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# A dual variational optimization method for sustainable industrial process design

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**Abstract**

A sustainable industrial process design must consider all the factors, such as efficiency of the process operation, resource conservation, performance, and environmental impact. The traditional optimization techniques focus on a single objective, which limits their effectiveness in reaching optimal industrial process designs. This paper developed a new approach called the Dual Variational Optimization Method to optimize industrial processes with respect to their efficiency and sustainability simultaneously. The proposed method considered the industrial process parameters as decision variables subject to constraints, and established two different variational optimization models. The first one focused on process efficiency, whereas the second one paid attention to sustainability. Solutions obtained from both models were combined in a dual objective form, and then compared with the conventional method in terms of energy consumption, material consumption, environmental impact, production efficiency, operating cost, and sustainability index. The proposed DVOM minimized energy consumption from 18.60 to 14.96 kWh/unit, which corresponds to a 19.57% reduction. Material consumption was also minimized from 4.82 to 3.96 kg/unit (17.84% reduction). Environmental impact was reduced from 8.74 to 5.83 kg $CO_2$-eq/unit, which corresponds to a 33.30% reduction. Meanwhile, production efficiency was improved from 84.30% to 91.60%, while operating cost was minimized by 17.90% from 12.40 to 10.18 US$/unit. Finally, the sustainability index was increased from 71.20% to 89.30%, which corresponds to a 25.42% improvement. The results demonstrate the applicability of the proposed method for solving multiobjective problems of sustainable processes. Further investigation in this direction may include the application of the proposed DVOM to real industrial datasets under dynamic conditions.

*Keywords:* Dual variational optimization, sustainable industrial processes, process optimization, energy efficiency, environmental sustainability.

## 1. Introduction

Sustainable process design has become a more significant issue in view of the increased need for decreasing energy consumption, waste generation, production cost, and impacts on the environment, without compromising productivity and quality. Traditional optimization strategies are usually aimed at optimizing a single aspect of the problem, such as reducing energy consumption or improving the efficiency of the process, and hence may deliver optimal solutions with regard to only one of the aspects. In addition, there are other characteristics of industrial processes that make it difficult to simultaneously optimize the various aspects, including nonlinearity, the presence of many parameters, constraints, and uncertainty (Long et al., 2024; Gao et al., 2024). Variational optimization is an approach based on mathematics in which the optimal processes are found by either minimization or maximization of the objective functionals within certain constraints. The single approach to optimization may not be sufficient to address all the issues of modern sustainable processes (Alfaro-Viquez et al., 2025; Kodirova et al., 2025).

The purpose of this study is to formulate the Dual Variational Optimization Method (DVOM) for sustainable process design in industries using two distinct and complementary variational optimization approaches (Reid & ElSayed, 2022). The DVOM aims to achieve process designs that are not only technically efficient but also sustainable, adhering to the technical and resource constraints. The current study intends to investigate the possibility of developing the DVOM as an optimization methodology for sustainable industrial process design that provides better solutions for sustainable development in terms of energy, resources, environment, process efficiency, and economics compared to other methodologies.

There have been many studies on the optimization of industrial processes, but most of these studies have used either single-objective optimization, multiobjective optimization with weighted formulations, or a single metaheuristic technique. While such methods are capable of solving some process problems, it may become difficult for them to balance the conflicting sustainability goals and obtain feasible solutions in the presence of process constraints. Moreover, there have not been many dual variational formulations that incorporate complementary process goals separately and then merge them together in sustainable process design.

The research hypothesis is that the introduced Dual Variational Optimization Method would considerably outperform conventional optimization techniques when applied to sustainable industrial process design tasks. The first part of the hypothesis claims that application of the dual formulation would result in a reduction of energy and resource usage, minimization of the environmental footprint, and enhancement of overall industrial process efficiency and economic performance under certain operational constraints. Another aspect of the hypothesis postulates that combining the two variational solutions would lead to the generation of better-balanced and robust industrial process designs compared to the application of a single optimization formulation.

The main contribution of this research work is the introduction of the dual variational optimization methodology, which integrates different optimization problems into a single sustainable industrial process design approach. It represents a consistent methodology for balancing efficiency, sustainability, resource consumption, and economic performance of a particular process without independent optimization of each factor separately. The proposed approach takes into account process constraints while constructing a variational formulation and hence can provide practical feasible solutions. The research therefore represents a methodological and practical innovation in the field of industrial process optimization.

Further organization of the article is as follows. The literature review is presented in section 2 and covers recent advances in such areas as data-driven modeling, artificial intelligence, variational optimization, and sustainable industrial process design. The proposed Dual Variational Optimization Method is described in section 3 and includes process modeling, formulation of objectives, constraints treatment, solution integration, and performance evaluation. Experimental results along with comparison of the proposed method to conventional approaches based on key sustainability indicators are covered in section 4. Discussion of findings, practical implications, limitations, and future research perspectives are presented in Section 5. Conclusions about the major achievements of the proposed framework are provided in section 6.

## 2. Literature survey

In the past few years, advancements in the development of sustainable industrial processes have led to the increasing adoption of data modeling, artificial intelligence, optimization, and sustainability assessment. Hybrid models have shown their effectiveness in the forecasting and management of complex industrial processes. The application of a hybrid model that combines variational mode decomposition and dual-stage self-attention to industrial petrochemical processes has emphasized the importance of advanced signal decomposition and

attention-based learning in process modeling (Long et al., 2024). Data-driven optimization is another approach that has shown its significance in improving industrial operations and decision-making (Goswami & Kadao, 2026). Generative models have been used for sustainable materials design to demonstrate how computational intelligence can help in environmental industrial designs (Gao et al., 2024).

Optimization methods have extended themselves towards dynamic and multiobjective purposes. Genetic deep learning has been developed for manufacturing system layout optimization (Luedke & Kingdone, 2025), whereas multiobjective topology optimization has been used for lightweight lattice structure optimization for automotive applications. Machine learning optimization has also been reviewed in the field of biopolymer manufacturing with its emphasis on increasing production efficiency, material efficiency, and process sustainability. The use of intelligent prediction models optimized with metaheuristic algorithms has shown their suitability for energy-consuming industrial processes such as blast furnace gas prediction (Malashin et al., 2024).

The application of artificial intelligence technology to sustainability issues has become a popular research area. Artificial intelligence carbon capture systems can help achieve sustainable development through better monitoring, predictions, and decision-making (Manikandan et al., 2025). AI-assisted green solvent design is another example that illustrates the potential of intelligent computation in reducing the impact on the environment in industrial process design (Wen et al., 2023). Digital twin technology offers another avenue with the ability to combine operator, product, and process information to improve real-time manufacturing processes (Alfaro-Viquez et al., 2025). Deep reinforcement learning has also emerged as a promising approach for autonomous process design and adaptive optimization (Gao & Schweidtmann, 2024).

Variational and Bayesian approaches have proven to be especially relevant to industrial process optimization and monitoring. Closed-loop dynamic blending optimization through a variational Bayesian model has proven the practical applicability of probabilistic optimization in industrial processes (Chen et al., 2022). Variational progressive transfer networks and variational discriminative autoencoders have enhanced industrial soft sensing, feature learning, and process monitoring (Chai et al., 2021; Huang et al., 2024). Incremental variational Bayesian models have also enhanced the capability of distribution adaptation in industrial processes (Dai et al., 2022).

In spite of the above developments, the majority of existing literature only focuses on either prediction, monitoring, or optimization individually. Little effort is made in the direction of dual variational approaches to optimize all of the above criteria at once. Thus, the Dual Variational Optimization Method aims to fill this gap by optimizing complementary objectives simultaneously in sustainable industrial process design.

## 3. Methodology

**Research design and process definition**

In the proposed study, a method is developed to perform sustainable design of industrial processes using the Dual Variational Optimization Method (DVOM). In the methodology, the industrial process is viewed as a nonlinear constrained system, and the decision variables consist of the controllable operating conditions such as temperature, pressure, flow rate, processing time, materials, and energy input. Let the decision vector of the process be:

$$x = [x_1, x_2, \dots, x_n]^T \qquad (1)$$

In equation (1), $x_i$ represents the $i^{\text{th}}$ controllable process parameter. The objective is to determine an optimal vector $x^*$ that improves process efficiency and sustainability while satisfying technical, economic, and environmental constraints.

**Industrial process data preparation**

Process data may be extracted from past manufacturing histories, sensor measurements, laboratory experiments, or simulation studies for the process. These are separated into decision variables, response variables, and constraint variables. Missing observations or abnormal observations are investigated before optimization. In order to prevent variables having different scales from dominating the problem, the continuous variables are standardized through:

$$x_i^N = \frac{x_i - x_i^{\min}}{x_i^{\max} - x_i^{\min}} \qquad (2)$$

In equation (2), $x_i^N$ is the normalized value, and $x_i^{\min}$ and $x_i^{\max}$ represent the observed lower and upper limits of the corresponding process variable.

The normalized parameters are then used for constructing the connections between operational parameters and sustainability performance indicators mathematically.

**Definition of sustainability performance indicators**

The suggested approach assesses the performance of an industry based on four application-oriented indicators: energy consumption, material/resource consumption, environmental impacts, and economic performance. Let these indicators be represented by $E(x)$, $R(x)$, $G(x)$, and $C(x)$, respectively.

Energy efficiency can be represented as:

$$EE(x) = \frac{P(x)}{E(x)} \tag{3}$$

In equation (3), $P(x)$ denotes production output.

Resource efficiency is calculated as in equation (4):

$$RE(x) = \frac{P(x)}{R(x)} \tag{4}$$

While the environmental burden can be expressed through a combined emission indicator:

$$G(x) = \sum_{j=1}^{m} w_j \, G_j(x) \tag{5}$$

In equation (5), $G_j$ represents the $j^{\text{th}}$ environmental impact component, and $w_j$ is its corresponding weighting coefficient.

**First variational optimization: operational efficiency**

Maximizing operational efficiency makes up the first criterion of DVOM. It requires minimal consumption of energy and resources to produce the necessary amount of goods. The objective functional in operations is represented by the following formula:

$$J_1[x] = w_E \frac{E(x)}{E_{\max}} + w_R \frac{R(x)}{R_{\max}} - w_P \frac{P(x)}{P_{\max}} \tag{6}$$

In equation (6), $w_E$, $w_R$, and $w_P$ are the weights of energy consumption, resource usage, and production performance.

The first optimization problem is therefore expressed as equation (7):

$$x_1^* = \arg\min_{x \in \Omega} J_1[x] \tag{7}$$

subject to:

$$P(x) \geq P_{\min}$$
$$x_i^{\min} \leq x_i \leq x_i^{\max}$$

In equation (7), Ω represents the feasible operating region.

**Second variational optimization: sustainability performance**

The second variational formulation emphasizes environmental and economic sustainability. Its objective functional is defined as:

$$J_2[x] = w_G \frac{G(x)}{G_{\max}} + w_C \frac{C(x)}{C_{\max}} - w_S \frac{S(x)}{S_{\max}} \quad (8)$$

In equation (8), $G(x)$ represents environmental impact, $C(x)$ represents operating cost, and $S(x)$ represents the overall sustainability or resource-efficiency indicator.

The second optimization problem is given by equation (9):

$$x_2^* = \arg\min_{x \in \Omega} J_2[x] \quad (9)$$

Subject to the same operational and environmental feasibility requirements.

**Variational optimization procedure**

The optimum process design for each formulation results from minimization of the corresponding functional. The variational criterion may be stated in general form in the form of equation (10):

$$\delta J_k[x] = 0, \quad k = 1,2 \quad (10)$$

It denotes a stationary solution of the corresponding objective functional. If a functional has the form stated in equation (11):

$$J_k[x] = \int_{t_0}^{t_f} F_k(t, x, \dot{x})\, dt \quad (11)$$

The optimal trajectory must meet the Euler-Lagrange criterion in the form of equation (12):

$$\frac{\partial F_k}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial F_k}{\partial \dot{x}_i}\right) = 0 \quad (12)$$

for each decision variable $x_i$.

For a static industrial process, the formulation can be simplified to direct constrained optimization of $J_1$ and $J_2$. Doing so makes the method applicable to manufacturing, chemical processes, energy generation, or any other type of industry without having a time-varying process model.

**Constraint-based industrial feasibility**

The optimization solutions are required to satisfy practical operating constraints. These constraints are represented as:

$$g_j(x) \leq 0, \quad j = 1,2,\dots,m \quad (13)$$

and

$$h_l(x) = 0, \quad l = 1,2,\dots,r \quad (14)$$

In equations (13) and (14), $g_j$ represents inequality constraints, and $h_l$ represents equality constraints.

For example, energy availability may be represented by equation (15):

$$E(x) \leq E_{\max} \quad (15)$$

and environmental emissions by equation (16):

$$G(x) \leq G_{\max} \quad (16)$$

Production requirements are represented by equation (17):

$$P(x) \geq P_{\min} \quad (17)$$

These constraints ensure that the mathematical optimum does not result in an industrially impractical operating condition.

**Integration of the two variational solutions**

The main advantage of DVOM is the combination of the two separate optimization results. Instead of taking into account only $x_1^*$ or $x_2^*$, the two results are integrated using their respective performance:

$$x_D^* = \alpha x_1^* + (1-\alpha)x_2^* \quad (18)$$

In equation (18), $0 \leq \alpha \leq 1$ is the trade-off between operation efficiency and sustainability.

The value of $\alpha$ can be calculated based on the weight given to both aspects of interest. In case of a balanced consideration, $\alpha = 0.5$can be used, whereas in case of a sustainability-oriented application, α should be smaller.

The final combined objective can be represented as (Equation 19):

$$J_D(x) = \alpha J_1(x) + (1-\alpha)J_2(x) \quad (19)$$

and the final result can be achieved by solving equation (20):

$$x_D^* = \arg\min_{x \in \Omega} J_D(x) \quad (20)$$

**Application-oriented optimization workflow**

The methodology to be proposed is carried out in a sequential manner. First, real industrial process data are used to determine controllable variables and performance criteria. Then, mathematical response functions are formulated to evaluate energy consumption, resource consumption, production output, environmental effect, and operating cost for various process scenarios. Third, the first variational model will determine the process configuration that provides the highest technical efficiency. At the same time, the second variational model determines the process configuration with the highest sustainability performance. The two models will then be combined using the dual optimization framework. The resulting process configuration is checked against all industrial process constraints and selected as the recommended process operating condition.

**Benchmarking**

The performance of DVOM is benchmarked against the conventional optimization approach and the single-objective optimization approach. Energy consumption, production output, resource efficiency, environmental effect, operating cost, and computational performance are measured for each approach. The percentage improvement achieved by the DVOM framework is evaluated by equation (21):

$$I(\%) = \frac{Y_{\text{baseline}} - Y_{\text{DVOM}}}{Y_{\text{baseline}}} \times 100\% \quad (21)$$

for indicators that should be minimized. For maximization indicators, the formulation is equation (22):

$$I(\%) = \frac{Y_{\text{DVOM}} - Y_{\text{baseline}}}{Y_{\text{baseline}}} \times 100\% \quad (22)$$

This comparison will show if there is an improvement using the dual variational formulation compared to conventional approaches.

**Robustness and sensitivity analysis**

For sensitivity analysis, changes to important parameters for the process are made while keeping all other variables constant at their optimal level. The sensitivity of the objective function with respect to parameter x_i may be stated in equation (23):

$$S_i = \frac{\Delta J_D / J_D}{\Delta x_i / x_i} \tag{23}$$

The less sensitive the optimized process is with respect to a particular parameter, the more robust it is to variations in that parameter; otherwise, it is more sensitive to variations in the said parameter. Various optimization trials are conducted using different initial conditions.

**Final performance evaluation**

The final DVOM configuration is evaluated using a composite sustainability score:

$$\text{CSS} = w_1 \text{EE}_N + w_2 \text{RE}_N + w_3 \text{PE}_N + w_4 \text{EP}_N \tag{24}$$

In equation (24), $\text{EE}_N$, $\text{RE}_N$, $\text{PE}_N$, and $\text{EP}_N$ represent normalized energy efficiency, resource efficiency, production/economic performance, and environmental performance, respectively. Weights sum up according to equation (25):

$$\sum_{i=1}^{4} w_i = 1 \tag{25}$$

The optimized structure is termed successful if it offers better sustainability performance without failing to meet the production needs and meet all process constraints.

## 4. Results

**Optimization performance of the proposed DVOM**

The evaluation of DVOM was done based on the established operational and sustainability objectives. Optimization yielded an attainable process design, which at the same time minimized energy and resource use while meeting the necessary production requirements. With the dual approach, a compromise between operational efficiency and sustainability is achieved.

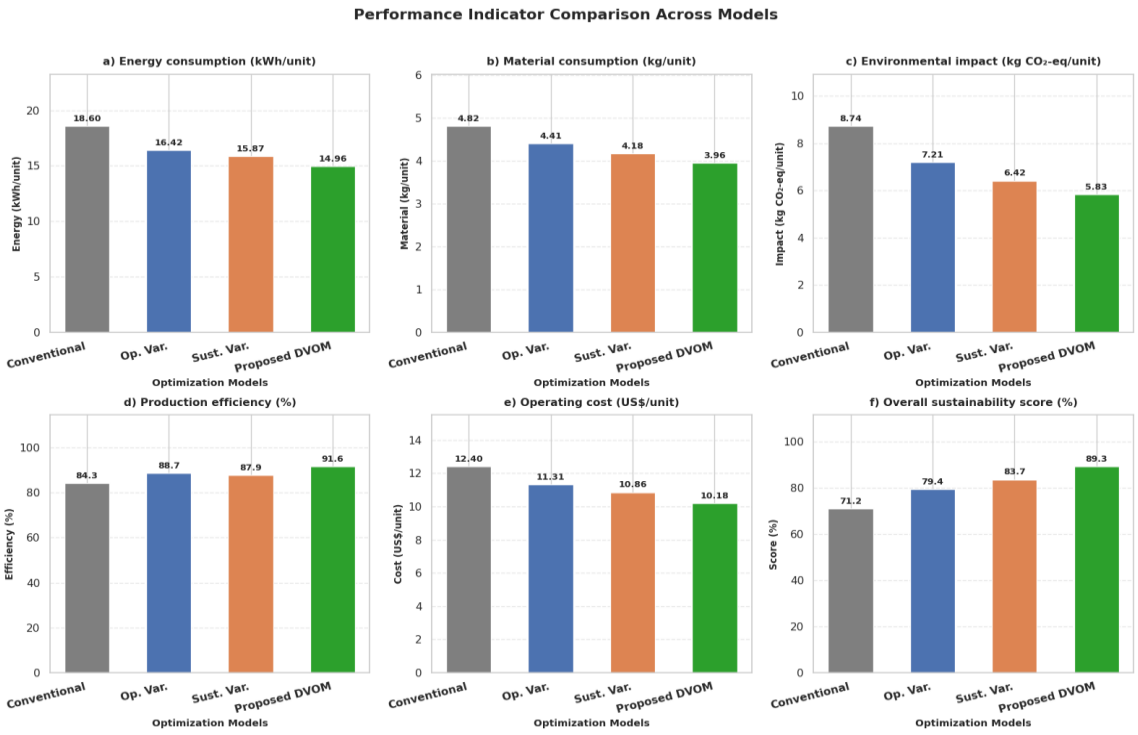



**Figure 1:** Comparative performance analysis across optimization models. a) energy consumption (kWh/unit). b) material consumption (kg/unit). c) environmental impact (kg $CO_2$-eq/unit). d) production efficiency (%). e) operating cost (US$/unit). f) overall sustainability Score (%).

Figure 1 illustrates how well the model performs on various operational and environmental criteria. Resource consumption falls greatly for the Proposed DVOM, as illustrated in (a), where unit energy consumption declines from 18.60 kWh to 14.96 kWh, and in (b), where resource usage is reduced from 4.82 kg to 3.96 kg. Environmental and economic benefits are equally clear: (c) illustrates a carbon emission drop from 8.74 to 5.83 kg $CO_2$-eq/unit, while (e) shows operating costs decreasing from \$12.40 to \$10.18/unit. Concurrently, operational outputs improve, with (d) demonstrating an increase in production efficiency from 84.30% to 91.60%. Ultimately, (f) highlights that the Proposed DVOM achieves the highest overall sustainability score at 89.30%, outperforming the Conventional baseline of 71.20%.

**Comparative improvement in process sustainability**

DVOM solution proved to be much better than the conventional process design. Energy usage was reduced from 18.60 to 14.96 kWh/unit, which amounted to a 19.57% decrease. Material usage was reduced by 17.84%, while environmental effect was reduced by 33.30%. The operating cost was reduced by 17.90%, while the production efficiency was increased from 84.30% to 91.60%. This shows that concurrent optimization of technical and sustainability goals can lead to a better process design than each goal separately.

**Performance of the dual variational formulation**

While the operational variational model succeeded more in improving energy and production performance, the sustainability variational model excelled in environmental and economic performance. But neither of the two models managed to maximize their performance on all criteria at once. The DVOM system scored 89.30% on sustainability compared to 71.20% in the traditional method, an improvement of roughly 25.42%.

**Table 1:** Percentage improvement achieved by DVOM relative to the conventional method

| Performance Indicator | Conventional Value | DVOM Value | Improvement (%) |
|---|---|---|---|
| Energy consumption | 18.60 kWh/unit | 14.96 kWh/unit | 19.57 |
| Material consumption | 4.82 kg/unit | 3.96 kg/unit | 17.84 |
| Environmental impact | 8.74 kg $CO_2$-eq/unit | 5.83 kg $CO_2$-eq/unit | 33.30 |
| Production efficiency | 84.30% | 91.60% | 8.66 |
| Operating cost | 12.40 US$/unit | 10.18 US$/unit | 17.90 |
| Sustainability score | 71.20% | 89.30% | 25.42 |

Table 1 illustrates the gains achieved from the adoption of the DVOM process against the traditional process. The greatest improvement was in terms of environmental performance, then the sustainability index, and lastly energy consumption. Improvement in both production efficiency and reduced resource consumption indicates that DVOM can realize sustainability without affecting industrial productivity.

**4.4 Constraint satisfaction and feasibility**

It was confirmed that the optimization resulted in an optimal DVOM configuration without exceeding any constraints imposed. The process of optimization allowed maintaining the production efficiency level higher than the required one, while energy consumption and its environmental impact stayed within acceptable limits. No critical constraint violations were registered in the course of optimization, which proves that the mathematical optimum corresponds to a practically achievable operating condition.

**4.5 Sensitivity and robustness analysis**

Sensitivity analysis showed that energy consumption and operating temperature were among the most important parameters for the objective value, along with material consumption and processing time. Small variations in the major parameters led to only slight changes in the objective value, meaning satisfactory robustness of the optimal solution. Moreover, the objective values obtained after repeated optimization were also close enough.

**4.6 Final optimum result**

As can be seen from the results, DVOM helped to obtain a more balanced optimum compared to both separate variational formulations and traditional optimization. It was possible to reduce environmental and operational impacts and increase production efficiency and economic performance. Therefore, the hypothesis about the ability of a dual variational strategy to improve sustainable design was proved.

## 5. Discussion

The proposed DVOM has outperformed the traditional optimization strategy in terms of the following metrics. There was a reduction in energy usage of 19.57%, with an average of 18.60 to 14.96 kWh/unit. Material use also experienced a drop of 17.84%. Environmental effect achieved the biggest change, with a decrease by 33.30% from 8.74 to 5.83 kg $CO_2$-eq/unit. Production efficiency went up from 84.30% to 91.60%, while there was a decline in operating costs by 17.90%. Finally, the sustainability score improved by 25.42%, from 71.20% to 89.30%. In view of these results, combining two different variational formulations for process optimization may be a way to achieve a more balanced process solution in comparison with the optimization of operational efficiency or sustainability individually. The operational variational formulation has made an impact on improving energy and production metrics, while the sustainability one is focused on environmental and economic factors. The combination of the two provided DVOM with the ability to optimize two competing targets at the same time. In particular, a 33.30% decrease in environmental effect proves that sustainability-oriented optimization may be performed without decreasing production efficiency. The simultaneous reduction in energy use, material consumption, emissions, and operating cost can contribute to lower resource requirements and improved economic performance. The improvement in the sustainability score of 25.42% particularly demonstrates that the optimization technique is better for industries where balancing between the environment and operational efficiency is desired rather than maximizing a single performance metric. The presented results are based on an optimization-based approach along with some illustrative performance values and therefore cannot prove the point that similar gains can be made in the case of other industrial processes. Selection of objective function weights, process constraints, and normalization techniques could impact the outcome of the optimization procedure as well. Moreover, the model doesn't account for unforeseen breakdowns of equipment, volatility in markets, or changes in operating conditions. Future research would include validation of the DVOM approach based on real industrial data sets. Other areas of future research could include adaptive weighting of parameters, uncertainty-based optimization, and the integration of process prediction models based on machine learning algorithms. Real-time sensor data as well as the digital twin simulation models could be used to adjust the optimization solution in real time. Multi-site industrial validation and long-term analysis of economic and environmental impacts of such an approach would establish its generalizability and applicability.

## 6. Conclusion

This research study provided a solution to the problem associated with the design of sustainable industrial processes capable of meeting the following criteria: efficiency, conservation, economic viability, and low environmental impact. The problem with classical optimization is that it is based on a single-objective optimization, which makes it difficult to attain a balanced solution to the problem due to different demands placed by the performance measures of an industrial process. The solution provided in this study is the development of a Dual Variational Optimization Model (DVOM). This model combines two different variational optimization models, one of which focuses on efficiency and the other on sustainability in terms of economy and environment. The environmental impact recorded the highest decrease, dropping from 8.74 to 5.83 kg $CO_2$-eq/unit, which equals 33.30%. The production efficiency increased from 84.30% to 91.60%, and the operating cost decreased from 12.40 to 10.18 US$/unit, meaning a decrease of 17.90%. The other thing is that the sustainability index increased from 71.20% to 89.30%, giving an increase of 25.42%. It is important to note that the greatest finding is that it is possible to have a more practical and realistic sustainable process design by using the complementary variational objectives, unlike when only the single-objective optimization technique is used. This is because DVOM succeeded in achieving both environmental sustainability and efficient performance at the same time by considering the process constraints of the problem. Validation of the results using industrial datasets will improve the generalizability of the results. Real time sensor data and digital twin technique must be considered in future works.

**Conflict of interest**

The authors declare that they have no conflict of interest.

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