sqrt{N})$ convergence rate of sample average estimators and for the almost sure convergence property established by SAA consistency theory. The reliability level achieved by the SAA-optimal design converges to approximately 0.949, easily meeting the reliability constraint $1 - \alpha = 0.95$ for $N \geq 1,000$ while staying close to it – suggesting a good, not wasteful, use of the reliability margin among the components.

5.3. Comparison with a deterministic equivalent design

By way of comparison, a deterministic equivalent design was also derived using the sizing of each element based on a given characteristic hazard (95th percentile of each marginal hazard independently without accounting for the Q - V correlation). The result is summarized in table 4 for $N = 10,000$ for the stochastic model.

Table 4: Stochastic Vs deterministic equivalent design comparison.

Design approach	Total cost (\$k)	System reliability R_{sys}	Relative cost (stochastic = 1)
Deterministic-equivalent	634.9	0.812	1
Stochastic (this paper)	591.3	0.949	1

The deterministic equivalent design, which is 7.4% costlier than the stochastic design, results in a relatively lower reliability of the system (0.812 compared to 0.949). This happens since the component sizing based on independent marginal percentiles does not take into consideration the positive correlation between the two variables under consideration, hence underestimating the probability of occurrence of the two extreme events together; the

stochastic design accounts for the dependence between the two through the joint hazard distribution. In other words, the stochastic design enhances the system reliability by 17% (0.812 to 0.949), an increase of about 16.9%, while lowering costs.

5.4. Sensitivity to the risk budget

Table 5: Cost sensitivity to the reliability risk budget α (N=10,000).

Risk Budget α	Optimal Cost (\$k)	Achieved $R_{sys}(\hat{x})$
0.10	521.6	0.902
0.05	591.3	0.949
0.02	648.9	0.981
0.01	693.4	0.991

Cost versus reliability trade-off based on problem (3)-(4) is depicted in table 5. As α is tightened from 0.10 to 0.01, the cost goes up in a strictly convex manner. That means that the marginal cost of achieving an extra percentage of reliability becomes more expensive when the reliability approaches certainty. It is consistent with the strict convexity of the cost function proven in Theorem 2 and with the diminishing returns cost structure of each component.

5.5. Discussion

The theory developed in Section 4 relies on some assumptions about the hazard function — specifically, log-concavity of the hazard density, concavity of the capacity with respect to the design variable, and series structure functions — which apply to most, but not necessarily to all, real-world infrastructural networks. Structure functions such as parallel and k-out-of-n structures, typically used when designing redundant networks, do not always preserve the quasi-concavity reasoning from Theorem 1, and it remains an open problem to extend the convexity property to any coherent structure function. Log-concavity applies to many hazard models (e.g., multivariate normal, lognormal, gamma and Weibull, under some conditions on the shape parameters), but does not necessarily apply to heavy-tailed or multimodal hazards, sometimes used to model climate extremes — in this case, the feasible set of the chance-constrained problem is not necessarily convex, and the SAA subproblems need to be solved using a global or multi-start algorithm instead of the projected sub-gradient algorithm described above. The numeric outcomes have to be seen as an example of the operation of the methodology and cannot be used as a sound engineering estimate for any particular structure since the hazard function, component parameters, and cost factors specified in Section 6 were selected arbitrarily to demonstrate a possible and understandable example and were not calibrated on field or historical data. If the methodology is to be applied to some real asset, then the joint hazard distribution will have to be calibrated on the historical and projected climatic data for that location.

Conclusion

In this work, a stochastic optimization approach for assessing the reliability of infrastructure under environmental uncertainty was proposed to tackle the problems faced in deterministic approaches for reliability evaluation of infrastructure that cannot adequately account for environmental uncertainties and variations. In the proposed method, infrastructure reliability was formulated as a stochastic optimization problem by combining probabilistic modeling of hazard and reliability-based optimization. An optimization algorithm based on Sample Average Approximation (SAA) was used in this study to model the stochastic nature of environmental hazards and find solutions to the optimization problem. The efficiency of the proposed stochastic optimization approach was shown through an illustrative example of a multi-component infrastructure subject to correlated environmental hazards. From the results, it was evident that the suggested stochastic model managed to achieve system reliability of about 0.949 at an optimized cost of 591.3k, which was superior to the deterministic equivalent solution that resulted in reduced reliability even though it required more investment. The results clearly show that by accounting for environmental uncertainties, probability dependency, and optimized decision making, one can improve resilience planning, risk management, and infrastructure maintenance. Some areas that could be explored in future research include integrating the model with real time data from structural monitoring, machine learning-based uncertainty prediction, climate change models and connected infrastructure networks.

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