

Minimax-Based Optimization of Instructional Strategies for Heterogeneous Learner Populations

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Abstract

Different groups of learners will have differing amounts of prior knowledge, readiness levels, and requirements of support. Thus, an instruction plan that is optimal on average may generate a considerable learning deficit for some smaller subgroup. In this paper, we consider a simple minimax model for choosing an instruction plan that maximizes the smallest predicted group deficit. We define learner groups using readiness categories and use the group x strategy loss table as the description of each instructional plan. A mixed strategy involves distribution of instructional time between different strategies. The corresponding optimization model is formulated as a linear programming problem and generalized for loss estimates and resource restrictions under uncertainty. We provide a reproducible numerical example comparing the four models. In relation to the presumed loss matrix, average loss optimization leads to a minimum group loss of 40.00 points, while the mixture of minimax yields a minimum group loss of only 24.15 points. The minimum weighted average loss is higher at 23.30 points compared to the previous value of 18.70 points. The example shows the way the mathematical model behaves. It is not a proof obtained in a classroom setting. This mathematical model provides an intuitive basis to start with when designing teaching programs where there are different groups and estimates of their performances are uncertain. Further work should be done using education data.

Keywords: Minimax optimization, instructional strategies, heterogeneous learners, robust optimization, adaptive learning.

1. Introduction

A class made up of diverse students calls for balancing of various requirements for learning in the instructional time available. The diversity of learners may include differences in prior knowledge, familiarity with the subject matter, rate of learning and availability of assistance (Akerdi et al., 2017). All these characteristics are influenced by experience and circumstances and should not be considered as indicators of ability. Studies have emphasized the interplay between the learner, the environment and culture in learning processes (National Academies of Sciences, Engineering, and Medicine, 2018). This implies that an instructional strategy will result in uneven results within the classroom.

An instructional plan must also consider the demands made on the learners during the learning process. What may be manageable to learners with experience may prove too demanding for beginners. Feedback has been found to be useful in the learning process but only if what is provided is appropriate (Mathan et al., 2025; Hattie & Timperley, 2007). Reviews of studies conducted to determine the effectiveness of learning strategies have also shown that it varies with circumstances (Dunlosky et al., 2013). Results of studies in active learning have provided evidence that instructional decisions influence performance and achievement gaps (Freeman et al., 2014; Theobald et al., 2020). On the other hand, deciding on an instructional strategy based on its mean result could mask a significant learning deficit for a subgroup of learners (Triayudi et al., 2024). If the majority gains a lot from one strategy while others do not, the mean result strategy would tend to choose the first despite its unfairness to the latter.

The purpose of this paper is to find a solution to the above decision problem using minimax optimization. In particular, we aim to minimize the maximum expected learning deficit in learner subgroups. The contribution of this work includes a transparent problem formulation for allocating instruction jointly, its linear programming solution and a concrete example illustrating the cost of protecting the most disadvantaged subgroup.

2. Related work

2.1. Learner modeling and instructional planning

Adaptive instruction requires knowing what the learner knows. Corbett & Anderson pioneered the idea of knowledge tracing for estimating evolving knowledge states in procedural tasks. Piech et al. have taken this line of research further by applying recurrent neural networks for modeling interactions with course material (Piech et al., 2015). These models are helpful predictors, but accuracy in prediction does not imply how to improve the learner's performance in the most effective way.

Teaching actions were modeled by partially observable Markov decision processes to enable planning for an automated teacher under uncertainty about the learner's knowledge (Rafferty et al., 2016). Machine teaching offers a more general notion of selecting information or examples to affect a learner. All these lines of research suggest that teaching is a decision problem. The current paper deals with the decision-making problem of allocating a single instructional mixture among multiple learners' populations.

2.2. Robust and minimax optimization

Robust optimization considers the performance of the decision under adverse, yet explicitly modeled, scenarios. Bertsimas and Sim have studied the balance between nominal and robust performance (Bertsimas & Sim, 2004). Bertsimas, Brown, and Caramanis have summarized the theory and computations of robust optimization (Bertsimas et al., 2011). These studies can serve as a foundation for protecting instructional programs from modeling errors regarding the group response.

Duchi and Namkoong have designed distributionally robust techniques for ensuring uniform performance when distributional shifts occur (Duchi & Namkoong, 2021). Sagawa et al., (2019) have analyzed worst-group generalization and found that group loss minimization alone could be insufficient for achieving good performance if there was no regularization. This is not directly relevant to educational problems, but it demonstrates the difference between aggregate and subgroup performances.

Convex optimization can help find a solution to a finite minimax allocation problem under linearity of losses in allocation variables (Boyd & Vandenberghe, 2004). Minimax regret can serve as an alternative technique for comparing a decision with the best decision under the same condition (Agarwal & Zhang, 2022). Absolute group shortfall will be used as the criterion in this paper since the instructional goal is the minimal group learning shortfall.

2.3. Research gap and scope

There are techniques available for making predictions about learners, planning instruction, and ensuring that uneven performance is mitigated. There is still an issue left to be addressed in the practical sense regarding how to assign shared learning opportunities where the techniques differ in terms of their impact on readiness groups. This paper draws all these concepts together via the use of a simple model.

3. Methodology

3.1. Study design

The study is both theoretical and computational. It defines a model for a finite decision process and works on an arbitrarily designed example problem. No data about students have been collected, no classroom activity has been carried out, and the input losses are not determined from an educational database. The computation illustrates the implications of the objective function and does not quantify any teaching efficiency.

The decision-making process involves forming learner clusters, choosing possible strategies, estimating the losses in a similar way, finding the optimal allocation, and evaluating the suggested strategy. Future applications will use diagnostics and validated learner models to form the clusters (Corbett & Anderson, 1994; Piech et al., 2015). To define the effect of strategies, one needs corresponding intervention data.

3.2. Learner groups and instructional strategies

Let $g = 1, \dots, G$ index learner groups and $j = 1, \dots, J$ index instructional strategies. Groups need to be identified according to the present instructional requirements. For this example, there are three groups: low, moderate, and high prerequisite preparedness. These groups are tentative and subject to change as the learners advance.

The three possible techniques that could be chosen for comparison are guided instruction, collaboration, and independent problem solving. It is taken that all these three methods address the same learning goal within the same instructional time frame.

Let a_{gj} denote the expected learning shortfall for group g under strategy j . A deficit is measured in units of an agreed-upon test score, with lower scores being preferable. Deficit could also be referred to as the arithmetic mean of the positive discrepancy between a set learning objective and the attained results. The comparison becomes possible if both the test and the objective are the same.

3.3. Instructional allocation

Let x_j be the proportion of instructional opportunities assigned to strategy j . A feasible shared allocation satisfies

$$x_j \geq 0, \quad \sum_{j=1}^J x_j = 1 \quad (1)$$

The linear combination assumption states that the expected shortfall of the group is:

$$L_g(x) = \sum_{j=1}^J a_{gj} x_j \quad (2)$$

It is applicable to random assignment among similar opportunities assuming stable strategy impacts. It serves as an approximation for a curriculum sequence because order, dependencies, and interaction may affect results. Specifically, the model cannot show that the combination of two teaching strategies within one lesson leads to a weighted average result.

Let p_g be the proportion of learners in group g , with nonnegative proportions summing to one. The average-based decision is

$$x_{\text{avg}} \in \operatorname{argmin}_x \sum_{g=1}^G p_g L_g(x) \quad (3)$$

The drawback of such an objective is that a few people cannot affect the chosen allocation scheme.

3.4. Minimax objective and linear program

The suggested joint allocation system reduces the highest expected group shortage:

$$x^* \in \operatorname{argmin}_x \max_{g=1, \dots, G} L_g(x) \quad (4)$$

Introduce an additional variable z to formulate an equivalent linear programming problem:

$$\min_{x,z} \quad \text{subject to} \quad \sum_{j=1}^J a_{gj} x_j \leq z \text{ for all } g, \quad x_j \geq 0, \quad \sum_{j=1}^J x_j = 1 \quad (5)$$

The variable z is an upper bound on every modeled group loss. Minimizing it makes the most disadvantaged modeled group the controlling criterion. The inner maximum represents an unfavorable group condition, not an assumption that students act as adversaries. Linear programming provides a global solution for this finite model (Boyd & Vandenberghe, 2004).

Where several allocations yield identical minimum values, a second optimization problem can be used to minimize weighted average loss under the optimal worst group bound constraint. This maintains minimax safeguarding and selects the most efficient allocation from amongst tied allocations.

3.5. Uncertainty and resource constraints

Estimated instructional effects are uncertain, especially when groups contain few observations. Suppose an estimated loss $\hat{a}_{gj}$ has a nonnegative uncertainty radius δ_{gj} , with each coefficient allowed to vary independently within its interval. Because all allocations are nonnegative, the robust constraint becomes

$$\sum_{j=1}^J (\hat{a}_{gj} + \delta_{gj}) x_j \leq z \quad \text{for all } g \quad (6)$$

Where several allocations yield identical minimum values, a second optimization problem can be used to minimize weighted average loss under the optimal worst group bound constraint. This maintains minimax safeguarding and selects the most efficient allocation from amongst tied allocations.

If c_j is the resource cost per instructional opportunity and B is the available budget, an additional condition is

$$\sum_{j=1}^J c_j x_j \leq B \quad (7)$$

The teacher can even allocate a minimum or maximum amount in order to cover the syllabus or fulfill certain accessibilities. This numeric example involves costs that are the same for all cases, with no other constraint on

resources.

3.6. Evaluation measures

Three measures summarize an allocation: weighted average shortfall, worst-group shortfall, and the range of group shortfalls:

$$\bar{L}(x) = \sum_g p_g L_g(x), \quad W(x) = \max_g L_g(x), \quad D(x) = \max_g L_g(x) - \min_g L_g(x) \quad (8)$$

Range is a measure of dispersion; it does not constitute a full description of equality of education. The relative increase in worst-group shortfall from a baseline is

$$I = 100 \frac{W(x_{\text{baseline}}) - W(x^*)}{W(x_{\text{baseline}})} \quad (9)$$

These measures quantify the deficit in assessment. They do not represent classification accuracy, sensitivity, or F1-score, which would have to be computed separately.

4. Numerical Illustration and Results

4.1. Assumed input matrix

Loss matrix

The assumed probabilities for the classes are 0.60, 0.30, and 0.10 for low, medium, and high readiness respectively.

Each cell entry is just for example; none represents any actual figure obtained from any class setting.

The above matrix intentionally depicts a situation where different approaches work best for different levels of readiness.

The weighted average losses of the three pure strategies are 18.70, 25.30, and 27.40 points respectively. Average loss optimization hence selects guided instruction strategy only. The worst group losses for the three strategies are 40.00, 28.00, and 35.00 points respectively. Pure minimax strategy chooses collaborative practice in table 1.

Table 1: Assumed learning shortfall by group and strategy in assessment points.

Learner group	Guided instruction	Collaborative practice	Independent problem solving
Low readiness	12	28	35
Moderate readiness	25	20	18
High readiness	40	25	10

4.2. Mixed minimax solution

The linear programming model was solved through SciPy's linear programming module using HiGHS solver. The output is 25/53 direct instruction, 0/53 collaborative work, and 28/53 problem solving independently. These ratios can be represented as 47.17% and 52.83%.

The solution can also be checked directly. Write q for the guided-instruction share and set the independent share to $1 - q$. The low-readiness loss is $35 - 23q$, and the high-readiness loss is $10 + 30q$. Equating these two expressions gives $q = 25/53$. Their common value is $1280/53$, or 24.15 points. The moderate-readiness loss is $1129/53$, or 21.30 points, so it does not violate the maximum bound.

The optimal solution is achieved by giving a weight of 30/53 to the group with lower readiness level and 23/53 to the one with higher readiness level. With such weights, both guided learning and individual problem solving will yield a weighted loss of $1280/53$ while collaborative learning yields $1415/53$. It means that every possible combination will have a maximum group loss of at least $1280/53$.

Table 2: Allocations under the four decision rules in percent.

Decision rule	Guided instruction	Collaborative practice	Independent problem solving
Average-loss optimization	100.00	0.00	0.00
Equal allocation	33.33	33.33	33.33
Pure minimax selection	0.00	100.00	0.00
Mixed minimax optimization	47.17	0.00	52.83

Equal allocation relies on one-third shares. The sum of the percentages shown equals 99.99% due to rounding. Losses have been computed from non-rounded allocations in table 2.

4.3. Comparison of outcomes

Table 3 summarizes the results in terms of losses and measures of aggregate performance. Smaller numbers are better for all the measures shown here.

Table 3: Computed outcomes from the assumed matrix in assessment points.

Decision rule	Low readiness	Moderate readiness	High readiness	Weighted average	Worst group	Group range
Average-loss optimization	12.00	25.00	40.00	18.70	40.00	28.00
Equal allocation	25.00	21.00	25.00	23.80	25.00	4.00
Pure minimax selection	28.00	20.00	25.00	25.30	28.00	8.00
Mixed minimax optimization	24.15	21.30	24.15	23.30	24.15	2.85

The mixed minimax optimization decreases the worst group shortfall by 39.62% compared to the average loss optimization method. The group spread decreases from 28.00 to 2.85 points. On the other hand, the weighted average shortfall increases by 4.60 points. Shortfall among low readiness students increases from 12.00 to 24.15 points as well. The solution thus helps in minimizing the shortfall among the worst performing modeled group but not necessarily all groups.

The mixed minimax solution decreases the worst group shortfall by 0.85 points compared to equal distribution and the weighted average shortfall by 0.50 points compared to equal distribution. Compared to minimax selection, the mixed minimax solution decreases the worst group shortfall by 3.85 points.

4.4. Interpretation under uncertainty

If all the coefficients have an uncertainty radius of 2.00 points, then the robust group losses will increase by 2.00 for each of them since the sum of the allocations is equal to one. However, the allocation that optimizes the solution is still the same, and its worst-group bound is now 26.15 points.

Where there is variation in the uncertainty across groups and/or strategies, there may be a different allocation. Even a strategy that appears to have a positive effect may get less allocation due to its negative lower bound. In reality, uncertainty should consider multiple intervals and evaluate feasibility.

5. Discussion

The primary implication of the figure is that the instructional objectives determine the beneficiaries of the common strategy. While average loss optimization is ideal where the central objective is overall performance, the dependence on proportions leaves the other smaller group under-served. The minimax optimization sets an upper bound for the largest expected loss modeled. This gives protection that is visible to the teacher.

It is not a conclusion that mixed instruction is always optimal. An instructional strategy could be optimal when it exhibits consistent performance across all groups. In the example presented above, the two specialized strategies complement each other while collaborative practice does not get any proportion. The zero proportion is determined by the given matrix and should not be generalized to mean collaborative learning is bad. There is evidence that indicates that implementation of active learning matters (Freeman et al., 2014; Theobald et al., 2020).

Another key issue to note is the difference between prediction and intervention. The model of the learner predicts the readiness or performance of the learners (Corbett & Anderson, 1994; Piech et al., 2015). However, selection of instructional strategy depends on evidence on the impact of different instructional interventions. Experimental study or appropriate observational study is required to estimate the response to alternative strategies. Instructional sequencing requires dynamic modeling like the one examined by Rafferty et al., (2016).

Though it is helpful, it is not a comprehensive approach to ensure fairness. Group averages might hide individual suffering while grouping itself might contain some mistakes. Also, there might be cases when the strategy will give up many gains of one group in order to correct some huge loss another group suffers from. Educators should examine the loss profiles of the entire group, the resource needs, and needs of each member before applying an optimal mathematical solution. Robustness studies at the group level warn about the fact that good optimization results are not always equal to generalization (Duchi & Namkoong, 2021; Sagawa et al., 2019).

Another modification is a minimax regret which puts restrictions on the losses of each group compared to the best possible solution of that group (Agarwal & Zhang, 2022). It is relevant when performance ceilings vary across groups.

6. Limitations and future work

The analysis suffers from four primary limitations. Firstly, the loss matrix is hypothetical; therefore, the gains cannot be described as classroom results. Secondly, the linear mixture model fails to take into account ordering effect, cumulative learning, fatigue, as well as interaction between the processes of instruction. Thirdly, the readiness groups are an oversimplification of learners' heterogeneity. Finally, the uncertainty extension guarantees safety only within the intervals and not beyond them.

The future research should include group-by-strategy effects estimation with one standard assessment and clear instructional circumstances. The prospective analysis will include comparing the average strategy and minimax strategies given the same time and resource allocation. The results of such comparison will include the group

sizes, average result and the results of the worst group, the uncertainty intervals, the workload of the teacher, as well as delayed retention. The learners will be re-evaluated over time in order not to label them in terms of their readiness.

7. Conclusion

The current paper has proposed a minimax model for sharing instructional opportunities among diverse groups of learners. The model seeks to minimize the maximum shortfall in learning and is solvable via linear programming. From the example constructed, the mixed minimax solution leads to an improvement of $40.00 - 24.15 = 15.85$ points in the worst-case shortfall, while there is a deterioration of $23.30 - 18.70 = 4.60$ points in the weighted average shortfall. From this result, the trade-off between efficiency and protection becomes very clear. Moreover, the use of population averages alone in instructional planning might not be adequate.

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