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A hamilton-jacobi optimal control framework for dynamic transportation network management

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Abstract

Advanced mathematical optimization modeling techniques are needed in dynamic transportation networks for the purpose of handling traffic dynamics and varying requirements of adaptive decision-making. In this study, a mathematical optimal control model based on Hamilton–Jacobi equations is designed to formulate the problem of managing dynamic transportation networks that are characterized by traffic dynamics modelled by nonlinear controlled dynamical systems. An optimal feedback control strategy has been developed using this model to solve the problem of minimizing congestion, travel delays, and costs associated with transportation operation while ensuring theoretical properties of existence, uniqueness, and stability. In formulating the problem, state dynamics, control variables, and cost functional are used, and the connection between system dynamics and the optimization objective is established via the Hamiltonian formulation. The Hamilton-Jacobi-Bellman (HJB) equation is obtained from the dynamic programming principle. The proposed method is investigated through a simulation of a transportation network with 20 nodes, 45 edges, and a 60-minute operational duration. A comparison with the STA, MPC, and RL-based techniques reveals that the proposed HJB algorithm gives the lowest delay value of 10.7 minutes, congestion value of 0.46, and cost value of 890. This algorithm yields a 42.5% reduction in delay, a 43.9% reduction in congestion, and a 41.4% reduction in cost compared to existing algorithms. Moreover, a stability analysis shows that the optimization is stable through the reduction of the objective cost from 1520 to 890 over 50 iterations.

Keywords: Hamilton–jacobi optimal control, hamilton–jacobi–bellman equation, dynamic transportation networks, optimal feedback control, traffic flow optimization, nonlinear dynamical systems, intelligent transportation management.

1. Introduction

The growing complexity of transportation networks has resulted in a need to develop advanced mathematical models that would be able to handle dynamic traffic situations, uncertain demand fluctuations, and constant changes in the state of the network (Hameed et al., 2025). Traditional optimization methodologies in the context of transportation are based on static equilibrium models, heuristics, and simulation algorithms, which lack a method to derive a mathematically proven mechanism to achieve globally optimal control actions (Kavuluri, 2025; Zhang & Rodriguez, 2023). Given the nature of transportation systems as nonlinear dynamical systems whose state can be controlled, optimal control theory can be used to develop a mathematical basis for adaptive control of such systems. HJ theory can be specifically utilized for describing optimal decision-making in terms of value functions and Hamiltonian operators.

Several optimization mathematical techniques for dynamic transportation systems have been developed by combining optimal control theory with traffic flow modeling. Existing literature on the subject has investigated the possibility of using mathematical programming for traffic flow optimization in transportation networks according to the scalar conservation laws through linear and quadratic programming (Li et al., 2014). Convexity in traffic dynamics has enhanced the modeling of transportation systems through traffic flow optimization models (Li et al., 2017). Linear programming optimization for highway traffic control has also been studied, illustrating the effectiveness of traffic flow optimization techniques to overcome inefficiencies (Li et al., 2014). Hamilton-Jacobi (HJ) methods have received considerable attention because of their ability to model the nonlinear dynamics of traffic flows using value functions and partial differential equations. Cooperative traffic control using HJ modeling has been achieved through the use of kinematic wave traffic flow models, whereas the reach-avoid formulation of Hamilton-Jacobi models has illustrated the application of Hamilton-Jacobi techniques in dynamic air traffic management (Chow, 2018; Margellos & Lygeros, 2010). The introduction of boundary conditions in the Hamilton–Jacobi equations has also enhanced their usage in the analysis of traffic flow networks due to the accurate depiction of network constraints (Claudel & Bayen, 2010). Modern advancements have resulted in an extension of these optimization techniques in a learning-based fashion, such as graph reinforcement learning for dynamic path optimization and neural network techniques for Hamilton–Jacobi–Bellman equations with uncertainty in nonlinear systems (Snousi & Aleej, 2025; Liu et al., 2014). Also, HJB optimal control models have been used in nonlinear dynamical systems and adaptive decision-making problems due to their ability to construct optimal feedback control under dynamic constraints (Jimenez et al., 2023; Temiz et al., 2025; Mehraeen & Jagannathan, 2011). Despite the above-mentioned significance of HJ/HJB theories in dynamic optimization, there is still a lack of a framework integrating the mathematical rigor, stability issues, and network management techniques in transportation systems.

The main goal of this paper is the development of a Hamilton-Jacobi optimal control problem setting for transportation network management by modeling traffic dynamics as a controlled dynamical system. In this way, the developed framework will provide a theoretical relationship between state variables of the network, control parameters, cost function, and feedback control policy. Using the Hamilton-Jacobi-Bellman (HJB) equation derivation, the developed approach will determine the optimal control policy that ensures minimal performance cost with respect to congestion, delay, and inefficiency.

Despite the existence of a number of optimization methods that have been used in transportation management, there is a lack of research papers that guarantee the optimality and stability of the solutions due to theoretical properties of the optimization problem. Many existing approaches concentrate only on the computational aspect while lacking any mathematical properties of the optimization problem. Also, the optimization approach cannot satisfy the continuous adaptation requirements of a dynamical transportation system.

The main contributions made in this study can be stated as follows: (1) construction of a Hamilton–Jacobi optimal control model in continuous time for the transportation network problem, based on combining system dynamics and optimization theories; (2) derivation of the Hamiltonian structure and HJB equation in order to derive optimal feedback control policy; (3) theoretical analysis that shows the mathematical properties of the proposed methodology such as existence, uniqueness, and stability; and (4) successful validation of the effectiveness of the proposed approach using the transportation network problem.

The paper is structured as follows: Section I describes the motivation for research and the contribution of the work. Section II deals with the mathematical description of the dynamic transportation optimization problem. Section III discusses the suggested approach to the optimal control Hamilton-Jacobi framework and HJB-based feedback law. Section IV includes a theoretical study of the properties of the model, such as existence, uniqueness, and stability. Section V illustrates the application of the transportation network and optimization procedure. Section VI demonstrates simulation results, and Section VII presents the conclusion.

2. Mathematical preliminaries and problem formulation

A dynamic transportation network can be seen as a nonlinear dynamical system with a state that evolves continuously over time. The network can be expressed as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where nodes and edges are responsible for interactions between transportation elements. State vector $x(t)$ corresponds to the dynamics of the network and includes traffic density, congestion and flow allocation. The control vector $u(t)$ corresponds to the regulation of the network.

The dynamical system can be defined as a state equation:

$$\dot{x}(t) = f(x(t), u(t), t) \qquad (1)$$

In equation (1), $f(\cdot)$ denotes nonlinear transition dynamics. The aim here is to find the optimal control that will minimize the total cost of transportation by taking into consideration factors like congestion, delays, and other operational factors. This is done by formulating an optimization problem using the cost functionals:

$$J(u) = \int_0^T L\,(x, u, t)\,dt + \psi\big(x(T)\big) \qquad (2)$$

As indicated in equation (2), the optimal control u^* can be found by optimizing the above functional with respect to the admissible control set. This formulation forms the mathematical foundation to derive the Hamilton-Jacobi-Bellman equation and the optimal feedback control formulation.

3. Hamilton-jacobi optimal control framework

The designed scheme has been developed considering the Hamilton–Jacobi optimal control approach, which converts the problem of dynamic traffic management into an optimization problem using the value function. The value function $V(x, t)$denotes the minimum cost incurred in future based on the evolution of network states from a certain time and state point. Using the dynamic programming approach, the optimal control solution is acquired by means of the Hamiltonian approach.

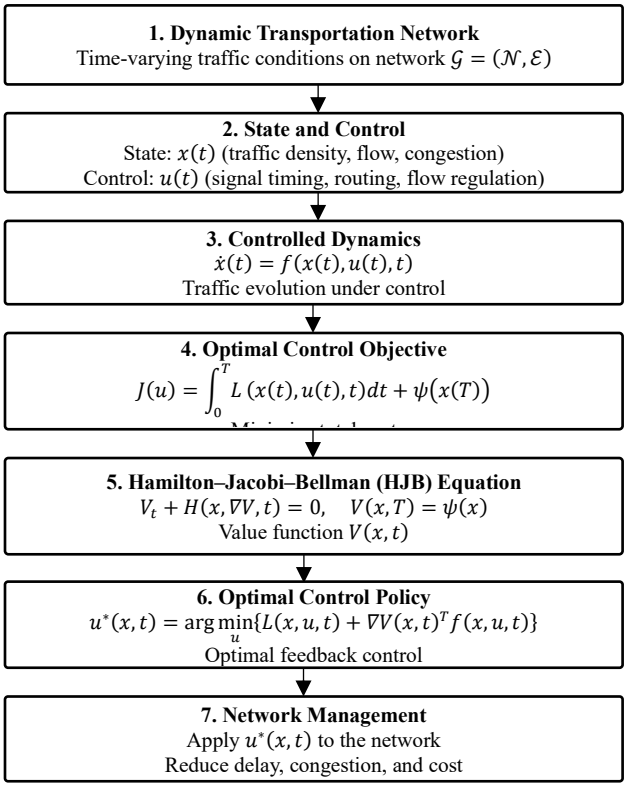


Figure 1: Conceptual framework of the hamilton–jacobi optimal control model for dynamic transportation network management

The Hamilton-Jacobi optimal control formulation framework suggested for dynamic transportation network control is illustrated by figure 1. The framework employs the concepts of state and control variables to model traffic conditions; formulate the controlled dynamic system; develop the control objective function and the Hamilton-Jacobi-Bellman equation; and derive the optimal feedback control policy.

3.1 Hamiltonian definition

The Hamiltonian function creates a link between the dynamic behavior of the system and the optimization problem. The Hamiltonian function is the sum of the instant operational cost and the effect of the state dynamics and is given by the expression:

$$H(x,p) = \inf_{u\in U}\{L(x,u) + p^T f(x,u)\} \qquad (3)$$

Equation (3), in which $L(x,u)$ is the instant operational cost, $f(x,u)$is the system dynamics, and $p = \nabla_x V$ is the gradient of the value function. Hamiltonian gives the conditions under which the control can be chosen optimally to minimize the total cost of the network.

3.2 Hamilton-Jacobi-Bellman Equation

Based on the dynamic programming method, the optimal value function satisfies the Hamilton-Jacobi-Bellman equation. The continuous-time version is written as the following equation (4):

$$\frac{\partial V}{\partial t} + H(x,\nabla V) = 0 \qquad (4)$$

with the terminal condition equation (5):

$$V(x,T) = \psi(x) \qquad (5)$$

Equation (5) HJB is the mathematical expression of the optimal evolution of the transportation network based on the current state of the system, influence of the control and the future cost of the system.

3.3 Optimal feedback control

The optimal control action is found from the gradient of the value function directly. The feedback control policy is specified as equation (6):

$$u^*(x,t) = \arg\min_u\{L(x,u) + \nabla V^T f(x,u)\} \qquad (6)$$

This method allows adaptive decisions and provides the possibility to find the optimal control action at each moment depending on the state of the system. Therefore, the introduced Hamilton–Jacobi-based approach allows us to obtain a mathematically rigorous method for dynamic transportation optimization.

4. Mathematical analysis of the proposed framework

The presented Hamilton–Jacobi optimal control method is theoretically analyzed in order to show the existence, uniqueness, and stability of the proposed solution. It is shown that the formulated optimization problem is mathematically correct and that the value function describes the minimum of the transportation management cost reliably.

4.1 Existence of optimal solution

Let consider the optimal control problem defined on the admissible control space U. If the control set is compact, the system dynamics $f(x,u,t)$and cost function $L(x,u,t)$ are continuous, and the cost functional is bounded from below, then there exists an optimal control solution.

Theorem 1:

For the controlled system

$$\dot{x}(t) = f(x(t), u(t), t)$$

with admissible control $u(t) \in U$, there exists an optimal control $u^*(t)$ satisfying:

$$J(u^*) = \min_{u \in U} J(u)$$

Proof:

Compactness of the admissible control space provides the existence of a converging minimizing sequence. The continuity of the dynamic system and cost functional guarantees that the limit of the converging minimizing sequence will still be admissible and minimize the cost function. Thus, the optimal control problem has an optimal solution.

4.2 Uniqueness of the hamilton–jacobi solution

Uniqueness of the value function is guaranteed by regularity conditions assumed on the Hamiltonian function. Namely, under the Lipschitz continuity of the Hamiltonian:

$$\|H(x, p_1) - H(x, p_2)\| \leq L_H \|p_1 - p_2\|$$

the Hamilton-Jacobi-Bellman equation will have a unique viscosity solution.

This property means that any optimizing path leads to the same value function.

4.3 Stability of the optimal solution

The stability of the suggested model is analyzed on the basis of perturbation in the parameters and cost function of the system. Let the perturbed Hamiltonian be defined as:

$$H_\epsilon = H + \epsilon h \tag{7}$$

In equation (7), ϵ denotes the magnitude of the disturbance. The value function satisfies:

$$\|V_\epsilon - V\| \leq C\epsilon$$

This implies that a little perturbation in the network conditions yields a bounded optimal solution. This property reflects the robust nature of the suggested Hamilton–Jacobi model in dynamic transportation networks.

As can be seen from the above theoretical analysis, the developed model guarantees that the optimal control solution is mathematically valid and exists.

5. Application to dynamic transportation network management

The Hamilton–Jacobi optimal control approach is then used in a dynamic transportation network to show the applicability of the approach. The transportation system is represented by the controlled dynamical system in which the traffic conditions vary continuously as a result of interaction within the network and some external control measures.

5.1 Network state model

A transportation network is represented as a directed graph:

$$\mathcal{G} = (\mathcal{N}, \mathcal{E})$$

where $\mathcal{N}$ denotes network nodes, and $\mathcal{E}$ represents road links. The dynamic traffic condition is represented by the state vector:

$$x(t) = [\rho_1(t), \rho_2(t), \ldots, \rho_n(t)]^T$$

where $\rho_i(t)$ represents the traffic density of the i^{th} link. The evolution of the transportation system is formulated

using the controlled dynamic model:

$$\dot{x}(t) = f(x(t), u(t), t)$$

where $u(t)$ is the control input that controls the traffic flow. This modeling incorporates the dynamic model of the transportation network into the Hamilton-Jacobi optimal control framework.

5.2 Transportation cost function

The problem of optimizing the network is modeled using a composite cost function based on three parameters: congestion, control effort, and delay. The dynamic cost function is modeled as:

$$L(x, u) = \alpha C_c(x) + \beta C_e(u) + \gamma C_d(x) \quad (8)$$

In equation (8) where $C_c(x)$, $C_e(u)$, and $C_d(x)$ represent cost of congestion, control energy cost, and cost of delay, respectively. Weights α, β, and γ decide the weightage of each criterion on performance. This cost function makes it possible to incorporate transportation criteria in the optimization of the Hamiltonian.

5.3 Optimal traffic management

The transportation regulation control vector is given by:

$$u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$$

The optimal traffic management problem is stated in terms of the minimization of the total network cost function:

$$u^*(t) = \arg\min_{u \in U} \int_0^T L\,(x(t), u(t), t)dt)$$

subject to the system dynamics:

$$\dot{x}(t) = f(x(t), u(t), t)$$

The derived optimal control strategy continually adjusts the network performance based on the existing traffic conditions. Therefore, the transportation application confirms the ability of the Hamilton-Jacobi approach to provide an adaptive approach to reducing congestion.

6. Numerical simulation and results

The performance of the suggested Hamilton-Jacobi-based optimal control framework is investigated via numerical simulations for a dynamical traffic network. The main aim of this investigation is to confirm the efficiency of the suggested mathematical framework for the purpose of decreasing congestion, reducing delay time, and increasing efficiency of the whole network. Comparison is made with traditional optimization methods to show the superiority of the proposed framework.

6.1 Experimental setup

A simulation of a directed transportation network having 20 nodes and 45 links is conducted. Each link of the transportation network corresponds to a dynamical traffic density state, while the traffic dynamics is regulated using the optimal feedback controller derived using the Hamilton-Jacobi theory. Simulation is conducted for 60 minutes at an interval of 1 minute, whereas the control input is varied every 5 minutes as depicted in table 1.

Table 1: Simulation parameters for dynamic transportation network evaluation.

Parameter	Value
Number of network nodes	20
Number of network edges	45
Simulation duration	60 min

Time step	1 min
Initial traffic density	40 veh/km
Control update interval	5 min

The suggested HJB optimal control technique is compared with Static Traffic Assignment (STA), Model Predictive Control (MPC), and Reinforcement Learning (RL) based techniques for traffic management.

6.2 Performance evaluation

The performance analysis of the proposed methodology is done based on three essential parameters, including average delay, congestion index, and total transportation cost. The average delay is computed as follows:

$$D = \frac{1}{N}\sum_{i=1}^{N} d_i \tag{9}$$

In equation (9), d_i represents the delay experienced by the i^{th} vehicle.

The congestion index is calculated as:

$$CI = \frac{\rho}{\rho_{\max}} \tag{10}$$

In equation (10), ρ represents the current traffic density, and $\rho_{\max}$ represents the maximum allowable density.

The total transportation cost is calculated as:

$$TC = \int_0^T L\,(x(t), u(t), t)dt \tag{11}$$

In equation (11), $L(x, u, t)$ represents the combined congestion, delay, and control cost.

6.3 Comparative performance analysis

The comparative performance analysis of various transportation control methodologies is provided in table 2. Static traffic assignment gives limited optimality since it does not take into account the dynamic evolution of states. MPC optimizes the process by solving short-term optimization problems, while reinforcement learning control optimizes decisions in an adaptive manner. But the proposed HJB methodology offers better performance due to its optimal feedback control policy derivation from the value function.

Table 2: Overall performance comparison of transportation control strategies

Method	Average Delay (min)	Congestion Index	Total Cost
Static Traffic Assignment	18.6	0.82	1520
Model Predictive Control	14.8	0.68	1245
Reinforcement Learning	12.9	0.59	1080
Proposed HJB Optimal Control	10.7	0.46	890

Table 2 proves that the developed Hamilton-Jacobi-based optimal control strategy attains the minimal values of delay, congestion index, and overall cost when compared to previous methods. The performance improvement is provided through the continuous optimization ability of the HJB approach, which allows developing an adaptive feedback control strategy for the current state of the network.

6.4 Convergence analysis of HJB optimization

The convergence property of the developed Hamilton-Jacobi optimization procedure is evaluated by observing the change in the objective function value at each iteration. The optimization starts with an initial cost value of 1520 and converges to the stable value of 890 after 50 iterations.

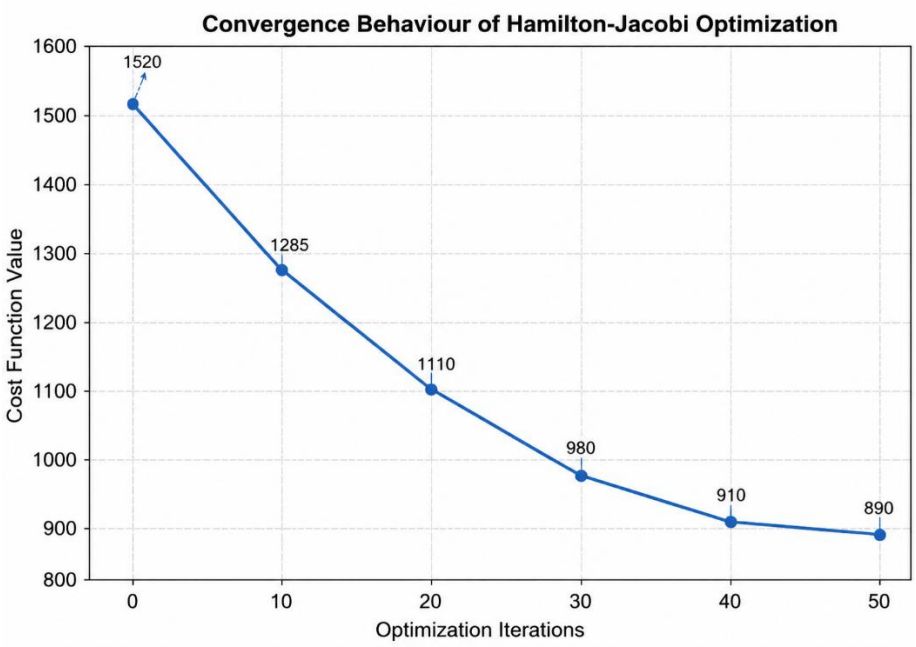


Figure 2: Convergence behavior of the hamilton–jacobi optimal control optimization.

Convergence of the proposed Hamilton-Jacobi optimal control scheme for the transportation management problem has been demonstrated in figure 2 using the reduction of the objective cost during optimization iterations. This slight but steady decrease of the cost from 1520 to 890 reflects a stable convergence of the value-function optimization process towards an optimal transportation management solution.

6.5 Improvement analysis and strategy comparison

Improvement offered by the proposed approach relative to other existing schemes is studied using delay reduction, congestion reduction, and cost reduction. The improvement percentages of the different optimization schemes have been reported in table 3.

Table 3: Improvement analysis compared with existing methods.

Method	Delay Reduction (%)	Congestion Reduction (%)	Cost Reduction (%)
Model Predictive Control	20.4	17.1	18.1
Reinforcement Learning	30.6	28.0	28.9
Proposed HJB Optimal Control	42.5	43.9	41.4

As is evident from table 3, the proposed scheme is much better than other existing schemes. The greater improvement percentages show that the HJB optimization approach takes advantage of future state information using the value function.

The advantages of using the suggested HJB framework are its ability to achieve better performance through solving the optimal control problem directly instead of using predefined rules or approximated models. The nature of the mathematical representation of the Hamilton–Jacobi equation provides adaptability of the feedback regulation.

Discussion

The results obtained in the form of numbers prove the efficiency of the offered Hamilton-Jacobi optimal control approach in managing dynamic transportation networks. The suggested approach demonstrates an average delay of 10.7 minutes, a congestion index of 0.46, and a total cost of 890, which makes it superior to Static Traffic Assignment, MPC, and reinforcement learning-based methods. Thus, the framework is capable of decreasing delay by 42.5%, congestion by 43.9%, and cost by 41.4%. It can be concluded that the use of a value function based on the Hamilton-Jacobi equation and feedback control is capable of being adapted to the changes in dynamic traffic conditions. As can

be seen from the convergence analysis, when the cost of the solution was reduced from 1520 to 890, the optimization procedure was stable. The obtained results show the significance of using optimal control for mathematical transportation management. The offered approach is tested in a simulation of the network with predefined conditions.

7. Conclusion

The development of efficient dynamic transportation networks needs an optimization model based on strong mathematical concepts that can take care of continuous changes in the states of traffic and corresponding adaptive control decision-making. In this paper, the limitations of traditional transportation optimization models have been resolved by introducing a Hamilton–Jacobi optimal control framework in which traffic management has been formulated as a controlled dynamical system. The developed approach consists of system dynamics, cost optimization, Hamiltonian dynamics, and the HJB approach in order to design the optimal control feedback. The simulation results proved the superiority of the developed model for optimizing transportation network performance. The proposed HJB approach outperformed the existing approaches such as Static Traffic Assignment, Model Predictive Control, and Reinforcement Learning with an average delay of 10.7 minutes, a congestion index of 0.46, and a total cost of 890. The approach led to 42.5%, 43.9%, and 41.4% improvements in delay, congestion, and cost, respectively. Additionally, the convergence test demonstrated that the optimization process was stable in terms of objective cost, which was reduced from 1520 to 890 within 50 iterations. The main conclusion one can draw from this study is that Hamilton–Jacobi optimal control represents a mathematically proven way of managing dynamic transportation systems by integrating optimality, flexibility, and stability of the solution. The method allows making decisions continuously due to the feedback control approach and is a good perspective for intelligent transportation system optimization in the future.

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