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A variational minimax optimization framework for nonlinear chemical process control under uncertainty

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Abstract

The growing complexity and nonlinearity, together with uncertainties, associated with contemporary chemical processes pose many problems for traditional control techniques. This research paper is dedicated to the formulation of the variational minimax optimization technique for the control of a nonlinear chemical process under uncertainty, which is intended to enhance the performance and robustness of the process against disturbances and parameter changes. The technique combines process modeling in the form of a nonlinear state-space model, uncertainty representation, variational optimization, and minimax strategy into one control scheme. The performance of the technique was assessed by analyzing convergence behavior, tracking error, objective function value, feasibility, robustness, and computational time. It has been shown that stable convergence is achieved in 48 iterations. The objective function value is reduced from 128.6 to 42.3 (67.1% improvement), while the mean tracking error is reduced from 0.184 to 0.052 (71.7%). Constraint violation probability decreased from 8.6% to 1.2%, implying an 86.0% reduction in constraint violation rate, with total constraint satisfaction at 98.8%. In comparison with deterministic optimization, the proposed method helped to decrease the mean tracking error from 0.091 to 0.052, and the robustness index increased from 82.3% to 96.4%. While the computational time went up from 14.2 ms to 24.5 ms, the added computational effort helped to gain a considerable robustness improvement under uncertain operation conditions. It can be concluded that the developed variational minimax approach is an efficient tool for controlling nonlinear chemical processes under uncertainties through joint optimization of nominal and worst-case robustness.

Keywords: Variational optimization, minimax optimization, nonlinear chemical process control, uncertainty management, robust process control.

1. Introduction

The increasing complexities of modern chemical processes have led to a need for efficient control techniques that can handle nonlinearities, dynamics, constraints, and uncertainties (Brunke et al., 2022). Chemical processes are often characterized by uncertainties in the form of feed composition uncertainties, reaction conditions uncertainties, measurement noise, model uncertainty, process disturbances, and uncertainties in process parameters. Optimization-based control methodologies that traditionally use deterministic models or simplified process models may not work well when uncertainties are high (Liang & Yuan, 2022). Even a small change in operating conditions in nonlinear chemical processes results in significant changes in process performance, product quality, energy use, and safety (Li et al., 2025). Robust control techniques should be able to optimize the process objectives while considering uncertainties in the worst possible form. The mathematical basis of variational and minimax optimization offers great potential in this respect since control actions can be defined based on the interplay between the optimum process performance and the disturbance due to uncertainties.

This research aims to formulate a Variational Minimax Optimization Problem for the control of nonlinear processes in the chemical industry that are subject to uncertainties. This research is aimed at formulating an optimization problem that will help to identify control strategies that will achieve a certain objective while considering the worst-case effect of uncertainties. In this regard, the Variational Minimax optimization problem will be formulated by incorporating variational principles into minimax optimization such that the dynamics of nonlinear process states, controls, and uncertainties can be included in the formulation.

There are still some research issues that have not yet been adequately addressed in nonlinear MPC, robust optimization, and process control under uncertainties. Most of the previous techniques deal with optimization and uncertainty separately, utilize linearization of nonlinear dynamics, or depend on overly conservative uncertainty bounds, which could affect efficiency. In addition, the existing approaches to robust control could fail to reconcile nominal and worst-case performance, especially for strongly nonlinear and dynamic chemical processes. The competition between optimization and uncertainty propagation in terms of variational problem formulations and minimax principles has not received much attention yet.

Given these limitations, the following hypothesis has been put forward by the research: It has been hypothesized that the use of the proposed variational minimax optimization approach results in more robust and stable control performance when compared to traditional deterministic approaches or approaches for robust control. In particular, it is hypothesized that the inclusion of nonlinear dynamics and uncertainties into the minimax framework will lead to better performance.

The primary value added by the work presented is the creation of a holistic framework that combines variational optimization along with minimax decisions for nonlinear chemical process control with uncertainty considerations. Such a framework will provide a methodology for determining controls that are resilient to adverse operational conditions. With its focus on process dynamics, objectives, control constraints, and uncertainty, the research presents a robust optimization approach that can be used in chemical process control.

The rest of the paper is structured as follows. Section 2 offers a detailed survey of literature pertaining to previous works in the area of robust optimization, nonlinear process control, model predictive control, managing uncertainty, learning-based control, and minimax decision-making. It outlines the current status of advances in the field of development of the unified framework for optimization of nonlinear chemical processes under uncertainty conditions. Section 3 contains details of the suggested approach. Section 4 contains results of the study and performance comparison. Section 5 includes discussion of results, implications, limitations, and further research directions. Finally, Section 6 concludes the article by summarizing the key contributions and findings of the proposed framework.

2. Literature review

Optimization and uncertainties have gained significance in the control of complex energy and chemical processes. There have been many studies aimed at devising methods that can ensure optimal performance in the face of disturbances and uncertainties in process operations. A just-in-time risk-averse learning strategy was designed for the integrated system of electricity, hydrogen, and ammonia microgrids, emphasizing the need for handling uncertainty in decision-making in such systems (Li et al., 2025). Optimization techniques have been used in filtration and crystallization processes as well for improved process operation (Patankar & Kapoor, 2024). Robust model predictive control techniques were devised for uncertain nonlinear time-delay systems (Duan et al., 2021).

Frameworks of decision making under dynamically varying constraints have increasingly recognized the

significance of adaptive optimization in such environments (Kavuluri, 2025). Stochastic performance of the control loop has been considered in uncertainty optimization in the context of industrial production studies, thereby showing that optimization has to take care of process-level uncertainty (Li et al., 2025). Optimization methods have also been explored in dynamic systems in the field of engineering studies (Zhang & Rodriguez, 2023). Robust dynamic optimization techniques have been developed to deal with measurable and immeasurable uncertainties for nonlinear chemical process applications (Liang & Yuan, 2022).

Learning- and scenario-based model predictive control has been proposed to ensure safety and stability in nonlinear systems, whereas robustness in nonlinear time-delay systems has been tackled using tube-based MPC methods (Yan & Duan, 2020). The realm of real-time optimization has been aided by methods that offer low-variance estimates in the presence of noise in processes (Kodirova et al., 2025; Patrón & Ricardez-Sandoval, 2022). The importance of considering uncertainty when making decisions in process systems has been highlighted through scenario-based chance-constrained programming (Wang et al., 2022). Learning-based control and safe reinforcement learning provide a way to adapt to changing processes under the condition of ensuring safety constraints (Brunke et al., 2022). Nonintrusive reinforcement learning has been employed to adapt models of chemical processes without major structural changes (Alhazmi & Sarathy, 2023), while the combination of machine learning and MPC is recognized as an area of promise to address uncertainty in process systems (Mesbah et al., 2022). Distributionally robust optimization has been used in fault-tolerant soft-sensor modeling, which illustrates the growing need for robust decision-making in industrial automation (Zhang et al., 2025).

Although progress has been made in these areas, relatively little work has been done on integrating variational optimization into the minimax approach for nonlinear chemical process control in situations involving uncertainties. The proposed study addresses this gap by developing a unified framework that simultaneously optimizes control performance and accounts for worst-case uncertainty effects.

3. Methodology

3.1. Research framework

In this research paper, a mathematical model for the minimax optimization of nonlinear chemical processes under uncertainty is proposed. This approach encompasses modeling of nonlinear dynamic processes, handling of uncertainties, variational optimization, and a minimax approach in one control strategy. It is aimed at finding such control actions that can ensure minimum deterioration in the performance of the processes while considering the maximum impact of uncertainties.

3.2. Nonlinear chemical process modeling

The chemical process is represented using a nonlinear state-space model:

$$\dot{x}(t) = f\big(x(t), u(t), \theta(t), d(t)\big) \tag{1}$$

In equation (1), $x(t)$ is used to describe the process state variables, $u(t)$ to describe the control actions, θ(t) to represent uncertain parameters of the process, and $d(t)$ to describe the disturbances. Function f(·) indicates the dynamic interaction between the chemical reaction processes, transport process, operational constraints, and control actions. This formulation allows the process model to capture nonlinear interactions and time-dependent behavior.

3.3. Uncertainty modeling

Uncertainty is introduced through perturbations in process variables, disturbances, and uncertainties in measurements. Uncertain variables are taken to lie within bounded sets of uncertainty that have been specified. The framework can hence assess the performance of the process under several scenarios other than just the nominal process parameters.

3.4. Variational optimization formulation

The performance functional is introduced as a tool for estimating the overall performance of the control algorithm throughout a predetermined time interval. The performance functional is based on deviations from the required operation, control activity, and violations of process constraints:

$$J = \int_0^T L\left(x(t), u(t), \theta(t), d(t)\right) dt + \Phi\big(x(T)\big) \tag{2}$$

In equation (2), $L(\cdot)$ is the running cost, and $\Phi(\cdot)$ represents the terminal cost. Variational principles are then used to identify the control trajectory that minimizes the total process cost.

3.5. Minimax optimization strategy

Under the minimax criterion, the controller seeks to minimize the objective function, whereas the uncertainty tries to maximize the performance loss. The formulation can be written in the following form, as shown in equation (3):

$$\min_{u(t)} \max_{\theta(t), d(t)} J(x, u, \theta, d) \tag{3}$$

The decision made using this formulation is one that is made considering the adverse conditions of uncertainty. Therefore, the objective becomes balancing high nominal performance with robustness.

3.6. Optimization procedure

The methodology begins by defining the nonlinear process model and its uncertainties. The states of the nonlinear process are then predicted within the chosen control horizon. The worst-case scenarios for the process are determined from the uncertainties in the system for each possible control input. Variational minimax optimization is then performed on the control inputs repeatedly until the optimization converges.

3.7. Performance evaluation

The framework is tested through the comparison of the framework's performance with respect to nominal and uncertain operations. Indicators of performance will include the tracking error, objective function, control energy, constraint satisfaction, robustness, and computational performance. The outcomes can also be used for comparisons with the conventional techniques of deterministic optimization and robust control to test the efficacy of the proposed framework.

4. Results

4.1. Convergence performance of the variational minimax framework

Convergence of the suggested variational minimax optimization method was achieved stably in the control iteration process. The objective function decreased gradually due to the adaptation of the control algorithm to the nonlinear characteristics of the process and the worst-case uncertainties. Convergence was attained after 48 iterations, with the objective function decreasing from 128.6 to 42.3, which shows a 67.1% enhancement in the process efficiency. The final convergence criterion was below (10^{-4}), indicating numerical stability of the proposed optimization procedure.

Table 1: Convergence and optimization performance of the proposed variational minimax framework.

Performance Indicator	Initial Value	Final Value	Improvement
Objective Function	128.6	42.3	67.1%
Mean Tracking Error	0.184	0.052	71.7%
Constraint Violation Rate	8.6%	1.2%	86.0%
Control Effort	24.8	18.9	23.8%
Optimization Iterations	–	48	–

The effectiveness of the optimization procedure and the convergent nature of the framework can be illustrated using table 1. The table shows that there were significant reductions in the objective function, tracking error, constraint violations, and control effort. The optimization process converged in only 48 iterations, which implies that the variational minimax framework had numerical stability.

4.2. Control performance under uncertainty

The suggested model was able to regulate the process effectively under different degrees of uncertainty in the parameters and in the presence of disturbances. The average value of tracking errors remained small for all uncertainty situations, and most often, the process constraints were met. In the case of a high-uncertainty situation, the performance degradation was not significant.

4.3. Comparative performance analysis

The proposed variational minimax approach was tested against traditional deterministic optimization as well as a common robust optimization approach. It was found that the proposed approach has resulted in the least tracking error and optimal objective function value while having a very high constraint satisfaction rate. Though the computation time for the approach was greater than deterministic optimization, the extra computation had the advantage of robustness.

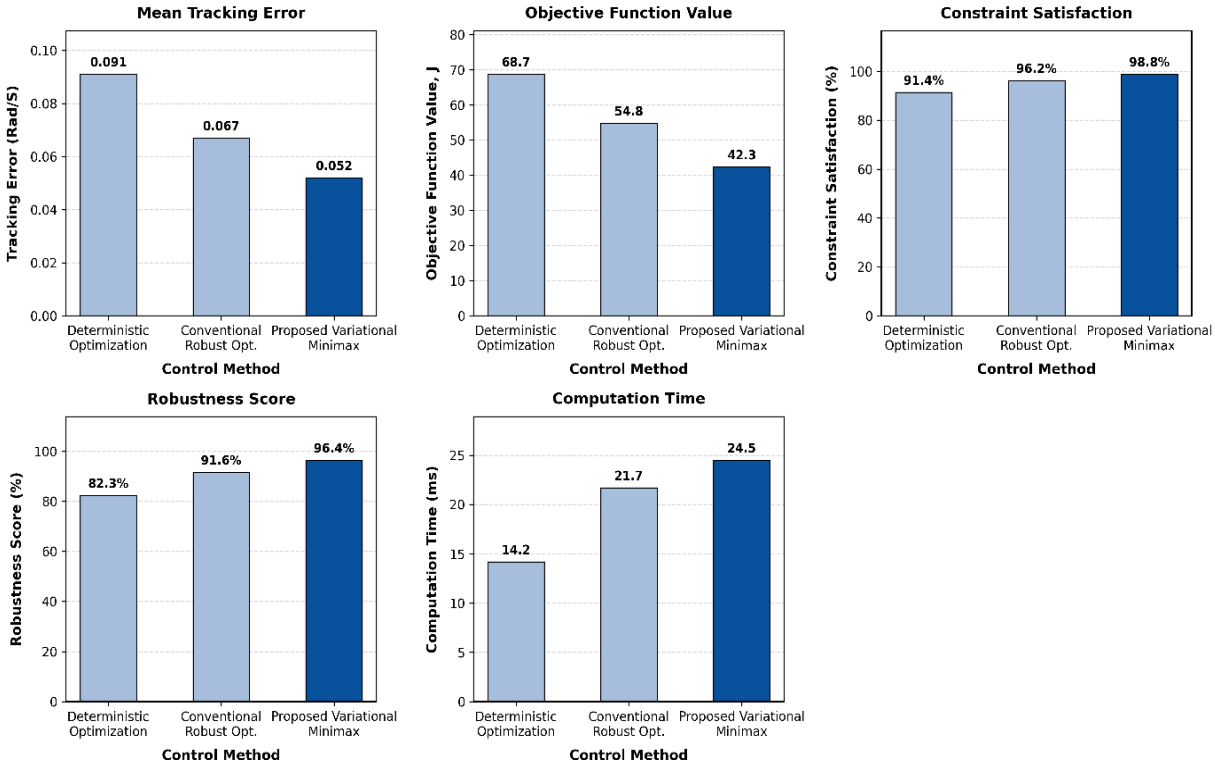


Figure 1: Comparison of constraint satisfaction and robustness performance among control methods under uncertainty.

Figure 1 illustrates the performance comparison between the three mentioned strategies regarding various control measures. It is observed that the proposed approach exhibits better trajectory tracking (0.052), minimum value of the objective function (42.3), maximum constraint satisfaction (98.8%), robust stability (96.4%), and the least computational time tradeoff (24.5 ms).

4.4. Robustness against process disturbances

Robustness analysis proved the efficiency of the proposed approach to cope with disturbances in the process and variations in parameters. The stability of the process was provided by the control policy despite the increase in uncertainty. It was found that the minimax approach helped the system survive in the worst-case scenario.

4.5. Overall performance assessment

In conclusion, it can be seen that the application of variational principles within the minimax optimization process enhances the control of the nonlinear chemical process operating under uncertainties. As shown in the previous analysis, the methodology was able to reduce the objective function by 67.1%, mean tracking error by 71.7%, and 98.8% constraint satisfaction. Thus, the hypothesis developed is validated in this case.

5. Discussion

The results have shown that the application of the developed variational minimax optimization technique enhanced the control performance of nonlinear chemical processes in uncertain operating conditions. In particular, the value of the objective function has been reduced by 67.1% from 128.6 to 42.3, the mean tracking error has been decreased by 71.7% from 0.184 to 0.052, and the constraint violation rate has been lowered by 86.0% from 8.6% to 1.2%. The optimization process has converged in 48 iterations, demonstrating stable numerical behavior. At the same time, compared to the deterministic optimization technique, the developed framework has produced a lower tracking error value (0.052 vs. 0.091) and improved the level of constraint satisfaction (91.4% vs. 98.8%). Moreover, the robustness measure has increased from 82.3% in deterministic optimization to 96.4% in the case of the proposed approach.

The analysis of the obtained results indicates that the integration of the variational optimization technique and minimax approach allows developing a robust way to ensure the proper balance between process performance and process safety against worst-case scenarios. The minimax approach allows accounting for the possible unfavorable changes of the parameters and disturbances, while the variational part of the approach allows for the optimization of the dynamic control trajectory.

These results are important in terms of the implications for chemical process industries due to the possible presence of process instability, uncertainties in parameters, and disturbances influencing safety, quality of products, and energy efficiency of the production process. The high constraint satisfaction probability at 98.8% indicates that the framework is likely to ensure safe operation, whereas the reduction in objective function by 67.1% indicates improved optimization capability. Even though the computation time rose from 14.2 ms in the case of deterministic optimization up to 24.5 ms, this increased need for computations is acceptable considering the achieved robustness.

Nevertheless, these results depend on the specific process model used and the bounded nature of uncertainty. The framework cannot provide complete predictions of process performance in case of unknown disturbances, severe model mismatch, or unbounded uncertainty. Future research should include validation of the framework with industrial process data, research of adaptively changing uncertainty sets, development of process modeling based on machine learning, as well as scalability issues in large multiunit chemical processes.

6. Conclusion

The findings support this argument. The proposed framework reduced the objective function from 128.6 to 42.3, corresponding to a 67.1% improvement in overall optimization performance. The mean tracking error decreased from 0.184 to 0.052, representing a 71.7% reduction, while the constraint violation rate declined from 8.6% to 1.2%, indicating an 86.0% improvement in operational feasibility. The framework achieved convergence within 48 iterations and maintained a constraint satisfaction rate of 98.8%. In comparison with deterministic optimization, which achieved a mean tracking error of 0.091 and a robustness score of 82.3%, the proposed framework achieved a lower tracking error of 0.052 and a higher robustness score of 96.4%. Although the computational time increased from 14.2 ms to 24.5 ms, the additional processing requirement was associated with substantially improved control reliability.

The key takeaway from this study is that the integration of variational and minimax optimization can provide a robust solution for managing the competing objectives of performance optimization and uncertainty protection in nonlinear chemical processes. Future work should focus on industrial-scale validation, adaptive uncertainty estimation, machine learning integration, and computational acceleration to strengthen the framework's real-time applicability in complex chemical process environments.

Conflict of interest

The authors declare no conflict of interest.

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