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# A Robust Mathematical Programming Approach for Sustainable Infrastructure Planning

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**Abstract**

Long-horizon capacity expansion decisions related to energy, water, and transport services are subject to significant uncertainty regarding demand growth, climate-induced degradation of performance, and construction cost. Capacity expansion programs using deterministic modeling that optimizes capacity expansion plans assuming a nominal forecast will often underperform when the actual conditions turn out to be different than the forecast, leading to service deficits at the moments when resilience is critical. In this paper, develop a robust mathematical programming approach to sustainable infrastructure planning by providing a mechanism for protecting capacity expansion decisions from budget-constrained parameter uncertainty and joint valuation of cost and embedded carbon emission. Using the budget-of-uncertainty model of Bertsimas and Sim, obtain a tractable linear formulation of a robust capacity expansion program in which the protection level, $\Gamma$, determines how many time periods can see worst-case performance degradation. The results show that the robust formulation is an exact linear transformation of the corresponding semi-infinite minimax problem and that the price of robustness of the problem is non-decreasing and concave in $\Gamma$. A finite sample probabilistic bound for satisfying robust constraints. For a particular instance of a problem involving six regions and five periods, In this plot the price of robustness curve, where it is demonstrated that in the worst case, a conservative plan has 25.4% higher cost and 27.4% more embedded carbon than a nominal plan, but the cost of extra protection decreases quickly as $\Gamma$ grows.

*Keywords:* Robust optimization, mathematical programming, sustainable infrastructure planning, minimax duality, uncertainty budget, capacity expansion.

## 1. Introduction

Capacity expansion of sustainable infrastructure systems, including generation of renewable energy, treatment of potable water, low-carbon transportation, and waste management, must be increased over a multi-decade planning horizon under conditions that planners are able to predict only approximately. Growth in demand, climate-induced degradation in delivered service performance such as reduced production of hydro power by drought, reduced efficiency of thermal plants due to heat waves, damage to power grid and transportation assets due to storms, fluctuations in costs of inputs, and changes in decarbonization targets all occur in a way that is, at best, bondable but not precisely predictable. Mathematical programs that formulate capacity expansion problems traditionally

optimize against a single nominal forecast of these parameters (Vázquez-Méndez et al., 2024). In the face of deviations from this forecast, which occur routinely, nominally optimal plans may fail to deliver on service goals when resilience matters most, or result in locking capital whose performance suffers from a changing climate (Sene et al., 2025; Hashemizadeh & Xie, 2026).

A solution to this problem lies in robust optimization, whereby a single-point forecast of uncertain parameters is replaced by an uncertainty set and the resulting optimization is against the worst-case outcome of the decision-maker's choice of protecting against. robust mathematical programming has become an established technique in operations research. The key property of robust optimization for infrastructure planning is that the robust counterpart of a linear program remains a linear program, ensuring that uncertainty protection does not come at the expense of computational tractability of mathematical programming (Vij & Nayak, 2025; Meidani & Ghanem, 2015).

Although the robust optimization methodology has reached a mature stage of development, its use in planning of sustainable infrastructure systems has remained limited to domain-specific, one-sector models, such as water-supply sequencing and production and scheduling under demand uncertainty. Such modeling efforts have not exposed the fundamental structure of robust programs in a way that is transferable to other infrastructure sectors (Aburasain, 2025; Zhang & Alipour, 2023). There exists, therefore, a need for (i) a robust formulation of capacity-expansion problem in terms of a sector-agnostic mathematical program that takes into account both cost of investment and embedded carbon, (ii) the exact characterization of tractability of the robust counterpart and its relationship

The contribution of this paper is four-fold. First, it introduces a deterministic program for sustainable capacity-expansion planning under a budget-constrained polyhedral uncertainty set for the effective service that can be provided by the capacity installed; second, it obtains the robust counterpart through a derivation based on duality theory for linear programs and shows that it provides an exact, not an approximate reformulation of the max-min problem (Skarbek & Kowalski, 2023). Third, it provides some results on the monotonicity and diminishing returns of the price of robustness, as well as a finite-sample probabilistic guarantee on feasibility. Fourth, it illustrates the use of the framework in a case study with six regions and five periods (Loaiza Quintana et al., 2026; Yang et al., 2016).

**Key contribution**

- A mathematical programming model based on optimization techniques is presented in order to facilitate sustainable infrastructure planning in the presence of uncertainty. This model incorporates resource allocation, cost reduction, sustainability and other constraints to facilitate reliable planning decisions.

- The proposed method includes robustness techniques in order to address uncertainties in the form of varying demands, resource constraints and environmental constraints. The method helps in finding out infrastructure plans which can cope with different scenarios of reality.

- The methodology involves multi-objective optimization approach in order to achieve a balance between economic efficiency, reduced environmental impact and increased resilience of infrastructure systems.

Section I starts with discussing the difficulties in sustainable infrastructure planning under uncertainties followed by the motivation and main contribution of the robust mathematical programming framework. Section II provides literature review on uncertainty modeling, robust optimization, and sustainable infrastructure planning. Section III formulates the deterministic capacity expansion problem and the uncertainty model for the infrastructure service degradation. Section IV gives formulation of the robust counterpart problem and shows the proof of its exact linear reformulation by applying duality theory. Section V analyzes the theoretical characteristics of the developed model which include the behavior of the robustness cost and the probabilities of feasibility of the solutions. Section VI applies the framework to solve the problem of sustainable infrastructure planning through a case study. Section VII discussion, Section VIII concludes the research by giving the summary and discussing possible future research directions including mixed integer and adjustable robust optimization frameworks.

## 2. Related work

The study of the impact of uncertainty in parameters in mathematical programming begins with who introduced linear programming under uncertainty as a decision problem as opposed to a sensitivity analysis exercise. The methods have evolved in two streams. The stochastic programming approach uses a probabilistic formulation in the form of scenarios and minimizes expectations or risk measures of costs; reviews the state of computation in this field. The robust optimization approach, on the other hand, uses deterministic and bounded sets for representing uncertainty and then minimizes the worst case over the set; no probabilistic formulation is required.

This provides a comparison between the two approaches using a problem of supply planning under uncertainty and shows that the robust approach can produce solutions that perform equivalently to the stochastic solutions in expectation but are better in the worst case.

Initial robust formulations, including the formulation, were robust to the entire box uncertainty set, where all uncertain parameters attain their worst-case value at the same time. While this ensures robust feasibility under any scenario in the box uncertainty set, it is overly conservative for practical planning because the chance of all the uncertain parameters being at their worst at once is remote. The tractable robust counterparts to ellipsoid and other convex uncertainty sets which reduce conservatism without losing convexity; To extend the counterpart derivation process to nonlinear uncertain inequalities in general. Applications of robust optimization in infrastructure planning have thus far focused on specific sectors. Which is prove that budget-of-uncertainty robust optimization is both computationally tractable and informative in infrastructure-related applications, but each has been formulated in relation to the constraints of a specific sector. The current paper generalizes the basic minimax structure in the context of a sector-neutral, multi-region and multi-period capacity expansion problem with a joint cost-and-emissions objective, and supplements the computational aspect of the cited papers by providing an exactness proof and characterizing the shape of the price-of-robustness frontier (Ugwu et al., 2006).

## 3. Problem formulation

### 3.1. Deterministic capacity-expansion program

Imagine a planner who decides to allocate capacity expansion investment over a number of regions i $\in \{1, \ldots, I\}$ and planning periods t $\in \{1, \ldots, T\}$. The decision variable $x_{it} \geq 0$ represents the new capacity that is created in region i in period t. It can be measured in terms of one capacity unit (e.g., service equivalent megawatts, megaliters per day, or passenger kilometers of transit throughput). Each unit of capacity invested in region i in period t is associated with a cost of $c_{it}$ (capital cost) and produces $e_{it}$ units of carbon (embodied carbon). The planner seeks to minimize the weighted sum of capital costs and embodied carbon,

$$min \sum_i \sum_t ( c_{it} + \lambda e_{it} ) x_{it} \qquad (1)$$

where $\lambda > 0$ is used for converting embodied carbon into a monetary unit, which can be either a shadow price on carbon or a policy-based weighting factor between environment and capital efficiency. Each unit of capacity installed in region i at time t provides $a_{it}$ amount of service; the sum of services provided should be greater than the minimum requirement of $h_i$ for each region during the planning period.

$$\sum_t a_{it} x_{it} \geq h_i , \; i = 1, \ldots, I \qquad (2)$$

Investment is additionally constrained by a capital budget B for the entire system and also by ceilings on construction throughput per period.

$$\bar{x}_{it} . \sum_i \sum_t c_{it} x_{it} \leq B , \; 0 \leq x_{it} \leq \bar{x}_{it} \qquad (3)$$

Equation (1)– (3) constitute a linear programming problem when the values of $a_{it}$, $c_{it}$, and $e_{it}$ are certain. The one parameter whose value is particularly vulnerable to forecasting errors is $a_{it}$. This is because the actual amount of service provided per unit of capacity depends on the climatic conditions (rainfall, temperature, storms) and demand-side behavior which cannot be forecast accurately over the horizon of several years which infrastructure investments have to be planned for. If assume a nominal value of $a_{it}$ as in the deterministic programming problem above, then will always understate the capacity needed to satisfy $h_i$, especially if the actual service provided turns out to be less than the nominal forecast.

### 3.2. Uncertainty model

Model the effective service coefficient as an uncertain parameter of the form

$$a_{it}(\xi) = \bar{a}_{it} - \xi_{it} \hat{a}_{it} , \; \xi_{it} \in [-1, 1] \qquad (4)$$

where $\bar{a}_{it}$ is the nominal (normal conditions) service coefficient and $\hat{a}_{it} \geq 0$ is the maximum possible level of service degradation caused by an unfavorable climate or demand realization in period t. In this do not assume that all coefficients can at the same time reach their extreme values, as in the approach of Soyster (1973), but use the budget of uncertainty approach developed by Bertsimas and Sim (2004): in any region i at most $\Gamma_i$ periods out of T can at the same time reach their worst-case value,

$$U_i(\Gamma_i) = \{ \xi \in \mathbb{R}^T : |\xi_t| \leq 1 \, \forall t , \; \sum_t |\xi_t| \leq \Gamma_i \} \qquad (5)$$

Protection parameter $\Gamma_i$ is one-dimensional and intuitive, meaning that $\Gamma_i = 0$ corresponds to the deterministic nominal program equation (4)–(5), $\Gamma_i = T$ corresponds to perfect box-uncertainty protection against simultaneous worst-case disturbances at every time period, and any value between the two corresponds to interpolation between these extremes, allowing one to bound the number of periods that can be simultaneously exposed to climatic and demand uncertainty without requiring the decision maker to define a joint probability distribution on $\xi_{it}$. Robust program of the decision maker is a modification of constraint (2), where constraint (2) is replaced by its worst-case version over $U_i(\Gamma_i)$.

## 4. The robust counterpart

The robust version of constraint (2) requires that region i's cumulative service target be met under every realization the planner protects against,

$$\sum_t \bar{a}_{it}\, x_{it} - \xi_{it}\, \hat{a}_{it}\, x_{it} \geq h_i\,,\ \forall \xi \in U_i(\Gamma_i) \qquad (6)$$

which is a semi-infinite constraint: it must hold for infinitely many realizations of $\xi$. Constraint (6) holds for every $\xi \in U_i(\Gamma_i)$ if and only if it holds for the realization that minimizes the left-hand side, so equation (6) is equivalent to

$$\sum_t \bar{a}_{it}\, x_{it} - \beta_i(x, \Gamma_i) \geq h_i\,,\ \ where\ \beta_i(x, \Gamma_i) = max\,\{\, \sum_t \xi_t\, \hat{a}_{it}\, x_{it} : \xi \in U_i(\Gamma_i)\,\} \qquad (7)$$

The function $\beta_i(x, \Gamma_i)$, called the protection function of Bertsimas and Sim (2004), is the optimal objective value of a linear program in $\xi$ for given $x$. As shown in Proposition 1, below, the inner maximization problem has an exact and finite dual linear program representation, transforming the semi-infinite constraint equation (7) into a set of linear constraints involving few additional variables.

**Theorem 1 (exact robust counterpart)**

For fixed $x \geq 0$ and $\Gamma_i \in [0, T]$, the robust constraint (6) holds if and only if there exist $z_i \geq 0$ and $p_{it} \geq 0, t = 1, \ldots, T$, such that

$$\sum_t \bar{a}_{it}\, x_{it} - \Gamma_i\, z_i - \sum_t p_{it} \geq h_i \qquad (8)$$

$$z_i + p_{it} \geq \hat{a}_{it}\, x_{it}\,,\ t = 1, \ldots, T \qquad (9)$$

Consequently, the robust counterpart of the deterministic program equation (8)– (10) is the linear program

$$min \sum_i \sum_t (\, c_{it} + \lambda\, e_{it}\,)\, x_{it}\ \ s.t. (8), (9), (3),\ x, z, p \geq 0 \qquad (10)$$

***Proof.*** Fix $x \geq 0$. The protection function $\beta_i(x, \Gamma_i)$ defined in equation (11) is the optimal value of the linear program

$$max \xi\ \sum_t \hat{a}_{it}\, x_{it}\, \xi_t\ \ s.t. \sum_t |\xi_t| \leq \Gamma_i\,,\ |\xi_t| \leq 1\ \forall t \qquad (11)$$

Writing $|\xi_t| \leq 1$ as the system of two linear inequalities, namely $\xi_t \leq 1$ and $-\xi_t \leq 1$, and $\sum_t |\xi_t| \leq \Gamma_i$ as a constraint on the auxiliary variable, to see that (11) is a bounded linear program (the feasible region is a nonempty polytope because $\Gamma_i \geq 0$ and the objective is linear, thus bounded on the polytope), and strong duality for linear programs holds. Introducing the dual variable $z_i \geq 0$ corresponding to the budget constraint $\sum_t |\xi_t| \leq \Gamma_i$ and the dual variables $p_{it} \geq 0$ corresponding to the T "box" constraints $\xi_t \leq 1$, the dual of equation (12) is

$$min\ over\ (z_i, p_{it})\ :\ \Gamma_i\, z_i + \sum_t p_{it}\ \ s.t. z_i + p_{it} \geq \hat{a}_{it}\, x_{it}\, \forall t\,,\ z_i, p_{it} \geq 0 \qquad (12)$$

which is precisely the right-hand side of equation (8)– (9) once minimized. By strong duality, the optimal value of equation (13) equals $\beta_i(x, \Gamma_i)$. Substituting this dual minimum for $\beta_i(x, \Gamma_i)$ in equation (7), the robust constraint holds if and only if

$$\sum_t \bar{a}_{it}\, x_{it} - min\{\, \Gamma_i\, z_i + \sum_t p_{it} : (9), z_i, p_{it} \geq 0\,\} \geq h_i \qquad (13)$$

which is true if and only if there exist $z_i \geq 0, p_{it} \geq 0$ satisfying (9) and $\sum_t \bar{a}_{it}\, x_{it} - \Gamma_i\, z_i - \sum_t p_{it} \geq h_i$, i.e., equation (8). The reason is that the minimum of a linear function on a nonempty polyhedron is attained, so demanding that the constraint be satisfied for the minimizing ($z_i$, $p_{it}$) is equivalent to demanding it be satisfied for some feasible $(z_i, p_{it})$ satisfying equation (8)– (9); each such feasible $(z_i, p_{it})$ gives an upper bound on the true minimum and hence a sufficient certificate, while the minimizing $(z_i, p_{it})$ is in particular one.

Theorem 1 shows that no conservatism is introduced by the reformulation: program equation (10) is not an approximation of the robust program associated with equation (6); it is its exact linear programming equivalent. The only conservatism in the formulation is the modelling choice of the uncertainty set $U_i(\Gamma_i)$ and the protection level $\Gamma_i$, both of which remain entirely under the control of the planner. The reformulation introduces $I(T+1)$ auxiliary variables (one $z_i$ and T $p_{it}$ per region) and $I(T+1)$ constraints to the deterministic program; thus, the robust program remains a linear program of comparable size to the nominal problem and can be solved by any general-purpose linear programming technique, including interior-point and simplex methods.

## 5. Theoretical properties

Two properties of the robust program (10) are of direct practical interest to a planner choosing a protection level: how the optimal cost changes as Γ increases (the price of robustness), and what probabilistic guarantee a given Γ confers once uncertainty is realized stochastically rather than Adversarially.

**Proposition 1 (monotonicity and concavity of the price of robustness)**

Let $z*(\Gamma)$ denote the optimal objective value of (10) for uncertainty budget $\Gamma = (\Gamma_1, \dots, \Gamma_i)$, uniformly extended to $\Gamma \in [0, T]$. Then z*(Γ) is non-decreasing and concave in each $\Gamma_i$.

***Sketch.*** Monotonicity is established since $U_i(\Gamma_i) \subseteq U_i(\Gamma_i')$ whenever $\Gamma_i \leq \Gamma_i'$; thus, the robust feasible region given by (8)– (9) is weakly nested and contracting in $\Gamma_i$ ;contraction of the feasible region in a minimization program cannot result in an objective value decrease. Concavity is due to the structure of the protection function $\beta_i(x, \Gamma_i)$ in (7): for fixed x, sorting the terms $\hat{a}_{it}\, x_{it}$ in descending order, $\beta_i(x, \Gamma_i)$ at integer $\Gamma_i$ is given by the sum of $\Gamma_i$ largest terms (Bertsimas & Sim, 2004), and hence the marginal gain from $\Gamma_i \; to \; \Gamma_i + 1$ is the $(\Gamma_i + 1)$-th largest term, forming a non-increasing sequence. Therefore, , $\beta_i$ is piecewise linear and concave in $\Gamma_i$ for fixed x, and $z*(\Gamma)$ (as a minimization program where $\Gamma$ appears only in the right-hand side of a single constraint, namely the one involving a subtraction of the $\beta_i(x, \Gamma_i)$ term, which is a concave function in $\Gamma_i$) is the pointwise infimum of a family of functions affine and non-increasing in $\Gamma_i$ for each fixed feasible x; while in general infimum of concave functions need not be concave, in the present case the fact that the feasible region is given by an inequality of the form (8), involving $\Gamma_i\, z_i$ with $z_i \geq 0$ and hence allowing adjustment of $z_i$, guarantees concavity of the optimal-value function as the concave conjugate-type construction typical of parametric linear programming; see Bertsimas & Sim (2004, Section 3) for the present class of budget-constrained robust counterparts.

Practically, the above proposition implies that the marginal price of increased protection is maximized when departing from the unprotected nominal plan and decreases as Γ goes close to its upper limit T. Thus, a planner extracts most of the potential robustness at a significantly lower price than the one of full box-uncertainty protection, as numerically shown in Section 7. Proposition 2 (Probabilistic guarantee under stochastic realizations).

Assume instead that the deviations $\xi_{it}$, rather than being the adversarial realization implied by ξ, are independent, symmetric, random variables uniformly distributed over [−1, 1]. Applying the result of Bertsimas and Sim (2004, Theorem 3) to the constraints of region i over its T time periods, the probability that the realized left-hand side of constraint (2) is less than the guaranteed level of the robust solution at protection $\Gamma_i$ is upper-bounded by a value that can be approximated for reasonably large T by

$$Pr\,[\,violation\,] \lesssim exp(-\Gamma_i^{\,2} / (2T)) \qquad (14)$$

Bound equation (14) is asymptotic and needs to be interpreted as such, not as a precise finite sample bound, while the combinatorial bound from Bertsimas and Sim (2004) should be employed when a numerical bound is needed. Nevertheless, the interpretation provided by the qualitative content of this bound is very valuable for practitioners: since the bound depends on $\Gamma_i^2$, but not on $\Gamma_i$ even relatively low values of protection that are much lower than the totally conservative level $\Gamma_i$ = T give some non-zero chance of violating constraints in stochastic cases.6. Solution Procedure

Program equation (10) is a linear program of size polynomial in I and T: it involves I•T decision variables $x_{it}$, I auxiliary variables $z_i$, and I•T auxiliary variables $p_{it}$, subject to I service-level constraints, I•T linking constraints, one budget constraint, and 2•I•T bound constraints. For problem instances of the sizes relevant to regional infrastructure planning, with a few tens to a few hundred region-periods, program equation (10) is routinely solved via standard interior-point or dual-simplex methods; in the example in Section 6 used a dual-revised - simplex solver by Huangfu & Hall (2018) available in a general-purpose linear programming software package (Palash & Dhurvey, 2024).

The overall procedure is as follows. First, the planner gathers nominal service coefficients $\bar{a}_{it}$, maximum degradations $\hat{a}_{it}$, service targets $h_i$, unit costs, $c_{it}$, unit embodied emissions $e_{it}$, construction ceilings $\bar{x}_{it}$, and system

budget B based on engineering and climatic data. Second, the planner chooses protection levels $\Gamma_i$, either uniform across regions or tailored to individual region's climate risk profile; see Section 7 for an illustration of the uniform sweep as well as the regional implications. Third, program equation (10) is solved for the chosen $\Gamma$; the exact nature of the reformulation in Theorem 1 makes it unnecessary to perform simulations or scenario enumeration to find a worst-case-feasible solution. Fourth, in the case that the planner cannot decide on the optimal protection level, the entire price-of-robustness frontier can be found by solving equation (10) again for a number of Γ values in a grid; the computational effort involved does not exceed a small multiple of the effort of solving a single instance, as each of the solves is a linear program and the feasible region only varies through the structure of the right-hand side in equation (8).

Two possible extensions are natural but not part of the linear-programming class addressed above. Should the location decisions have a discrete nature, that is, the selection of one out of a number of possible facility locations, then the $x_{it}$terms are constrained to be integers, and formulation equation (10) turns into a mixed-integer linear program. Theorem 1 holds even for the relaxation of the robust constraints to the continuous case; hence, the reformulation can be readily included within the standard branch-and-bound scheme. In case of capacity decisions depending on realized information in time and not solely defined at t=0, then the model generalizes to adjustable robust optimization based on affine decision rules, or by column-and-constraint generation (Zhang et al., 2016).

## 6. Case Study

### 6.1. Data and setup

To apply the proposed framework to a synthetic yet representative planning problem involving I = 6 zones with different levels of climate exposure, namely Coastal Zone, Urban Core, Rural District, Industrial Corridor, Highland Basin, and River Delta, through T = 5 periods. Climate-related degradation factors $\hat{a}_{it}$ and nominal service rates $\bar{a}_{it}$ were sampled to be consistent with realistic climate derating patterns (12% to 30% reduction in worst-case service relative to nominal), build costs $c_{it}$and embedded-emissions rates $e_{it}$ were taken to exhibit modest year-on-year improvements, and per-period build caps $\bar{x}_{it}$ were taken to be consistent with realistic permits and supply chains. The total capital budget is B = USD 2,650 million and the emissions weight is $\lambda = 18$ (USD million per kt $CO_2e$). Complete parameter values are available from the authors on request; the conclusions below hold up to modest changes in parameter values because are based on the structural properties described in Section 5.

### 6.2. Price-of-robustness frontier

For problem (10), the optimal solution was calculated for the constant protection level $\Gamma_i = \Gamma \in \{0, 1, 2, 3, 4, 5\}$ for all six regions, employing the dual-simplex algorithm proposed by Huangfu and Hall (2018). Table 1 shows the value of the build cost and its marginal increment for different Γ values.

**Table 1.** Price-of-robustness frontier: total and marginal cost and emissions by protection level Γ

| Γ | Total cost (USD million) | Marginal cost (USD million) | Total emissions (kt $CO_2e$) | Marginal emissions (kt $CO_2e$) |
|---|---|---|---|---|
| 0 | 1,854.80 | – | 250.85 | – |
| 1 | 2,063.39 | 208.59 | 280.64 | 29.79 |
| 2 | 2,176.44 | 113.05 | 297.52 | 16.88 |
| 3 | 2,269.41 | 92.97 | 311.05 | 13.53 |
| 4 | 2,314.35 | 44.94 | 317.57 | 6.52 |
| 5 | 2,326.73 | 12.38 | 319.64 | 2.07 |

Transition from the nominal strategy ($\Gamma = 0$) to complete protection ($\Gamma = 5 = T$) implies that total build cost rises by 25.4% – from USD 1,854.8 million to USD 2,326.7 million, while embodied emissions increase by 27.4% – from 250.8 to 319.6 kt $CO_2e$. In accordance with Proposition 1, both cost and emissions functions are concave with respect to Γ.The price of the first unit of protection ($\Gamma: 0 \rightarrow 1$) equals USD 208.6 million, whereas the price of the fifth unit (Γ: 4→5) is just USD 12.4 million – a seventeen-fold decrease in marginal price.

### 6.3. Regional reallocation of capacity

Table 2 compares the nominal ($\Gamma = 0$) and fully robust ($\Gamma = 5$) capacity allocation by region.

**Table 2.** Regional capacity allocation under the nominal and fully robust plans

| Region | Service target $h_i$ | Nominal capacity ($\Gamma = 0$) | Robust capacity ($\Gamma = T$) | Increase |
|---|---|---|---|---|
| Coastal Zone | 90 | 100.67 | 127.48 | +26.6% |

| | | | | |
|---|---|---|---|---|
| Urban Core | 130 | 150.44 | 195.30 | +29.8% |
| Rural District | 70 | 70.38 | 82.75 | +17.6% |
| Industrial Corridor | 110 | 105.85 | 130.86 | +23.6% |
| Highland Basin | 60 | 59.53 | 71.68 | +20.4% |
| River Delta | 85 | 93.94 | 104.40 | +11.1% |

Two trends can be observed. The first one is that the redistribution of capacity to the protection effort is not uniform among regions: Region River Delta, which has a nominal allocation with the largest cushion to its service target (93.94 units of capacity versus a target of 85 units), has the smallest proportional increase in capacity under the full protection (11.1%), while Region Urban Core, which has a nominal allocation relatively close to its target, has the largest proportional increase (29.8%). This follows directly from the region-specific protection functions $\beta_i(x, \Gamma_i)$in (7): regions with a larger variance of the uncertain coefficients $\hat{a}_{it}\, x_{it}$ are subjected to a larger penalty function for the same level $\Gamma_i$ and therefore, need a relatively higher level of capacity to stay feasible. The second trend is that each region has a nominal allocation exceeding its service target because of the capital budgeting constraint (3) tying all six regions together, with the optimal allocation moving the marginal capacity to lower cost units even prior to robustification.

### 6.4. Sensitivity to the protection level

Since Theorem 1 establishes that program (10) will always remain a linear program regardless of $\Gamma$, re-running the whole grid from Table 1 took just a fraction of a second, demonstrating that the computation of the whole price of robustness curve is a task computationally undemanding beyond solving the nominal program itself. This is an important practical feature because it allows a decision-maker to produce the whole trade-off curve as a decision support artifact and choose the value of $\Gamma$ (or $\Gamma_i$ for each region separately) based on either risk aversion of the organization or regulatory requirements, without having to pay the modeling costs for that.

## 7. Discussion

There are several ways in which the model developed in this paper provides a planner with an advantage over purely deterministic capacity expansion modeling on the one hand and heuristic and simulation approaches to modeling uncertainty on the other. First, $\Gamma$ is a simple, auditable parameter with a clear operational interpretation, the number of worst-case time periods that may simultaneously occur, rather than an abstract risk aversion coefficient or a probability distribution that is hard to justify to a stakeholder without mathematical training. A regulator, board of directors or rate-setting authority can be shown the whole price-of-robustness curve from and asked to pick a protection level that corresponds to its resilience policy (Ruiz-Vélez et al., 2024). Second, since Theorem 1 proves that the robust formulation is equivalent to the linear programming model, a planner receives an accurate guarantee rather than an approximation, while the computational complexity of obtaining it is essentially equal to that of the deterministic model, which requires neither Monte Carlo simulation nor building a scenario tree; it requires just a small increase in the number of decision variables discussed in Section 6. Finally, the cost and emission objective function (1) results in the price-of-robustness curve being stated in terms of carbon dioxide emissions, which is more useful for sustainability reporting (Zhang et al., 2023).

The example also raises an important caveat regarding constraint (3), the capital budget limit for the entire system: since the budget constraint applies to all regions together, changes in capacity due to increased protection are not proportional to the individual exposures of the regions involved, as table 2 demonstrates. Thus, a region, even one with low uncertainty parameters in isolation, may experience a change in its capacity allocation due to changes in how the capital budget is allocated in response to the worst-case scenario being drawn by other regions. Practitioners wishing to use the model for systems with independent budgets for each region should be careful to separate constraint (3) when applying the model.

There are several limitations in how the results of the analysis can be applied. First, the model assumes the decisions on capacity are continuous and single-period; therefore, it does not allow for discrete location decisions on facilities, requiring the use of the mixed-integer extension (Section 6), or for capacity decisions in subsequent periods based on service realization in previous periods, requiring the use of adjustable robust optimization (Riepin et al., 2022). Second, the uncertainty model (4)-(5) allows uncertainty only in the effective service coefficient $a_{it}$; however, the extension of the budget-of-uncertainty approach to correlation in uncertainty in cost $c_{it}$ or emissions $e_{it}$ is trivial by a similar argument as in Theorem 1 but was not considered here in order to keep the focus clear. Finally, the case study is conducted using synthetic data based on realistic values and does not correspond to a particular named system of infrastructure. Therefore, while the qualitative observations (price-of-robustness function's concavity and uneven regional reallocation in the presence of a shared budget) are valid based on the results of Section 5 and should apply to real systems, the particular tradeoff shown in table 1 should only be interpreted as an example.

## 8. Conclusion

The development of a mathematical programming framework for robust sustainable capacity expansion is accomplished, where the balance between cost efficiency and protection from bounded budget-constrained parameter uncertainty is mathematically formalized. The proposed robust capacity expansion is based on the Bertsimas and Sim (2004) approach for constructing polyhedral uncertainty sets and is shown to be exactly equivalent to the linear program reformulation of the underlying semi-infinite minimax problem, while the price of robustness is demonstrated to be a monotonic and concave function of the level of protection; moreover, probabilistic guarantees for satisfying uncertain constraints in the face of stochastic realization of uncertainties are provided. The price of robustness is calculated for a six-region and five-period numerical example where it was demonstrated that the computation of the full price of robustness frontier entails negligible computational overhead beyond that for solving once the deterministic program, that the cost of the fully robust solution amounts to roughly 25% of the cost of the nominal solution both in capital investment and emissions terms, and that the marginal cost of protection quickly approaches zero. Further developments include mixed-integer robust optimization to address discrete location decisions, adjustable robust optimization through affine decision rules and/or column-and-constraint generation, and distributionally robust optimization which does not assume a known bound on the support of distributions.

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