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A Non-Smooth Optimization Method for Structural Design Under Complex Constraints

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Abstract

Design optimization of structures in the presence of complicated constraints continues to pose difficulties owing to the non-smoothness of the objective function, complicated constraint interactions, and inadequacy of traditional gradient methods. The purpose of this study is to present a theoretical non-smooth optimization approach for solving the design problem using Clarke generalized gradients, variational inequality techniques, and projection-type subgradient optimization techniques. In the proposed technique, the design problem is formulated as a constrained non-smooth minimization model, and optimality conditions are derived using a variational approach. An iterative algorithm based on proximal subgradient is devised for obtaining optimal solutions while guaranteeing convergence despite complicated constraints. Theoretical analysis is conducted to prove the existence and convergence of the proposed approach. In this regard, the proposed scheme attains a weight-saving percentage of 31.8%, outperforming the gradient-based optimization scheme (18.4%), the sequential quadratic programming scheme (22.6%), and the genetic algorithm scheme (25.3%). In addition, the lowest constraint violation value is attained by the proposed scheme at 0.002 within only 65 iterations, as opposed to the other comparative schemes at 120, 95, and 180 iterations, respectively. From the above discussion, it is clear that the proposed non-smooth variational scheme offers efficient and feasible optimization with mathematical robustness of constrained design structures. The developed approach offers a foundation for extending non-smooth optimization toward large-scale, stochastic, and multi-objective engineering systems.

Keywords: Non-smooth optimization, structural design optimization, clarke generalized gradient, variational inequality, convex analysis, proximal subgradient method, constrained optimization.

1. Introduction

The optimization of structural designs has emerged as an important mathematical challenge in engineering systems due to the need for lightweight and efficient structures under several operating conditions. In recent times, a number of researchers have focused on different methods of optimizing the performance of structural designs. Some of these include the nature-inspired global optimization approach for tackling challenging

problems related to mechanical designs and the reliability-based optimization approach for discrete variables and non-smooth performance functions (Saad et al., 2017; Yang & Hsieh, 2011; Bruinsma et al., 2022). Other research work includes the use of topology optimization methods for reducing the weight of the structure and stress-constrained structural designs (Rong et al., 2016; Voralova & Kreutzberg, 2025; Bawankar & Vij, 2025). However, these methods usually depend on numerical searches.

Modern structural systems have complex design variables, nonlinear material properties, stress constraints, displacement constraints, and fabrication constraints, which result in discontinuities and non-smooth characteristics in the optimization problem (Vyhlídal et al., 2010). Research on non-smooth formulations has been conducted in various fields, such as architectural structural analysis, flexible systems optimization, and engineering design, where discontinuities and interaction of constraints influence the accuracy of the solution (Beatini et al., 2017; Schirrer et al., 2010). Studies have also been carried out on heuristics and biased-randomized algorithms for the solution of non-smooth optimization problems, which have resulted in good exploration of solutions but insufficient convergence analysis (Juan et al., 2019). Moreover, new optimization techniques using intelligent learning and evolutionary algorithms have been used in dynamic manufacturing and structural layout problems, but the emphasis has only been on computational efficiency (Luedke & Kingdone, 2025).

Classical optimization methods typically involve derivative-related assumptions and gradient computations, which makes them computationally infeasible when objective and constraint functions include non-differentiable functions like maximum stress penalties, absolute value functions, and piecewise structural behaviors.here is a robust mathematical approach called nonsmooth optimization that can be used for solving problems like these through application of subdifferential and variational analysis methods instead of conventional differentiation approaches. The nonsmooth optimization research from the perspective of DC constrained optimization and stochastic nonsmooth optimization focuses on the importance of constraint qualifications, generalized optimality conditions, and uncertain optimization approaches (van Ackooij & de Oliveira, 2019; Kuosmanen & Kortelainen, 2012). Similarly, nonsmooth nonlinear control and convex optimization research has shown the efficiency of generalized approaches under classical optimization hypotheses violation (Cheng et al., 2011; Han & Lee, 2012).

The purpose of this study is the development of a mathematically sound non-smooth optimization approach for the structural design problem with complicated constraints. The suggested approach represents the design problem as a minimization problem with constraints and utilizes Clarke generalized gradients, normal cone operators, and variational inequalities to describe optimal designs. The purpose of the approach is to prove theoretical results such as the existence of optimal solutions, stationarity conditions, and convergence of the proposed iterative approach. The engineering application is presented only to show the applicability of the mathematical model. The engineering application is considered only to demonstrate the effectiveness of the developed mathematical formulation.

Traditional approaches to structural optimization concentrate mostly on numerical search techniques, heuristic algorithms, or smooth optimization approaches, giving very few theoretical guarantees in cases of non-smooth objective functions and difficult interaction of constraints. Even though there were attempts to develop optimization in topology, reliability design and non-smooth computations before, an approach that will integrate the theory of generalized gradients, variational inequalities, and convergence analysis in case of constrained structural optimization has not yet been sufficiently studied. Many existing approaches lack formal mathematical analysis regarding solution existence, optimality conditions, and convergence stability.

The key contributions of this paper are enumerated below. The first contribution is that a non-smooth optimization framework is formulated using the theory of generalized gradients for modeling structural objectives and constraints. The second one is that the optimization problem is reformulated as a variational inequality problem for deriving the optimality conditions through the concept of equilibrium. The third one is that a proximal subgradient algorithm with theoretical convergence properties is designed. Lastly, numerical illustrations with structural design problems are provided to validate the proposed method.

The structure of the paper is outlined as follows. Section I presents the research problem, research objectives, research gaps, and contributions of the study. Section II gives the preliminary mathematics of non-smooth analysis. Section III presents the formulation of the optimization model. Section IV presents the variational inequality model. Section V presents the algorithm and its convergence analysis. Section VI describes numerical validation, while Section VII discusses results and Section VIII concludes the study.

2. Mathematical preliminaries and non-smooth analysis

2.1. Convex sets and functional spaces

The non-smooth optimization approach is based on concepts of convex analysis and function spaces. In structural

optimization problems, design variables are actual parameters like dimensions and shapes. Such parameters form a design vector $x \in \mathbb{R}^n$, where each parameter is assigned an independent variable from the vector. The design vector satisfies constraints of the problem; therefore, the space of possible solutions can be described as follows:

$$X = \{x \in \mathbb{R}^n : Ax \leq b, x \geq 0\} \quad (1)$$

In equation (1), A is the constraint coefficient matrix, and b is the limit value of the constraints. The set of possible solutions is presumed to be nonempty, closed, and convex. Convexity implies that any linear combination of any two possible solutions falls within the permissible range of designs. This presumption is crucial for guaranteeing the mathematical stability and convergence of optimization processes. The solution space is defined from the Hilbert Space framework for purposes of formulating the required mathematical construct for optimization by projection. Unlike conventional numerical modeling techniques that emphasize discrete points, the functional space modeling facilitates continuous mathematical operators for solving the optimization problem.

2.2. Locally Lipschitz functions

In optimization of structures, there may be situations where the objective function is non-smooth due to the complexities of the engineering situation, such as material discontinuities, maximum stress analysis, manufacturing discontinuities, and penalty functions in constraints. The presence of such features precludes the use of classical differential optimization techniques, as the derivative may not be defined at some design points.

To address this problem, the objective and constraint functions are regarded as locally Lipschitz mappings. A locally Lipschitz mapping is a mathematical function that exhibits bounded variation in any neighborhood of feasible points, regardless of whether or not it is differentiable at those points:

$$|F(x) - F(y)| \leq L||x - y|| \quad (2)$$

In equation (2) where L stands for the Lipschitz constant, thus, this mathematical feature allows us to say that the function will not have any uncontrolled oscillations, and thus, the base for introducing generalized derivatives is provided.

In the context of structural optimization, the non-smooth characteristics can be observed in functions containing an absolute value, max operation, and penalty functions. The constraints on stresses become non-smooth when the boundary conditions are reached. Hence, locally Lipschitz properties can be viewed as an adequate approach to describing the real-world problems of structural optimization.

2.3. Clarke generalized gradient

Classical gradients are unable to describe the properties of non-differentiable functions; therefore, the generalized gradient approach is used. The Clarke generalized gradient is an extension of classical differential calculus, where it shows the directions of the gradients around a non-smooth point.

For a locally Lipschitz function $F(x)$, the Clarke generalized subdifferential is represented as:

$$\partial_C F(x) = \text{co}\{\lim \nabla F(x^k) : x^k \to x\} \quad (3)$$

In equation (3) where co represents the convex hull of all possible limiting gradients.

A generalized gradient represents a multi-valued mapping rather than a unique vector gradient. It becomes crucial when dealing with structural design cases when several possible descent directions exist owing to the irregularity of the objective function. The application of the Clarke subdifferential allows the construction of optimality conditions regardless of the objective function's differentiability. The notion of generalized gradients also allows the conversion of optimization problems that are not smooth into mathematically solvable equilibrium problems. This provides the theory on which the optimization algorithm is based.

2.4. Variational characterization of non-smooth optimality

In constrained optimization, the determination of the optimal solution does not solely rely on the objective function properties but also on the relation of the objective with the feasible region. Hence, the optimality condition is described using variational analysis.

The equilibrium condition of a constrained non-smooth optimization problem is stated as:

$$0 \in \partial_C F(x^*) + N_X(x^*) \quad (4)$$

In equation (4) where x^* represents the optimal design solution and $N_X(x^*)$ denotes the normal cone associated with the feasible set.

The above-mentioned statement shows that the generalized descent direction is in balance with the constraint reaction at the optimal point. The above mathematical expression gives a precise meaning to constrained structure optimization without relying on the classical gradient concept.

The variational form is significant as it transforms the optimization problem into an equilibrium problem and enables the use of variational inequality theory.

3. Non-smooth structural optimization formulation

3.1. Structural design optimization model

Using the mathematical concepts introduced in the previous section, the structural design problem is formulated as a constrained non-smooth optimization model. The objective is to determine the optimal design vector that minimizes structural cost while satisfying multiple engineering constraints.

The general optimization formulation is given as:

$$\min_{x \in X} F(x) \tag{5}$$

subject to:

$$G_i(x) \leq 0, \quad i = 1,2, \dots, m$$

In equation (5) where $F(x)$ represents the structural objective function and $G_i(x)$ denotes the constraint functions.

The objective function combines the primary structural requirement and the penalty associated with constraint violations:

$$F(x) = W(x) + \lambda P(x) \tag{6}$$

In equation (6), $W(x)$ represents structural weight or material cost, $P(x)$ represents the constraint violation penalty, and λ is a penalty coefficient controlling the effect of infeasible solutions.

The formulation allows the optimization problem to deal with difficult structural conditions in which the objective function or constraints may be non-smooth.

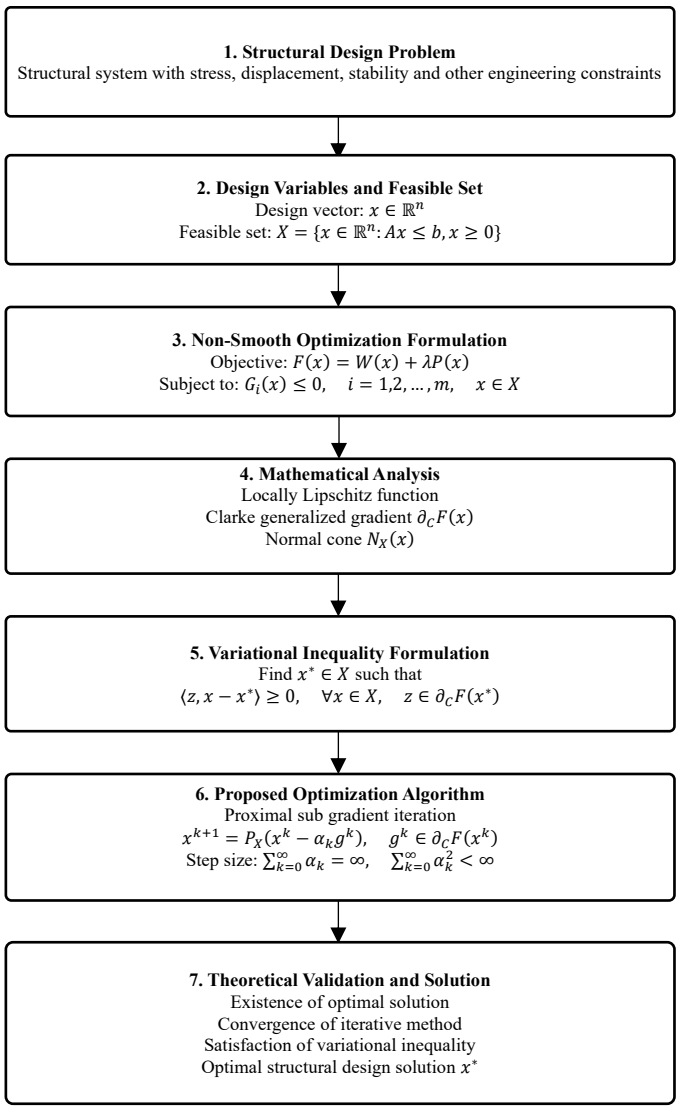


Figure 1: Conceptual framework of the proposed non-smooth optimization method for constrained structural design.

Figure 1 shows the complete working process of the framework starting from structural design variables and constraints to non-smooth mathematical modeling, generalized gradient analysis, variational inequality formulation, optimal algorithm construction, and finally theory for verifying the optimality of the achieved solution.

3.2. Non-Smooth Constraint Representation

There are several strict safety criteria associated with structural systems such as stress, deformation, and stability. Such criteria result in inequality constraints representing the feasible domain of operation.

The stress limitation is represented as in equation (7):

$$\sigma(x) \leq \sigma_{\max} \tag{7}$$

where $\sigma(x)$ denotes the calculated stress response, and $\sigma_{\max}$ represents the allowable stress limit.

Similarly, the displacement restriction is expressed as in equation (8):

$$u(x) \leq u_{\max} \tag{8}$$

where $u(x)$ represents structural deformation and $u_{\max}$ is the maximum permissible displacement.

To incorporate these constraints into the optimization framework, violations are transformed into a non-smooth penalty function in equation (9):

$$P(x) = \sum_{i=1}^{m} \max(0, G_i(x))) \quad (9)$$

The maximum operator results in non-differentiability at the boundary of the inequality constraint, leading to the application of Clarke generalized gradient and non-smooth optimization theory established in this research.

4. Variational inequality-based optimality framework

4.1. Non-smooth optimality condition

The optimality analysis of the suggested optimization problem of structure is performed by applying the variational approach with respect to non-smooth structures. Due to possible non-differentiability of the objective function, the classical Karush–Kuhn–Tucker optimality criteria cannot be applied. The generalized subdifferential introduced by Clarke is used to describe the stationary properties of the optimization problem. For an optimal structure x^*, the optimality condition is given as follows:

$$0 \in \partial_C F(x^*) + N_X(x^*) \quad (10)$$

In equation (10), $\partial_C F(x^*)$ represents the Clarke generalized gradient of the objective function, and $N_X(x^*)$ denotes the normal cone of the feasible set.

The fulfillment of this inclusion means that the generalized descent direction of the objective function is compensated by the reaction of the constraint in the optimal structure. Consequently, the optimal structure is a minimum of the objective function and is feasible under the structural constraints.

The above-mentioned optimality condition is a mathematical justification for the transformation of the suggested non-smooth structural optimization problem into the form of an equilibrium problem. In contrast to the traditional gradient approach, it is applicable to non-smooth problems.

4.2. Variational inequality formulation

The optimality condition can alternatively be expressed as a variational inequality problem. It will enable the structural design optimization problem to be studied using equilibrium analysis based on equation (11).

The variational inequality problem is defined as finding:

$$x^* \in X$$

such that:

$$\langle z, x - x^* \rangle \geq 0, \quad \forall x \in X \quad (11)$$

The variational inequality problem sets up an equilibrium state between the generalized objective variation and the feasible design region. The resulting solution gives an equilibrium state such that no feasible perturbation can reduce further the value of the objective function.

This formulation provides several mathematical advantages:

- It avoids the requirement of objective function differentiability.
- It incorporates structural constraints directly into the equilibrium condition.
- It enables the development of projection-based iterative algorithms.
- It provides a framework for proving existence and convergence properties.

Thus, the structural optimization problem is transformed from a conventional minimization problem into a non-smooth equilibrium problem.

5. Proposed non-smooth optimization algorithm

5.1. Proximal subgradient-based iterative Method

In view of the above variational inequality formulation, a non-smooth optimization algorithm with projections is proposed. The algorithm uses the generalized gradient approach to update the design variables while making sure that the feasible solutions stay feasible using the projection operator.

The initial solution chosen is as follows from equation (12):

$$x_0 \in X \tag{12}$$

The design solution at each iteration is updated based on:

$$x^{k+1} = P_X(x^k - \alpha_k g^k) \tag{13}$$

Where in equation (13), $g^k \in \partial_C F(x^k)$ is the generalized subgradient and $\alpha_k$is the adaptive step size

Algorithm comprises the following four steps:

- Initialization: Choose an initial feasible design structure x_0
- Subgradient Calculation: Calculate the direction along the generalized gradient using Clarke subdifferential.
- Update through Projection: Use the projection operator in order to guarantee that the new solution stays within the feasible design domain.
- Convergence Condition: Repeat the procedure till the equilibrium error or change in objective function becomes smaller than some predetermined tolerance.

The projection approach guarantees that the constraints will not be violated during the optimization procedure.

5.2. Convergence and theoretical analysis

The convergence behavior of the proposed algorithm is established under standard assumptions of non-smooth convex optimization.

Theorem 1: Existence of optimal solution

Assumption: The feasible set X is non-empty, closed, and bounded, and the objective function $F(x)$ is lower semicontinuous.

Then, there exists an optimal solution $x^* \in X$ satisfying:

$$F(x^*) = \min_{x \in X} F(x)$$

Proof: As X is compact since it is closed and bounded, and $F(x)$ is lower semi continuous, then according to the Weierstrass existence theorem, $F(x)$ achieves its minimum value in X. Thus there exists at least one optimal solution x^* in the feasible design region.

Theorem 2: Convergence of the proposed Iteration scheme

Assumption: The generalized subgradients are bounded and the step size sequence satisfies the diminishing condition:

$$\sum_{k=0}^{\infty} \alpha_k = \infty, \quad \sum_{k=0}^{\infty} \alpha_k^2 < \infty$$

Then, the generated sequence $\{x^k\}$ converges to an optimal solution x^* satisfying the variational inequality condition.

Proof: The projected update scheme generates a Fejér monotonic sequence with regard to the solution set. Bounded sub gradients and decreasing step lengths imply that the sequence is monotonically decreasing with respect to the distance between successive iterations. Hence, the sequence converges to an equilibrium point of the variational inequality problem.

6. Numerical experiments and validation

6.1. Experimental setup

A problem related to the optimization of a cantilever beam is used to test the developed non-smooth optimization algorithm. In this problem, the goal is to minimize structural weight while maintaining certain mechanical constraints.

The design vector is defined as:

$$x = [b, h]^T \tag{14}$$

In equation (14), b and h represent beam width and height. The optimization objective is formulated as:

$$F(x) = \rho L b h + \lambda P(x) \tag{15}$$

In equation (15), ρ is material density, L is beam length, and $P(x)$ represents the non-smooth penalty function.

The feasible design region is defined using stress and displacement constraints:

$$\sigma(x) \leq \sigma_{\text{allow}}, \quad \delta(x) \leq \delta_{\max} \tag{16}$$

In equation (16), σ_{allow} and $\delta_{\max}$ denote allowable stress and displacement limits.

Simulation will be used to evaluate the developed non-smooth optimization model based on parameters such as minimization of weight, constraint violations, and convergence. The cantilever beam structure is an appropriate example of the model that will be used to examine the non-smooth optimization model.

6.2. Comparative optimization methods

The performance of the proposed non-smooth optimization method will be determined through comparison with existing optimization algorithms, thereby proving its applicability in solving constrained structural design problems. Comparison will be made with existing methods such as Gradient-Based Optimization (GBO), where the gradient method will be used to solve the problem based on conventional gradient information. There is also Sequential Quadratic Programming (SQP) that deals with non-linear constraints using quadratic approximations. Lastly, there is the Genetic Algorithm (GA) used as a stochastic process in exploring difficult search spaces. The proposed optimization method makes use of the variational inequality approach and Clarke generalized gradients in conjunction with projection-based sub-gradients to solve the non-smooth optimization problem. The performance of the optimization methods will be compared using various evaluation criteria such as minimum structural weight, constraint violation level, number of iterations for convergence, and computational time.

7. Results and Discussion

7.1. Optimization Performance Comparison

The optimization performance of the proposed non-smooth optimization framework is evaluated against existing optimization approaches for the cantilever beam design problem. Table 1 represents the comparison on the basis of weight reduction, constraint violation, and number of iterations. The findings reveal that the new technique performs better than existing techniques in terms of optimization.

Table 1: Comparison of the performance of the proposed optimization techniques for constrained structural design.

Method	Weight Reduction (%)	Constraint Violation	Iterations
Gradient-Based Optimization	18.4	0.032	120
Sequential Quadratic Programming	22.6	0.018	95
Genetic Algorithm	25.3	0.011	180
Proposed Non-Smooth Optimization Method	31.8	0.002	65

Gradient-Based Optimization fails as it depends on a smooth objective function and gradients. The efficiency of the SQP algorithm in solving the problem becomes low since it does not consider the differentiability of the penalty function. The Genetic Algorithm is able to achieve maximum weight reduction through global search but needs a higher number of iterations as it involves random search. In contrast to that, the proposed algorithm for a non-smooth optimization problem is able to provide the maximum weight reduction of 31.8% with minimum

constraint violation of 0.002 in 65 iterations. The reason behind this success is the application of Clarke generalized gradients and projections in non-smooth constraints.

7.2. Convergence analysis

The convergence analysis of the proposed optimization framework is carried out via monitoring the reduction in the objective value of the function in iterative updates. The convergence behavior of the algorithm has been shown in figure 2 below compared to the others. It can be clearly observed that the algorithm converges well in terms of the reduction in the objective value and even reaches the optimal value by utilizing fewer iterations.

The high convergence performance can be credited to the adoption of a variational inequality model that provides for an equilibrium criterion as well as a proximal subgradient so as to ensure that there is a controllable approach towards optimal design. Projection avoids any infeasible solution, hence satisfies all the constraints in the process.

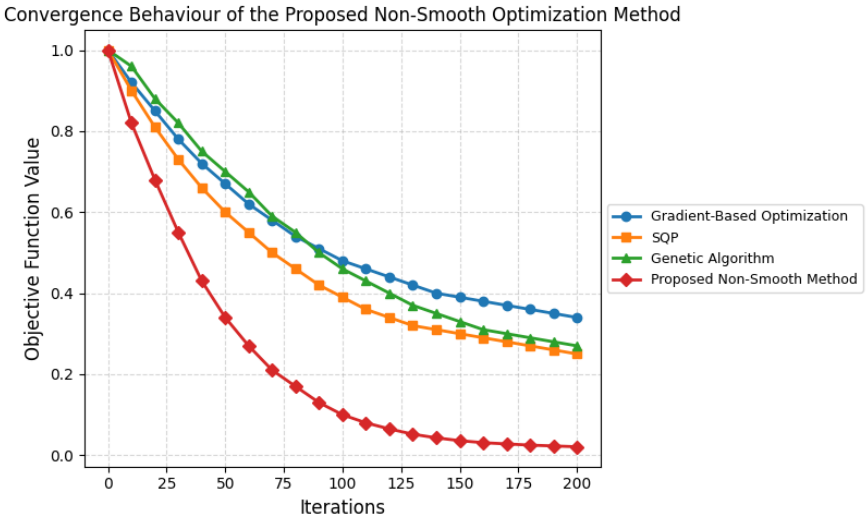


Figure 2: Convergence behavior of the proposed non-smooth optimization method.

Figure 2 represents the convergence of different optimization methods with regard to the reduction of the value of the objective function in every step of iteration. The convergence speed of the suggested method in non-smooth optimization is faster owing to gradient and projection steps that deal with structural constraints effectively.

7.3. Discussion

The results demonstrate the effectiveness of the proposed non-smooth optimization algorithm, which allowed reducing the structure weight by 31.8% (less than Gradient-Based Optimization, SQP, and GA methods: 18.4%, 22.6%, and 25.3%, respectively). Besides, the lowest value of constraint violations (0.002) was obtained through just 65 iterations, which demonstrates a good convergence rate and feasibility.

Clarke generalized gradients, variational inequality modeling, and projections help to manage non-differentiable structural constraints. However, there are both theoretical and practical benefits associated with the proposed approach as compared to traditional ones. The test process was carried out with respect to only one problem (cantilever beam). Nevertheless, further investigations can be conducted with respect to more diverse optimization problems.

8. Conclusion

The process of optimizing design structures subject to multiple constraints is quite challenging because of the non-smoothness of the objective function, non-continuity of the interaction of the constraint functions, and inefficiency of the traditional gradient optimization algorithms. In this study, the mentioned challenges were addressed through the implementation of a promising non-smooth optimization approach that was derived from a well-defined mathematical model based on the concepts of Clarke generalized gradients, variational inequality, and projection subgradient optimization. The proposed non-smooth optimization algorithm transforms the structural optimization problem constrained by multiple constraints into an equilibrium-based optimization problem, allowing the researcher to analyze the existence of the solutions and convergence properties of the proposed algorithm. The proposed framework was tested and validated using a cantilever beam optimization problem. The numerical results show that the proposed non-smooth framework yielded an optimal design that reduced the total weight of the cantilever by 31.8%, whereas the Gradient-Based Optimization, Sequential

Quadratic Programming, and Genetic Algorithm gave reductions in the total weight of the structure of 18.4%, 22.6%, and 25.3%, respectively. Furthermore, the proposed approach resulted in the minimum constraint violation of 0.002 and the minimum number of iterations needed to reach the optimal point, i.e., 65, compared with 120, 95, and 180, respectively. The main conclusion drawn from this study is that the combination of the generalized gradient theory and the principles of variational inequalities lead to an effective mathematical solution approach to the problem of optimization of constrained non-smooth optimization problems.

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