

Robust Optimization of Personalized Learning Pathways Under Uncertainty in Student Performance

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Abstract

Personalized learning systems have the potential to be more effective by designing the order of instruction for each learner, but the decisions are based on noisy measures of the performance of the learners. Inference of mastery probabilities from few interactions, varied engagement level, and imperfect measurements will be uncertain, and thus the optimal path generated might become ineffective due to the noise. In this paper, we propose a robust optimization approach for constructing personalized learning paths that maintain effectiveness under various performance realizations of the learners. Our proposed method considers the selection of personalized learning paths as a sequential decision problem with uncertain mastery-transition parameters, where uncertainty is represented using budgeted uncertainty sets, and then construct. The proposed model is structured in a three-step pipeline of: knowledge tracing for estimating the level of mastery and associated uncertainty, creating an uncertainty set based on those estimations, and robust optimization of the learning path considering the worst-case realization of that uncertainty set. We propose a chance-constrained variation that ensures a specific probability of fulfilling the constraints associated with the learning objectives. We utilize benchmark information on performance prediction and optimization of learning pathways from the existing literature to motivate the proposed framework, and a symbolic example will show how the robustness budget affects the suggested learning pathway.

Keywords: Personalized learning pathways, robust optimization, student performance uncertainty, knowledge tracing, chance-constrained optimization.

1. Introduction

The goal of adaptive learning systems is to offer a learner-specific instructional path different from the one-size-fits-all approach to classroom education (Chen et al., 2018). A key component of the adaptive learning system is a recommendation engine that can choose the next learning material video lecture, exercise or assessment based on the results of psychometric tests and other attributes (Chen et al., 2018). There is great potential since personalized paths of learning are meant to save time for learning and boost learning achievements through a more appropriate sequence for heterogeneous learners (Rakhimova et al., 2026; Sethuraman, 2026; Jiang et al., 2022).

However, there are two common shortcomings of most pathway construction algorithms used in adaptive learning systems that inspired this research paper. The first drawback of the majority of pathway construction algorithms is that they pay attention either to students' characteristics or knowledge structures, disregarding the importance of their dynamic learning states (Jiang et al., 2022). Earlier studies on pathway construction have considered the prior knowledge about prerequisite relations in the content of a subject in order to determine the sequence of instructions without any regard for learning outcomes of the students (Lin, 2016). The second and more general disadvantage of most pathway construction algorithms is that they use uncertain input to construct the recommended sequences of learning materials (Cheng et al., 2025).

This uncertainty is important since an optimal path according to a point estimate of the ability of a student can turn out to be suboptimal when the actual performance of the learner varies from the point estimate. A student whose ability is estimated to have mastered the prerequisite might not have done so; therefore, the path will become invalid. Robust optimization provides the right framework in that it is meant to work with a lack of complete information about the uncertainties involved, modeling problems in a way they can be efficiently solved and providing performance guarantees of the solution (Ben-Tal et al., 2009).

The contribution of this paper is in three areas. First, we define the personalized pathway selection as a robust sequential decision-making problem with uncertain mastery transition parameters. Second, we build budgeted uncertainty sets using knowledge tracing outcomes and come up with a tractable robust counterpart of the formulation by employing the polyhedral uncertainty method that produces linear robust counterparts with controlled conservatism (Bertsimas & Sim, 2004). Third, we come up with a chance-constrained counterpart of the formulation in situations where the decision maker requires probabilistic guarantees rather than worst-case performance guarantees of the solution (Nemirovski & Shapiro, 2007).

The rest of the paper is structured as follows. Section 2 covers previous works in personalized pathways, knowledge tracing, and robust optimization. Section 3 problem formulation. Section 4 outlines the proposed robust optimization framework. Section 5 presents the results and benchmarks. Section 6 addresses the implications and limitations, while Section 7 concludes.

2. Related Work

2.1 Personalized learning pathways

Choosing the path which a learner would follow to finish his/her learning is another important issue for personalized learning (Lin, 2016; Fauzi & Aisyah, 2023). An important research stream has formulated the pathway choice issue as an optimization issue based on competencies and learner's learning result, revealing that the optimization issue of choosing such a path is essentially a Markov Decision Process that enables the application of decisions to choose the best paths which ensure optimal learning result of learners (Lin, 2016).

In more recent research, this idea of sequencing was applied again. An algorithm of dynamic personalized learning trajectory planning recommends suitable knowledge points for online learners, taking into account their current learning state and knowledge point difficulty, where the difficulty is estimated automatically first, and then a dynamic knowledge mastery model is constructed using learning behavior data and test results standardized (Abidova et al., 2026; Jiang et al., 2022). The experiment proved that the proposed algorithm could construct personalized trajectories effectively, which could enhance effective behavior rate, course passing rate, and learning efficiency (Jiang et al., 2022). Reinforcement learning has been one of the major paradigms for this sequence decision making problem: the recommendation task could be formulated within the Markov decision problem formulation framework, and a reinforcement learning method had been proposed to solve it (Tang et al., 2019). An efficient recommendation system makes tradeoffs between making the most optimal recommendations according to current knowledge and exploring new learning trajectories (Tang et al., 2019).

2.2 Knowledge tracing and uncertainty in student performance

Pathways rely on mastery estimates from knowledge tracing. The two most popular methods are Bayesian knowledge tracing and deep knowledge tracing (Ilić et al., 2023). Bayesian knowledge tracing uses probability and machine learning approaches to model the learning process of students, assuming that the student knowledge can be modeled using binary variables, one for each skill that indicates mastery of the skill (Ilić et al., 2023). These models rely on point estimates, and thus ignore the uncertainty in interactions and do not model the true knowledge state of students (Cheng et al., 2025). Recently, there have been efforts to directly tackle this problem: a knowledge tracing model that takes into account uncertainty represents the uncertainty in student interactions using stochastic distribution embeddings, and incorporates a self-attention approach to capture the state distribution transition through the interactions (Cheng et al., 2025).

The uncertainty associated with student performance does not only exist due to modeling assumptions but is also predictable from empirical data. The studies on early warning based on interaction data collected from online learning showed that unsuccessful students can be detected by means of the k-nearest-neighbor approach with the

accuracy equal to 89% at the end of term and with the probability of 74% already in three weeks into a course (Akçapınar et al., 2019). Similar research on LMS log files has revealed that the early prediction of performance is possible with the completion of 25% of the course with the AUC of 0.756 for at-risk students and with the accuracy of 78.2% for high performers (Santos & Henriques, 2023).

2.3 Robust optimization

The robust optimization method solves optimization problems under uncertainty by guaranteeing the quality of the solution under incomplete information (Ben-Tal et al., 2009). The history of robust optimization contains uncertain linear programming, relationships between the formulation of the uncertainty sets and the classical chance-constrained (probabilistic) approach, as well as generalization to conic quadratic, semidefinite, and multistage problems (Ben-Tal et al., 2009). One of the key issues in robust optimization is that of being too conservative, and the way out of this situation is the special technique developed for polyhedral uncertainty, which results in linear robust counterparts controlling the degree of conservativeness of the solution (Bertsimas & Sim, 2004). In this technique, uncertain parameters are distributed symmetrically around their nominal value, and the budget parameter controls the trade-off between the robustness and conservatism of the approach: the larger is , the more robust is the solution (Bertsimas & Sim, 2004).

In the case where probabilistic guarantees are desired over worst-case guarantees, the approach of chance-constrained programming becomes useful; the goal here is to minimize a convex function over solutions that satisfy a set of randomly disturbed convex constraints with a close-to-one probability and convex conservative approximations such as the Bernstein approximation ensure that this type of problems can be solved efficiently (Nemirovski & Shapiro, 2007). With respect to the application of robust optimization in the education sector, uncertainty has already been considered in personalizing the learning experience using chance-constrained programming under the assumption of independently and normally distributed uncertain parameters (Aslan et al., 2023).

3. Problem Formulation

3.1 Setting and notation

Consider a curriculum of knowledge components (KC) and a learner whose progress through this is measured at each time point. At each time point, the system recommends a task to be performed using one of the available resources, and the learner responds to this task from which mastery is inferred. Let m_t be the mastery of KC at time t as predicted by the knowledge tracing model (Ilić et al., 2023). Let d_t be the decision vector of the path, where $d_{t,k}$ is recommended at time point t and zero otherwise.

3.2 Nominal model

For the point estimate of mastery framework, the pathway optimization problem becomes one of minimizing the cost function that consists of time to mastery, number of prerequisites violated, and performance loss:

$$\begin{aligned} \min_x c^T x \quad \text{s.t.} \quad Ax \leq b, x \in \{0, 1\}^{nT} \\ \min_x c^T x \quad \text{s.t.} \quad Ax \leq b, x \in \{0, 1\}^{nT} \end{aligned}$$

with A representing prerequisite and sequencing constraints. This model belongs to the same family of models that becomes an MDP when the decision rule depends on state (Lin, 2016), and it reflects deterministic pathway models while ignoring learning state dynamics.

3.3 Uncertainty model

The important parameters are the mastery-transition probabilities, which dictate the pace of advancement by the learner. Rather than assume that these are known, we model each transition probability as an unknown parameter within the symmetric interval, with $\{ij\}$ being the half-width determined by the spread of the knowledge-tracing posterior. This definition is motivated by the practical observation that performance of students contains uncertainty that can be quantified to some degree (Akçapınar et al., 2019).

3.4 Robust counterpart

Using a budget of uncertainty approach, the robust counterpart of the nominal model focuses on the most adverse outcome possible in the uncertainty set. The budget of uncertainty determines the conservatism of the obtained solution, and the higher the value of, the more conservative the solution. When a polyhedral uncertainty set is used, the robust counterpart still represents a linear program. This is crucial for our purpose since robust optimization approaches are only helpful when applied to the specific data and produce computable solutions (Ben-Tal et al., 2009).

3.5 Chance-constrained variant

If a probabilistic guarantee is preferred over the worst-case guarantee, then the hard robustness requirement is substituted with a chance constraint stating that the learning objective constraint is to be satisfied with at least probability $1-\alpha$. In accordance with the theory of convex approximation of chance-constrained programs, this problem can be recast into a computationally tractable deterministic problem given an affine dependence on the perturbations (Nemirovski & Shapiro, 2007). This problem formulation resembles the already established chance-constrained model of personalized learning decisions under normal distribution (Aslan et al., 2023).

4. Proposed Robust Framework

The framework operates as a closed loop with three stages.

Stage 1 — Knowledge tracing. Knowledge tracing involves estimating the learner's competency for each KC as well as the dispersion of the posterior distribution. We use the uncertainty-aware model where the stochastic distribution embeddings model the uncertainty in student responses; the classical Bayesian knowledge tracing offers the baseline model where mastery is binary (Ilić et al., 2023).

Stage 2 — Uncertainty-set construction. Posterior spreads for stage 1 are turned into interval half-widths, while the budget is chosen according to how risk-averse the decision-maker is. Tunes robustness directly, hence it is the only control knob of the whole system.

Stage 3 — Robust path optimization. This solution of the robust counterpart (or, alternatively, the chance-constrained version of it) determines the next task; the agent reacts; and the process starts all over again. The sequential state-dependent nature of the algorithm is in line with the approach used for the path planning MDP setting (Lin, 2016), whereas the reinforcement learning perspective influences the exploration-exploitation trade-off.

5. Results

This section highlights (i) benchmark results from the literature which justify the use of this approach, and (ii) an example of the impact of the robustness budget on the optimal path.

5.1 Benchmark evidence

Table 1. summarizes the results of research on performance forecasting and pathway optimization. These numbers are taken from the references directly and outline the empirical territory on which our framework operates.

Study	Context	Reported outcome
Akçapınar et al. (2019)	Online Computer Hardware course (76 students)	kNN predicted end-of-term unsuccessful students with 89% accuracy; unsuccessful students predicted with 74% accuracy as early as 3 weeks (Akçapınar et al., 2019)
Santos & Henriques (2023)	Information management school, one year of LMS data	AUC of 0.756 for at-risk students; 78.2% accuracy for high-performing students; early prediction possible as early as 25% course completion (Santos & Henriques, 2023)
Aslan et al. (2022)	Precedence-constrained knapsack for hybrid personalized learning	Approximate cutting-plane methods achieved very small optimality gaps overall, and even found solutions with no optimality gaps as the number of items increased; pre-processing was particularly effective when precedence relations were dense (Aslan et al., 2023)
Jiang et al. (2022)	MOOC cognitive-diagnostic path planning	Personalized paths improved effective behavior rates, course completion rates, and learning efficiency

The three key conclusions are: first, that student performance is predictable well before and accurately (Akçapınar et al., 2019; Santos & Henriques, 2023), making uncertainty sets based on posterior knowledge tracing reasonable. Secondly, that uncertainty aware formulations of personalized learning policies are computationally feasible, with small and possibly negligible optimality gaps when preprocessing data (Aslan et al., 2023). And finally, that personalized learning paths generate tangible improvement in completion and efficiency, the results we wish to protect.

5.2 Illustration of the robustness budget

The illustration presented next is a metaphorical one; it depicts the process of the robust counterpart.

Suppose that there is a prerequisite relationship with the nominal transition factor and half-width. Under the nominal model (), the planner schedules the downstream KC right after its prerequisite, since the point estimate shows that the learner is prepared to do so. However, as ϵ grows larger, the robust counterpart guards against the event occurring in the budgeted uncertainty set. Consequently, the suggested approach becomes closer to recommending the insertion of the consolidation task prior to the downstream KC when the worst-case transition factor is lower than the threshold above which advancing right away is possible.

The following two behavioral properties emerge. First, as ϵ approaches the nominal path becomes robust. The second is that as the value of ϵ increases, the path becomes more conservative, i.e., more review, less acceleration, in exchange for expected mastery time vs. security from poor performance realizations. It is important to notice that the monotone property of the budget vs. conservatism is exactly the one proved for polyhedral robust optimization (Bertsimas & Sim, 2004). For a probabilistic guarantee, the chance-constrained version replaces the worst-case realization by a probabilistically satisfied constraint with probability at least $1 - \alpha$ (Nemirovski & Shapiro, 2007).

5.3 Expected performance profile

With respect to the results obtained from the benchmarks, the framework will achieve its maximum benefits right in those cases where the uncertainty is maximum, i.e., when the estimate of mastery of the learner is most uncertain, when the deterministic path fails the most. This is consistent with the objective of uncertainty-aware knowledge tracing, which aims at addressing the problem with deterministic models failing to represent the actual knowledge state (Cheng et al., 2025). Since the approach to optimizing pathway under uncertainty has already demonstrated very small optimality gaps on structured instances (Aslan et al., 2023), one does not expect any prohibitively high computational costs in employing the robust formulation.

6. Discussion

In essence, this paper's main idea is the following: the variability of student performance is not an inconvenience to average away, but a property to consider when choosing pathways. Incomplete representations of knowledge do not take into account the variability of students during their interaction, hence, cannot accurately represent the true knowledge state (Cheng et al., 2025), and, therefore, pathways based on such representations lack this property. Robust optimization fills the gap: it considers incompleteness, provides efficient modeling, and delivers performance guarantees (Ben-Tal et al., 2009).

There are three major practical consequences of adopting this approach. Firstly, there is a clearly stated budget parameter that allows instructors and system developers to set the degree of risk aversion, thereby, controlling the conservativeness of the pathway (Bertsimas & Sim, 2004). Secondly, since early performance predictions can be made by the third week, when the prediction reaches 74% accuracy (Akçapınar et al., 2019) and even earlier, with predictions possible at 25% of course completion (Santos & Henriques, 2023), the uncertainty sets can be updated frequently. Finally, this approach is agnostic about the domain in terms of curricula, which is in line with the pathway techniques, not being bound to any particular course.

Two important issues must be mentioned. Firstly, the figure shown in Section 5 is a hypothetical representation, so the relative size of improvement over baselines must be quantified using a controlled experiment. Secondly, the performance of a reliable path is conditioned by calibration of the knowledge-tracing model because poorly calibrated posterior results in poorly sized confidence interval, which is one of the reasons for further research in uncertainty-based tracing.

7. Conclusion

The paper presented a robust optimization-based approach for generating personalized learning pathways in the presence of uncertainties in the performance of the learners. Through the use of uncertain mastery transition parameters, budgeted uncertainty sets based on knowledge tracing posteriors, and the robust counterpart formulation (or an alternative chance-constrained formulation), the approach produces pathways that would be robust with respect to possible uncertainties in the actual performance of the learners. The robust optimization-based approach was founded on the sound theories of robust optimization, chance constrained program approximations using convexity, and the empirical observation that there is noise and partial predictability in student performance.

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