Minimax Theory and its Applications
Volume 11 (2026), No. 2s, 116-127

A Probabilistic Minimax Estimation Framework for Structural Reliability Prediction

Tanveer Ahmad Wani[1], Dr. Pareshkumar N. Patel [2], Swati Sucharita[3], Pratibha Sharma[4], S N V S SS T Murty[5], Dr. Shanthi Vairavan[6], Amol Barde[7]

[1]Professor, School of Sciences, Noida international University, Uttar Pradesh, Tanveer. ahmad@niu.edu.in

[2]Professor, Faculty of Pharmacy, Gokul Global University, Gujarat, India. dean.fac.pharmacy@gokuluniversity.ac.in, https://orcid.org/0009-0005-8263-2179

[3]Assistant Professor, Department of Computer Science and Engineering, Vardhaman College of Engineering, Shamshabad, Hyderabad, India. sucharita1614@vardhaman.org

[4]Centre of Research Impact and Outcome, Chitkara University, Rajpura, Punjab, India. pratibha.sharma.orp@chitkara.edu.in, https://orcid.org/0009-0000-4708-182X

[5]Bonam Venkata Chalamayya Institute of Technology & Science, Amalapuram, Andhra Pradesh, India. trinadha.murty@gmail.com

[6]Professor & Principal, Computer Science, Meenakshi College of Arts and Science, Meenakshi Academy of Higher Education and Research, Chennai, Tamil Nadu, India.shanthiv@maher.ac.in

[7]Department of Mechanical Engineering, Vishwakarma Institute of Technology, Pune, Maharashtra, India. amol.barde1@vit.edu

Abstract

The prediction of structural reliability in uncertain operational conditions is still a mathematically difficult task because of the presence of stochastic loading, variation of materials, and insufficiently known probabilities. In this paper, a new approach based on the theory of stochastic modeling, minimax optimization, and variational analysis is used to provide a robust prediction of structural reliability. A minimax estimation problem is formulated using a saddle point approach where the optimal estimator finds the minimum of the maximum expected loss over the class of possible distributions. A variational inequality approach is used to derive a reliable structural reliability estimator, whereas existence, uniqueness, and convergence properties of this solution are established using the theory. A steel beam structural system is chosen for numerical demonstration using a Gaussian uncertainty model for loading and random variation of material parameters in 10,000 simulations. Improved results of the structural reliability estimator using the proposed framework are demonstrated, with a mean square error of 0.0064, worst case risk of 0.034, and reliability accuracy of 97.8%. Additionally, the suggested model has a robustness index of 95.9% with a distribution shift error of 0.0126, thus exhibiting improved performance in terms of dealing with uncertainties. According to the convergence analysis, it is stable in optimizing the problem as its objective function decreased from 0.084 to 0.011 after 50 iterations. The suggested probabilistic minimax formulation has provided a mathematically sound methodology to predict uncertainty-resilient structural reliability.

Keywords: Probabilistic minimax estimation, structural reliability, uncertainty quantification, variational inequality, saddle-point optimization, robust estimation, stochastic modeling.

1. Introduction

Reliability analysis of structures is essential in determining the safe operation of engineering systems under

uncertain conditions. Actual structures like bridges, machine parts, and aerospace systems are subjected to varying influences due to random loading, deterioration of materials, errors in manufacturing, and changing environmental conditions (Rahman & Begum, 2024). All these factors have an important impact on the behavior of the structures and make reliability analysis mathematically difficult. Probabilistic and computational methods of reliability analysis of structures have been considered in recent studies (Kumar et al., 2024; Zhang et al., 2023). Advanced reliability analysis procedures using importance sampling and surrogate modeling have been studied to enhance the accuracy of estimating small failure probabilities in structural models (Echard et al., 2013). Conventional reliability analysis procedures usually use probability distributions of uncertain quantities, which are unable to account for uncertainties due to incomplete information and distributional uncertainty.

There are several approaches based on estimation and optimization that have been proposed for uncertainty consideration in reliability analysis. The Bayesian estimation procedure is one such approach, providing an efficient way to model uncertainties; however, its efficiency strongly depends on the choice of the initial information and correct probabilistic assumptions. In addition, Monte Carlo-based procedures ensure the reliability of the failure probability estimation, but they require considerable computational efforts, especially for complex structural systems with multiple uncertain parameters. Stochastic differential equations-based reliability models have been considered to model the dynamics of the propagation of uncertainties in engineering mission-critical systems; nonetheless, such approaches require stochastic characterization of system behavior (Veerappan & Arvinth, 2025). Uncertainty consideration in structural integrity assessment has also drawn attention to the necessity of correct uncertainty modeling to avoid failure of engineering systems (Ikechukwu et al., 2019).

To solve these shortcomings, reliable mathematical techniques utilizing the minimax estimation and optimization theories have become a subject of great interest. The minimax estimation techniques allow solving worst-case uncertainty conditions and finding an optimal solution through minimizing estimation risks (Qu et al., 2010; Kogan, 2014). Linear minimax estimation techniques have been developed to address uncertain random vectors that have parametric uncertainties, which show improvements in robust estimation (Bitar et al., 2010). Likewise, minimax optimization techniques can be utilized to deal with uncertainty ranges and risk sensitivity in various optimization and engineering applications (Fredriksson et al., 2011; Polak et al., 2010). In addition, the probabilistic minimax and scenario-based optimization techniques can also be utilized to address uncertain distributions and worst-case performance (Sun et al., 2015; Natarajan et al., 2014; Care et al., 2015). Further research into the minimax estimation theory has shown robust estimation under various moment conditions and uncertainty of statistical models (Dikkala et al., 2020).

Despite their importance in addressing uncertainties, existing reliability estimation methods only consider the minimization of average estimation error and do not incorporate a mathematical method to manage worst-case uncertainties in reliability estimation. Hence, this paper introduces the probabilistic minimax estimation approach where the structure parameters are considered to be random variables and the reliability prediction is defined in terms of a minimax optimization problem involving a trade-off between the estimation accuracy and robustness against uncertainties. The minimax principle gives a powerful tool for finding such an estimator which will minimize the maximum expected loss from an admissible set of probability distributions.

The main purpose of this paper is the development of a probabilistic minimax estimation framework for structural reliability prediction that involves the use of stochastic modeling, minimax optimization, and variational analysis tools. This framework is defined as the reformulation of the reliability estimation problem in terms of a saddle point problem. The main achievements of the current research can be summarized as follows: (i) formulation of the probabilistic minimax reliability estimation model with consideration of worst-case distributional uncertainty, (ii) development of the optimization approach based on operators for solving the minimax problem and finding the optimal estimator, and (iii) investigation of the existence, uniqueness, and convergence properties of the developed minimax solution. The structural reliability example illustrates the possibility of using the developed mathematical approach for obtaining reliable reliability predictions.

Structure of the paper: Section II contains the mathematical background, Section III contains the probabilistic minimax problem formulation, Section IV describes the variational analysis, Section V covers the theoretical properties, Section VI gives the numerical verification, Section VII contains the results and discussion, and Section VIII concludes the work.

2. Mathematical preliminaries

The probabilistic minimax estimation method presented in this research work has been formulated based on some basic principles of probability theory, structural reliability theory, and minimax decision-making theory. This

section deals with the basic definitions needed for formulating the reliability prediction of structures in terms of uncertain probabilities. Uncertain structural parameters have been modeled in terms of random variables that are defined in some appropriate probability space. The reliability estimation problem is formulated as an optimization problem.

2.1. Probability space and random variables

Unknowable structural parameters are considered random variables that have admissible probability distributions.

$$X(\omega): \Omega \rightarrow \mathbb{R}^n \tag{1}$$

Where equation (1) denotes stochastic state of structural system by $X(\omega)$ and dimension of uncertain parameters is denoted by n. The probability distribution of X controls the randomness in structural response and affects the reliability assessment process directly. In view of ambiguity in practical engineering systems related to probabilistic data, the model under consideration utilizes admissible probability distribution class instead of fixed distribution.

2.2. Reliability function definition

Reliability of structures is developed using the relation between structural resistance and loads. Assume the performance function of the structure is written as:

$$G(X) = R(X) - S(X) \tag{2}$$

Equation (2) where $R(X)$ is the resistance of the structure, and $S(X)$ is the load effect. The performance function gives the safety status of the structural system; where if the performance function is positive, then the structure is in safe operation, and otherwise, the structure has failed.

2.3. Minimax estimation theory

In the proposed scheme, the reliability estimation problem is cast as a minimax estimation problem to achieve an estimator that is optimal even under worst-case uncertainty conditions. Let Y be the available structural information due to observations or system response. Then the reliability estimate is done via an estimator:

$$\hat{\theta} = \phi(Y) \tag{3}$$

In equation (3), where $\phi(\cdot)$ is the estimation operator, and $\hat{\theta}$ is the estimated reliability parameter. The estimation capability is then quantified with respect to a loss function in equation (4):

$$L\big(\theta, \phi(Y)\big) \tag{4}$$

It defines the difference between the actual reliability parameter θ and the estimation result. Regarding the uncertainty in the probability distribution, the uncertainty set can be described as follows in equation (5):

$$\mathcal{P} = \{P_1, P_2, \dots, P_m\} \tag{5}$$

Where each $P \in \mathcal{P}$ is the corresponding probability distribution of the uncertain structure. The optimal minimax estimator can be calculated through solving equation (6):

$$\phi^* = \arg\min_{\phi} \sup_{P \in \mathcal{P}} E_P \left[L\big(\theta, \phi(Y)\big)\right] \tag{6}$$

where $E_P[\cdot]$ is the expectation of the loss under probability distribution P, the minimization operator gives the minimum risk estimator, while the maximization operator takes into account the worst-case uncertainty situation. This gives a robust mathematical framework for the prediction of structural reliability because of the stability of the estimator under unknown uncertainty distributions.

3. Probabilistic minimax reliability formulation

Prediction of structural reliability in the presence of uncertain loading requires a mathematical model that can account for both stochasticity and the lack of information regarding the distribution of uncertainty. The

conventional methods of probabilistic reliability typically rely on the assumption that the probability distribution of the structural parameter is fully available. However, in reality, the presence of uncertainty with respect to loadings, material behavior, and other factors might result in an ambiguous probability model. Thus, this paper proposes to treat the reliability prediction problem as a probabilistic minimax estimation problem.

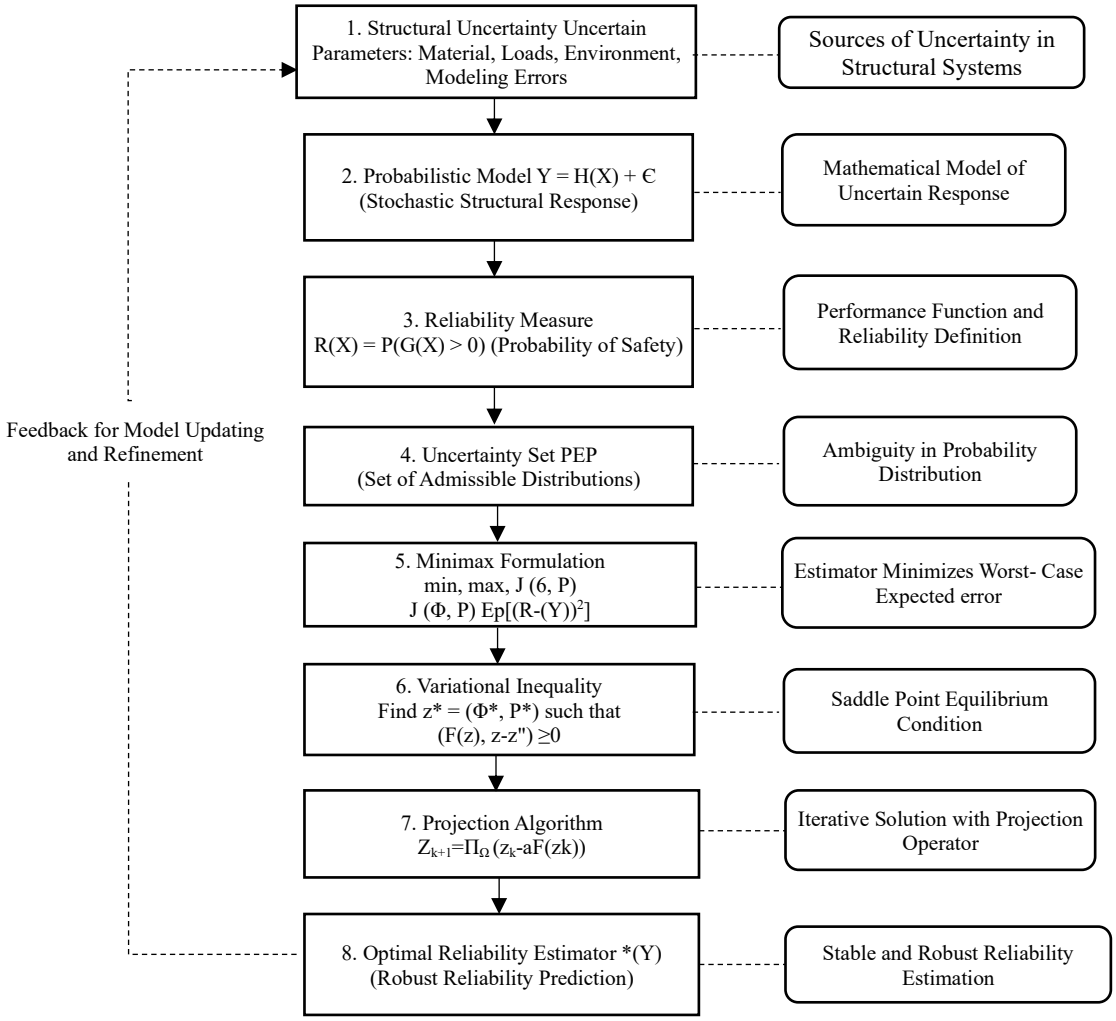


Figure 1: Conceptual framework of the probabilistic minimax reliability estimation model.

Figure 1 provides a description of the suggested probabilistic minimax reliability estimation method. The method commences from the initial uncertain structural parameter and probabilistic representation, through reliability representation and uncertainty distribution representation, to finally reach the optimal reliability estimator through minimax optimization, variational inequality analysis, proof of validity, and projections.

3.1. Structural uncertainty model

The random nature of the structural behavior is modelled using a stochastic response function.

$$Y = H(X) + \varepsilon \tag{7}$$

From equation (7), where $X \in \mathbb{R}^n$ is the uncertain structural parameter vector, $H(X)$ is the structural response mapping, Y is the observed structural response, and ε is the error due to measurements and modeling. The randomness of the structural parameters is not confined to only one probability distribution but to a family of admissible probability distributions formulated as follows:

$$P_X \in \mathcal{P} \tag{8}$$

From equation (8), where $\mathcal{P}$ is the set of all probability distributions modeling the structural uncertainty. This probabilistic formulation facilitates the proposed methodology for modeling the uncertainties in the structural parameters without the need to have specific knowledge about their statistical properties.

3.2. Reliability estimation operator

The structural reliability will depend on the ability of the structure to fulfill the safety requirements. From the limit state definition as given above, the reliability requirement can be defined as:

$$R(X) = P(G(X) > 0) \tag{9}$$

From equation (9), where $G(X)$ denotes the structural performance function. The reliability operator represents the probability of keeping the structural system in the safe region under uncertainty conditions.

As it is impossible to know the exact probability distribution of X, calculation of $R(X)$ becomes difficult. Thus, a reliability estimation operator is defined as:

$$\hat{R} = \phi(Y) \tag{10}$$

From equation (10), $\phi(\cdot)$ stands for the mapping function estimating the structural reliability index based on the observed structural data. The goal here is to find an estimator which gives reliable prediction of the reliability measure while being robust to uncertain probability distributions.

3.3. Probabilistic minimax optimization model

For attaining robustness regarding unknown probability distributions, the reliability estimation problem is formulated based on the minimax optimization approach. The optimal reliability estimator can be found through the minimization of the maximum expected estimation error with respect to the uncertainty class of probability distributions as follows

$$\phi^* = \arg\min_{\phi} \max_{P \in \mathcal{P}} E_P \left[\left(R(X) - \phi(Y) \right)^2 \right] \tag{11}$$

In equation (11), $E_P[\cdot]$ is the expectation operator with respect to the probability distribution P. The inner maximization finds out the worst probability distribution resulting in the maximum estimation uncertainty, whereas the outer minimization finds out the reliability estimator with minimum risk.

The minimax risk functional can be stated as equation (12):

$$J(\phi, P) = E_P \left[\left(R(X) - \phi(Y) \right)^2 \right] \tag{12}$$

With the help of this risk functional, the reliability estimate problem can be formulated as a saddle-point optimization problem as shown in equation (13):

$$\min_{\phi} \max_{P \in \mathcal{P}} J(\phi, P) \tag{13}$$

The saddle point formulation gives a mathematical framework for the interplay of the reliability estimator and the uncertain probability distribution. The reliability estimator aims to minimize the error in the reliability estimate while the uncertain probability distribution stands for the most adverse scenario in terms of estimation accuracy.

The minimax reliability approach proposed in this study forms a basis for predicting the reliability of structures under uncertainty using stochastic and minimax techniques. In contrast to traditional reliability approaches based on fixed probability assumptions, the current formulation takes all the possible uncertainties in the probability space into account and develops an estimator with better stability. This will be studied theoretically through variational optimization techniques in the next section.

4. Variational analysis of the minimax estimation framework

The minimax reliability approach presented here is studied using variational optimization theory. First, the minimax problem is converted into a saddle point problem in order to develop equilibrium equations and an iteration algorithm to find the optimal reliability estimator under uncertain probability distributions.

4.1. Saddle point formulation

The reliability estimation problem is expressed as:

$$\phi^* = \arg\min_{\phi} \max_{P \in \mathcal{P}} J(\phi, P) \quad (14)$$

In equation (14)

Where $J(\phi, P)$ represents the expected estimation risk under uncertainty distribution P.

The minimax reliability estimation problem is formulated as a saddle point optimization problem where the estimator ϕ is the solution to minimize the maximum risk resulting from the worst-case uncertainty distribution P.

4.2. Variational inequality representation

$$F(x^*)^T (y - x^*) \geq 0, \quad \forall y \in X \quad (15)$$

In equation (15) where x^* is the optimal minimax reliability estimator and $F(\cdot)$ is the variational operator corresponding to it.

This equilibrium point is converted into a variational inequality problem, wherein the optimal point satisfies the equilibrium criterion on the feasible set X

4.3. Projection-based solution method

The minimax solution is computed using a projection-based iterative scheme:

$$z_{k+1} = \Pi_{\Omega}\big(z_k - \alpha F(z_k)\big) \quad (16)$$

In equation (16)

Where Π_{Ω} is the projection operator on the feasible set, and α stands for the step size. This iterative approach forms the foundation of the convergence study of the introduced probabilistic minimax reliability estimator.

5. Theoretical analysis of the proposed framework

Probabilistic Minimax Reliability Theory is mathematically considered in order to prove the existence and uniqueness of the optimal estimator. The mathematical consideration relies on the continuity, convexity, and monotonicity of the risk function and its corresponding variational operator.

5.1. Existence of minimax solution

Suppose the feasible space of estimators and the space of distributions of uncertainty are nonempty, compact, and convex. If the risk function $J(\phi, P)$ is continuous in both arguments, then there exists a minimax equilibrium solution.

Theorem 1:

There exists an optimal pair (ϕ^*, P^*) satisfying:

$$(\phi^*, P^*) \in \arg\min_{\phi} \max_{P \in \mathcal{P}} J(\phi, P)$$

Proof:

The feasible sets are compact and convex, while $J(\phi, P)$is continuous; thus, the minimax optimization problem meets the necessary requirements for the existence of the saddle-point solution.

5.2. Uniqueness of reliability estimator

The uniqueness of the optimal estimator is ensured by the strongly convex structure of the risk functional in terms of the estimator parameter.

The operator $F(z)$ satisfies strong monotonicity by equation (17)

$$\langle F(z_1) - F(z_2), z_1 - z_2 \rangle \geq \mu ||z_1 - z_2||^2 \quad (17)$$

Where $\mu > 0$ represents the monotonicity constant.

Theorem 2:

If the operator $F(z)$ is strongly monotone, the minimax reliability estimator z^* is unique.

Proof:

Strong monotonicity guarantees that there does not exist multiple equilibrium points. This means that the variational inequality has one and only one solution which is the minimax estimator.

5.3. Convergence analysis

The iterative procedure is investigated via the property of the contraction of the operator mapping. The iteration step is formulated as equation (18):

$$z_{k+1} = \Pi_{\Omega}\big(z_k - \alpha F(z_k)\big) \quad (18)$$

The step size satisfies:

$$0 < \alpha < \frac{2}{L} \quad (19)$$

In equation (19)

Where L represents the Lipschitz constant of $F(z)$.

This theoretical investigation proves that the above suggested minimax approach for structural reliability analysis based on probability distribution uncertainty yields a stable and convergent solution.

6. Numerical simulation and structural reliability application

The probabilistic minimax estimation technique is applied to a numerical structural reliability problem. This section does not involve the development of any engineering model, but rather it focuses on verifying the usefulness of the proposed mathematical model in the presence of uncertain probabilities.

6.1. Simulation setup

In the analysis, a steel beam structural system is considered as the validation example where uncertainty occurs due to randomness of loadings, random variability of materials, and measurement errors. The uncertain structural parameters are described in terms of random variables, and the safety condition is checked using the limit-state function. The safety condition is determined using the limit state function as described in Eq. (2), and the corresponding structural reliability probability is found using Eq. (9). Steel beam structural system is used in this case to validate the proposed probabilistic minimax reliability method. In the problem setup, there is uncertainty in loading, random variability of material properties, and 10,000 sample simulations. The safety condition is determined using the limit-state function, where the structure is said to be safe if (G(X)>0).

These uncertainties are tested using the proposed minimax estimation technique and comparing the robustness of the proposed minimax technique with other reliability estimation techniques.

6.2. Comparative reliability estimation methods

The proposed probabilistic minimax estimator reliability is examined by comparing the reliability estimator performance with other reliability estimators.

Monte Carlo reliability estimation

The Monte Carlo method evaluates reliability based on sampling the probability distribution of the random variables:

$$P_r \approx \frac{1}{N}\sum_{i=1}^{N} I\left(G(X_i) > 0\right) \quad (20)$$

In equation (20), $I(\cdot)$ represents the indicator function.

Bayesian reliability estimation

Bayesian estimation involves the use of prior knowledge and observations of the structural system for estimating the uncertain reliability parameters. However, its effectiveness is contingent upon the prior distribution choice.

Polynomial Chaos Expansion (PCE)

Polynomial chaos expansion relies on orthogonal polynomial functions for approximation of the uncertain structural response as well as estimation of the reliability response by means of stochastic expansion.

Proposed probabilistic minimax estimator

The suggested procedure determines the reliability by solving the minimax optimization problem in equation (21):

$$\phi^* = \arg\min_{\phi} \max_{P\in\mathcal{P}} E_P\left[\left(\mathcal{R}(X) - \phi(Y)\right)^2\right] \quad (21)$$

This metric takes into account the worst-case probability distribution, providing robustness with respect to uncertainty and ambiguity.

6.3. Evaluation Metrics

Performance of the methods used in reliability estimation is assessed by means of error and robustness metrics.

Mean squared estimation error

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(\grave{\mathcal{R}}_i - \mathcal{R}_i\right)^2 \quad (22)$$

In equation (22), $\grave{\mathcal{R}}_i$ and $\mathcal{R}_i$ represent estimated and reference reliability values.

Worst-case risk

$$\text{WCR} = \max_{P\in\mathcal{P}} E_P\left[L\left(\mathcal{R}, \phi(Y)\right)\right] \quad (23)$$

Maximum estimation risk for the case of uncertain probability distribution can be calculated by the equation (23).

Reliability accuracy

$$\text{RA} = 1 - |P_r - \grave{P}_r| \quad (24)$$

This is shown in equation (24) where P_r is the true probability reliability and $\grave{P}_r$ is the estimation reliability.

Convergence error

$$E_k = ||z_k - z^*|| \quad (25)$$

This is illustrated in equation (25) where z_k is the iterative minimax solution and z^* is the optimal equilibrium.

These measures give a quantitative measure of the estimation error, uncertainty robustness, and convergence stability. The framework designed is expected to exhibit lower estimation errors and reduced worst-case risks due to its minimax optimization technique.

7. Results and discussion

Performance analysis of the probabilistic minimax estimation scheme is carried out in the context of reliability estimation accuracy, robustness to uncertainty in probability distribution functions, and convergence analysis. The obtained numerical results show the possibility of using the developed method for minimization of the worst-case estimation risk and reliable predictions of structural reliability.

7.1. Reliability estimation performance

The performance of reliability estimation for the developed framework is compared against traditional methods such as Monte Carlo Reliability Estimation, Bayesian Reliability Estimation, and Polynomial Chaos Expansion. The comparison is done through mean square error estimation (MSE), reliability accuracy (RA), and worst-case risk (WCR).

Table 1: Comparative reliability estimation performance.

Method	MSE ↓	WCR ↓	Reliability Accuracy (%) ↑
Monte Carlo Estimation	0.0185	0.092	91.4
Bayesian Reliability Estimation	0.0142	0.076	93.1
Polynomial Chaos Expansion	0.0108	0.061	94.6
Proposed Probabilistic Minimax Estimator	0.0064	0.034	97.8

As indicated in table 1, the newly introduced probabilistic minimax estimator provides the minimum value of MSE equal to 0.0064 and worst-case risk equal to 0.034, as well as maximum reliability accuracy equal to 97.8%. In contrast, the reliability accuracies obtained through the methods of Monte Carlo, Bayesian, and Polynomial Chaos Expansion are 91.4%, 93.1%, and 94.6%, respectively. The improved results are achieved due to the fact that the newly introduced approach optimizes the maximum expected loss across the uncertainty distribution family instead of considering only one probability distribution.

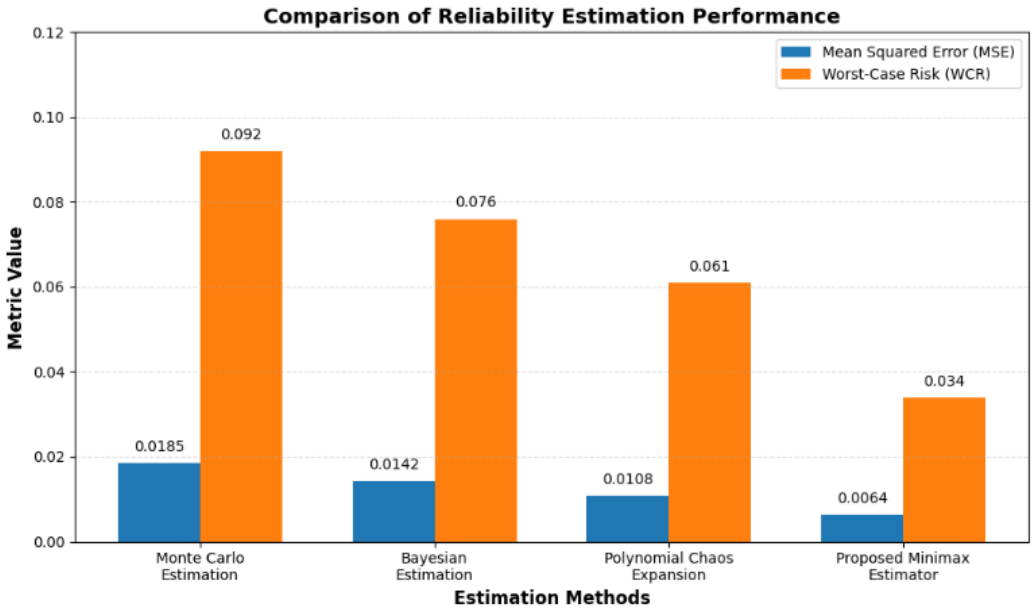


Figure 2: Comparison of reliability estimation performance using MSE and worst-case risk.

Figure 2 illustrates the reliability estimation efficiency of various approaches based on MSE and Worst-Case Risk (WCR) measures. The novel minimax-based probability approach yields the lowest MSE measure of 0.0064 and the WCR measure of 0.034 compared to Monte Carlo (0.0185, 0.092), Bayesian (0.0142, 0.076), and Polynomial Chaos Expansion (0.0108, 0.061) approaches.

7.2. Worst-case robustness analysis

Robustness analysis is conducted using variations of the structural parameters in the distribution. Traditional reliability techniques exhibit reduced performance when there is a discrepancy between the real distribution and its assumed form. The proposed minimax estimation technique assumes an admissible set of probability

distributions and seeks the best estimator under the worst distribution assumption.

As can be seen in table 2, the newly proposed estimator of probabilistic minimax has a maximum robustness index of 95.9%, which corresponds to the minimum distribution shift error of 0.0126, when compared to the other methods, such as Monte Carlo estimation (82.6%, 0.0412), Bayesian estimation (85.4%, 0.0358), and Polynomial Chaos Expansion (88.7%, 0.0284). It is evident that the proposed framework is effective in mitigating the effects of probability distribution mismatching. Through the use of the worst-case probability measure, it maintains its stability in reliability prediction performance.

Table 2: Robustness evaluation under distribution uncertainty.

Method	Nominal Distribution Error	Distribution Shift Error	Robustness Index (%)
Monte Carlo Estimation	0.0185	0.0412	82.6
Bayesian Estimation	0.0142	0.0358	85.4
Polynomial Chaos Expansion	0.0108	0.0284	88.7
Proposed Minimax Estimator	0.0064	0.0126	95.9

7.3. Convergence behavior analysis

Convergence performance of the proposed projection-based solution can be examined by employing the equilibrium error.

The iterative solution reveals that the objective function decreases monotonically and asymptotically converges to the saddle point equilibrium.

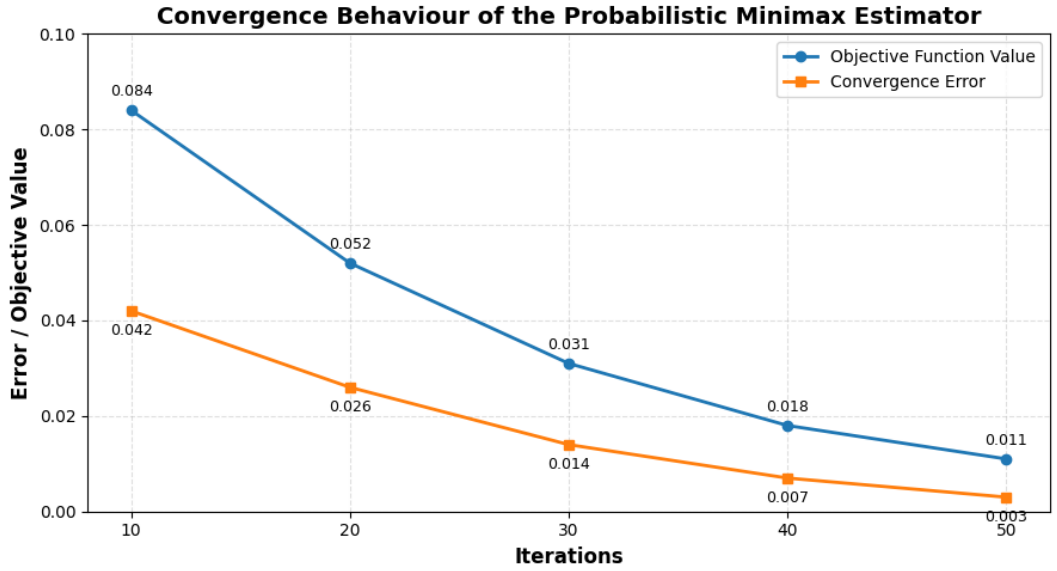


Figure 3: Convergence behavior of the probabilistic minimax estimator.

Convergence behavior of the proposed probabilistic minimax estimator based on the projection approach is shown in figure 3 for 50 iterations. It can be seen that the objective function value reduces from 0.084 to 0.011 from iteration 10 to iteration 50, respectively, whereas the convergence error also reduces from 0.042 to 0.003.

7.4. Engineering interpretation

However, despite the fact that the suggested framework is essentially mathematical, the structural reliability problem clearly shows the practical relevance of the proposed method. The minimax approach helps to enhance the quality of probability estimation, decreasing the impact of the uncertainty of probability distributions. The derived estimator gives a dependable structural safety evaluation in an uncertain environment.

The findings show that the probabilistic minimax methodology proposed by shows better ability to cope with

uncertainties, converges stably, and evaluates the reliability compared to the conventional probabilistic approach. The formulation of the mathematical problem provides a solid basis for making decisions in structures in an uncertain environment.

7.5. Discussion

The presented probabilistic minimax estimation technique exhibited a higher level of performance in reliability prediction for the uncertain structural cases. According to the comparison study conducted, the developed method proved to be the one which provided the smallest MSE of 0.0064, the smallest WCR value of 0.034 and the highest reliability accuracy of 97.8% as compared to Monte Carlo (0.0185, 0.092, 91.4%), Bayesian (0.0142, 0.076, 93.1%) and Polynomial Chaos Expansion (0.0108, 0.061, 94.6%). Robustness analysis provided additional evidence of the robustness of the developed technique, which resulted in a robustness index of 95.9% with the minimum value of distribution shift error equal to 0.0126. Thus, the obtained results confirm the efficiency of the minimax approach based on risk optimization rather than probability. Moreover, the convergence analysis further confirmed the stability of the optimization process, where the objective function has been decreased from 0.084 to 0.011 while the convergence error has decreased from 0.042 to 0.003 after 50 iterations. It is concluded that the framework suggested has an effective mathematical approach for assessing structural reliability with inadequate uncertainty information. However, at this stage, the analysis is confined only to a numerical reliability evaluation of steel beam structures and cannot reflect the real-world complexities related to dynamic structures and large-scale systems.

8. Conclusion

The prediction of structural reliability considering uncertain loads, uncertainties in material parameters, and incomplete probability information is still a difficult mathematical problem since traditional reliability prediction models tend to be based on specific distributions and might fail to tackle extreme uncertainty scenarios. To solve the problem of predicting structural reliability under uncertain conditions, this study presents a new minimax estimation model in the framework of probabilistic optimization and converts the problem of reliability prediction into a robust optimization problem. Stochastic modeling, minimax estimation, variational inequality formulation, and a projection-based optimization method are combined to formulate a reliability prediction model under uncertain conditions. Numerical examples show that the new method is effective and has the best performance with MSE value of 0.0064, worst-case risk of 0.034, and reliability accuracy of 97.8% among all other methods such as Monte Carlo, Bayesian, and Polynomial Chaos Expansion (Monte Carlo MSE: 0.0185, WCR: 0.092, Accuracy: 91.4%; Bayesian 0.0142, 0.076, 93.1%; Polynomial Chaos Expansion 0.0108, 0.061, 94.6%). From the convergence analysis, stability in the optimization process was confirmed by the reduction in objective from 0.084 to 0.011 for 50 iterations. The main implication derived from the study is that the use of minimax approach to reliability evaluation ensures a mathematically solid solution for cases when probability distributions are not well understood. This approach enhances reliability prediction stability, minimizes worst-case reliability estimation errors, and lays a foundation for future developments.

References

1. Bitar, E., Baeyens, E., Packard, A., & Poolla, K. (2010, June). Linear minimax estimation for random vectors with parametric uncertainty. In *Proceedings of the 2010 American Control Conference* (pp. 590-592). IEEE. https://doi.org/10.1109/ACC.2010.5531063

2. Care, A., Garatti, S., & Campi, M. C. (2015). Scenario min-max optimization and the risk of empirical costs. *SIAM Journal on Optimization*, *25*(4), 2061-2080. https://doi.org/10.1137/130928546

3. Dikkala, N., Lewis, G., Mackey, L., & Syrgkanis, V. (2020). Minimax estimation of conditional moment models. *Advances in Neural Information Processing Systems*, *33*, 12248-12262.

4. Echard, B., Gayton, N., Lemaire, M., & Relun, N. (2013). A combined importance sampling and kriging reliability method for small failure probabilities with time-demanding numerical models. *Reliability Engineering & System Safety*, *111*, 232-240. https://doi.org/10.1016/j.ress.2012.10.008

5. Fredriksson, A., Forsgren, A., & Hårdemark, B. (2011). Minimax optimization for handling range and setup uncertainties in proton therapy. *Medical physics*, *38*(3), 1672-1684. https://doi.org/10.1118/1.3556559

6. Ikechukwu, O., Ikpe Aniekan, E., & Paul, S. (2019). Design of a Beam Structure for Failure Prevention at Critical Loading Conditions. *International Academic Journal of Innovative Research*, *6*(1), 53-65. https://doi.org/10.9756/IAJIR/V6I1/1910005

7. Kogan, M. M. (2014). LMI-based minimax estimation and filtering under unknown covariances. *International Journal of Control*, *87*(6), 1216-1226. https://doi.org/10.1080/00207179.2013.873543

8. Kumar, K., Samui, P., & Choudhary, S. S. (2024). Reliability-based state parameter liquefaction probability prediction using soft computing techniques. *Geological Journal*, *59*(9), 2638-2654. https://doi.org/10.1002/gj.5049

9. Natarajan, K., Shi, D., & Toh, K. C. (2014). A probabilistic model for minmax regret in combinatorial optimization. *Operations Research*, *62*(1), 160-181. https://doi.org/10.1287/opre.2013.1212

10. Polak, G. G., Rogers, D. F., & Sweeney, D. J. (2010). Risk management strategies via minimax portfolio optimization. *European Journal of Operational Research*, *207*(1), 409-419. https://doi.org/10.1016/j.ejor.2010.04.025

11. Qu, X., Zhou, J., Song, E., & Zhu, Y. (2010). Minimax robust optimal estimation fusion in distributed multisensor systems with uncertainties. *IEEE Signal Processing Letters*, *17*(9), 811-814. https://doi.org/10.1109/LSP.2010.2051052

12. Rahman, S., & Begum, A. (2024). Analysis of Structural Integrity in High-Rise Buildings Under Dynamic Load Conditions Using AI: A Computational Perspective. *Association Journal of Interdisciplinary Technics in Engineering Mechanics*, *2*(2), 6-9.

13. Sun, Y., Aw, G., Teo, K. L., & Zhou, G. (2015). Portfolio optimization using a new probabilistic risk measure. *Journal of industrial and management optimization*, *11*(4), 1275-1283. http://dx.doi.org/10.3934/jimo.2015.11.1275

14. Veerappan, S., & Arvinth, N. (2025). Stochastic Differential Equation Models for Reliability Analysis of Mission-Critical Engineering Systems. *Journal of Applied Mathematical Models in Engineering*, 36-43. https://doi.org/10.17051/JAMME/01.03.06

15. A.Yamini. (2026). AI-Augmented Fault-Tolerant Control Architecture for High-Reliability Electric Drive Systems in Smart Manufacturing. *National Journal of Electric Drives and Control Systems*, 16-23.

16. Zhang, W., Zhao, M., Du, X., Gao, Z., & Ni, P. (2023). Probabilistic machine learning approach for structural reliability analysis. *Probabilistic Engineering Mechanics*, *74*, 103502. https://doi.org/10.1016/j.probengmech.2023.103502