Minimax Theory and its Applications
Volume 11 (2026), No. 2s, 508-516

Mathematical frameworks for non-smooth and nonconvex systems under uncertainty: applications in optimization, control, and game-theoretic equilibria

Saniya Khurana[1], Tanveer Ahmad Wani[2], D. Balaji[3], Kaustubh Milind Utpat[4], Vundela Swathi[5], Arsh Goyal[6], Dr. Chetansinh R. Vaghela[7]

[1]Centre for Research Impact & Outcome, Chitkara University Institute of Engineering and Technology, Chitkara University, Rajpura,Punjab, India. saniya.khurana.orp@chitkara.edu.in, https://orcid.org/0009-0005-3659-5362

[2]Department of Physics, Noida international University, Greater Noida, Uttar Pradesh, India, tanveer.ahmad@niu.edu.in

[3]Assistant Professor, Department of Humanities and Sciences/ Mathematics Division,Aarupadai Veedu Institute of Technology, Vinayaka Mission's Research Foundation(DU), Tamil Nadu, India. balaji@avit.ac.in, https://orcid.org/0000-0003-2858-2578

[4]Department of Mechanical Engineering, Vishwakarma University, Maharashtra, Pune, India. kaustubh.utpat@vupune.ac.in

[5]Centre for Multidisciplinary Research, Anurag University, Hyderabad, Telangana, India. VundelaSwathi2024@protonmail.com, https://orcid.org/0009-0009-4991-1511

[6]Quantum University Research Center, Quantum University, Roorkee, Uttarakhand, India. aarshgoyal@gmail.com*, https://orcid.org/0009-0009-1847-4064*

[7]Associate Professor, Faculty of Science, Gokul Global University, Sidhpur, Gujarat, India. crvaghela.maths.gsc@gokuluniversity.ac.in,

https://orcid.org/0000-0001-5692-5190

Abstract

Many optimization, dynamical systems, and game-theoretic models of interest in learning, control, and economics involve objectives that are non-smooth, nonconvex, and stochastic. Smooth stochastic approximation theory and non-smooth analysis each have tools to study different aspects of this complex terrain, but a unifying formalism applicable to all three categories of problems, under one set of regularity assumptions, has yet to emerge. In this paper, we provide such a formalism by viewing the three problem types as examples of a differential inclusion of stochastic processes defined by expected (Aumann integral) Clarke subgradients. We show existence of stationary points for the class of models satisfying a.s. Clarke regularity and integrable upper subgradient bounds; prove the convergence of a projected stochastic subgradient algorithm to the stationary set at Robbins-Monro rates via differential-inclusion ODE techniques; and extend the argument from Rosen on diagonal strict concavity of potential games to prove existence of generalized equilibria for nonconvex potential games with stochastic disturbances. Three applications to piecewise-smooth learning in noisy environments, viability-controlled stochastic switched systems, and selection of equilibria in threshold-toll congestion games demonstrate how the three theorems apply in distinct settings.

Keywords: non-smooth analysis, stochastic approximation, differential inclusions, generalized Nash equilibrium, Clarke subdifferential, nonconvex optimization.

1. Introduction

The combination of non-smoothness, non-convexity, and randomness is no longer an exception but a rule in practical

computation. Neural networks with rectification or piecewise linearity are inherently non-smooth; switched and hybrid control systems have set-valued dynamics that depend on the mode; and problems related to equilibrium in transportation, energy, or market design often have kinked costs that arise due to thresholds or constraints. The classical theory of first order, which includes the Karush-Kuhn-Tucker optimality conditions, the smooth stochastic approximation approach in (Robbins & Monro 1951), and the variational inequality formulation of Nash equilibrium, all rely on the assumption of differentiability.

Two distinct bodies of literature have separately attempted to bridge this gap. First, deterministic non-smooth and variational analysis, pioneered, provides generalized derivatives and calculus rules sufficient for nonconvex, locally Lipschitzian functions. Second, stochastic approximation theory, going back to Robbins & Monro, (1951), and extending up to Benaïm et al., (2005) and beyond, considers the almost sure convergence properties of noisy recursive algorithms, whether the recursions are controlled by differential inclusions rather than vector fields. The gap to be filled is to articulate a notion of an expected generalized subdifferential of random non-smooth functions, and state the least regularity condition under which such an object supports a theory of existence of stationary points, as well as one of optimization, control and equilibrium formulation.

Contributions. The present paper (1) introduces a stochastic, non-smooth, and non-convex setting based on the Aumann integral of the Clarke subdifferential of a random integrand; (2) establishes an existence result for stationary points of the expectation of the objective under almost-sure Clarke regularity and an integrable subgradient condition (Theorem 1); (3) demonstrates almost-sure convergence of a projected stochastic subgradient method to the stationary points set under the Robbins-Monro step-size assumptions by employing the ODE method in differential inclusion form (Theorem 2); (4) generalizes Rosen's, (1965) existence proof for concave games to nonconvex potential games where the potential is almost-surely Clarke regular and strategy correspondences are set-valued and Aubin continuous (Proposition 3); and (5) illustrates the above three theorems in three applications from machine learning, control, and game theory domains to demonstrate that in all three cases the regularity assumptions suffice.

1.1 Related work

Deterministic non-smooth calculus is well advanced: Generalized gradient forms the basis of most of the Lipschitzian optimization theory, whereas variational analysis extends the calculus to the case of non-regular and extended real-valued functions at the expense of losing convex-valuedness. The corresponding stochastic theory is that and its generalization, which considers noisy recursions with mean field being a single-valued and usually continuous vector field (Robbins & Monro, 1951). The extension of this stochastic recursion theory to include set-valued mean fields, provides the rigorous background for stochastic approximations in the context of non-smooth optimization, yet it has not been used for analyzing equilibrium problems (Benaïm et al., (2005),. On the other hand, variational inequalities, complementarity problems and the recent survey of generalized Nash equilibrium problems are mostly deterministic and do not use non-smooth calculus when dealing with non-convexity issues (Facchinei & Kanzow, 2010). The recent literature in discusses stochastic subgradient algorithms for weakly convex functions that form a subset of the set of Clarke regular functions and are contained in the present context (Davis & Drusvyatskiy, 2019; Duchi & Ruan, 2018). Studies have taken uncertainty-aware optimization a step further using intelligent approaches. proposed an uncertainty-aware deep cooperative learning scheme for spectral sensing and channel access optimization, proving the significance of adaptation to uncertain network settings (Ferrari et al., 2025). On the other hand, (Sindhu, 2023) considered dynamic systems under interval uncertainties, whereas Wesam Ali, (2025) proposed physics-informed neural architecture for uncertainty-aware adaptive control.

2. Preliminaries and notation

Throughout this paper, $(\Omega, \mathcal{F}, \mathbb{P})$denotes a complete probability space carrying a random vector ξ, and $\mathbb{E}$represents the expectation operator with respect to $\mathbb{P}$. A function $f: \mathbb{R}^n \times \Xi \to \mathbb{R}$is called **Carathéodory** if $f(\cdot, \xi)$is locally Lipschitz for $\mathbb{P}$-almost every ξ, and $f(x, \cdot)$is measurable for every x.

For a locally Lipschitz function $g: \mathbb{R}^n \to \mathbb{R}$, the **Clarke generalized directional derivative** at $x$in the direction v, and the corresponding Clarke subdifferential, are defined as equation (1)

$$g^{\circ}(x; v) = \limsup_{\substack{y \to x \\ t \downarrow 0}} \frac{g(y + tv) - g(y)}{t},$$
$$\partial g(x) = \{\zeta, \in \mathbb{R}^n : \langle \zeta, v \rangle \leq g^{\circ}(x; v) \ \forall v\}. \tag{1}$$

A function $g$is said to be **Clarke regular** at $x$if the classical one-sided directional derivative exists and coincides with $g^{\circ}(x;\cdot)$for every direction v.

A set-valued mapping $F\colon \mathbb{R}^n \rightrightarrows \mathbb{R}^m$satisfies the **Aubin (pseudo-Lipschitz) property** (Aubin and Frankowska, 1990) at $(\bar{x},\bar{y}) \in \operatorname{gph} F$if there exist neighborhoods $U$of $\bar{x}$, $V$of $\bar{y}$, and a constant $\kappa \geq 0$such that

$$F(x) \cap V \subseteq F(x') + \kappa \parallel x - x' \parallel \mathbb{B},$$

for all $x, x' \in U$.

The Aumann integral of a random set-valued mapping $x \mapsto \mathcal{G}(x,\xi)$is defined as

$$\mathbb{E}[\mathcal{G}(x,\xi)] = \{\mathbb{E},[\gamma(\xi)],:\gamma(\xi) \in \mathcal{G}(x,\xi) \text{ a.s.}\gamma \text{ measurable}\mathbb{E} \parallel \gamma \parallel < \infty\}.$$

This integral is well-defined and compact-convex-valued when $\mathcal{G}(x,\cdot)$is measurable and admits an integrable bound, according to the Kuratowski–Ryll-Nardzewski measurable selection theorem (1965).

A trajectory of the differential inclusion

$$\dot{x}(t) \in H\big(x(t)\big)$$

which is a continuous function with compact-convex values and is locally bounded is called absolutely continuous function for which holds inclusion almost everywhere. The Aumann-integrated Clarke subdifferential and its corresponding differential inclusion are the basic mathematical elements that are used in this paper.

Two calculus rules are repeatedly applied in the following sections. First, the **Clarke sum rule** states that if $g_1$and $g_2$are Clarke regular at x, then $g_1 + g_2$is also Clarke regular and

$$\partial(g_1 + g_2)(x) = \partial g_1(x) + \partial g_2(x).$$

The regularity condition of both functions is necessary because, for non-regular functions, the Clarke subdifferential only satisfies the inclusion

$$\partial(g_1 + g_2)(x) \subseteq \partial g_1(x) + \partial g_2(x).$$

Second, the **Clarke max rule** states that for

$$g(x) = \max\big(g_1,(x)g_2(x)\big),$$

where $g_1$and $g_2$are continuously differentiable near x, the function $g$is Clarke regular. If $g_1(x) = g_2(x)$, then

$$\partial g(x) = \operatorname{conv}\{\nabla, g_1(x)\nabla g_2(x)\},$$

whereas for a strict maximizer,

$$\partial g(x) = \{\nabla g_i(x)\}.$$

These calculus rules are utilized respectively for composing penalty terms in Section 5.1 and differentiating threshold-based toll cost functions in Section 5.3.

3. A unified stochastic non-smooth framework

3.1 Stochastic non-smooth, nonconvex optimization

Consider the stochastic optimization problem

$$\min_{x\in\mathbb{R}^n} F(x) := \mathbb{E}_{\xi}[f(x,\xi)],$$

where $f(\cdot,\xi)$is locally Lipschitz and Clarke regular for $\mathbb{P}$-almost every ξ, with an integrable local Lipschitz bound. Since Fmay be neither differentiable nor convex, the classical gradient operator $\nabla F(x)$is replaced by the expected

generalized subdifferential.

The relationship between the subdifferential of the expected objective and the expected subdifferential of the stochastic function is given by

$$\partial F(x) \supseteq \mathbb{E}_{\xi}[\partial f(x,\xi)], \tag{2}$$

where $\mathbb{E}_{\xi}[\partial f(x,\xi)]$represents the Aumann integral of the random set-valued mapping $\partial f(x,\xi)$. Equality holds when $f(\cdot,\xi)$is Clarke regular almost surely and the generalized gradients are dominated by an integrable Lipschitz constant.

Interchangeability of expectation and subdifferential in equation (2) is the basis for the stochastic analog of the envelope theorem. Such property allows us to use sample-based generalized gradients

$$\zeta(x,\xi) \in \partial f(x,\xi),$$

as unbiased estimates for elements of the expected generalized subdifferential ∂F(x). In other words, even when F is nonsmooth and nonconvex, we can use stochastic optimization approaches to solve the problem through sampled subgradients.

3.2 Stochastic control with set-valued dynamics

Consider a controlled stochastic process evolving within a time-varying viability set $C(t) \subseteq \mathbb{R}^n$and governed by the differential inclusion

$$\dot{x}(t) \in \mathcal{A}\big(x,(t)u(t)\big) + \sigma\big(x(t)\big)\frac{dW(t)}{dt},$$

where A is the set-valued map for dynamics of the switched or hybrid system, σ (x(t)) represents the diffusion term, and W (t) is the Wiener process. In order to ensure feasibility of the constraints, which are dynamic, time-varying, and possibly nonconvex, the normal-cone correction of the dynamics yields the projected differential inclusion

$$\dot{x}(t) \in \mathcal{A}\big(x,(t)u(t)\big) - N_{C(t)}\big(x(t)\big),$$

where $N_{C(t)}\big(x(t)\big)$represents the normal cone associated with the constraint set. The corresponding stochastic optimal control problem is formulated through the cost functional

$$J(u) = \mathbb{E}\left[\int_0^T \ell\left(x,(t)u(t)\right)dt\right],$$

where $\ell\big(x,(t)u(t)\big)$may be non-smooth with respect to the state variable. As a result, the stationarity condition of the optimal control problem is not expressible in terms of the classical derivatives, but rather in terms of an inclusion of the Clarke subdifferential. The generalised stationarity condition is formulated based on the theory developed in Section 3.1 and is applied along the trajectories of the state.

3.3 Nonconvex generalized Nash games

Consider a game involving Nplayers, where each player $i$selects a strategy $x_i \in X_i(x_{-i})$. Here, $X_i$denotes a possibly nonconvex and set-valued strategy correspondence that depends on the strategies of other players $x_{-i}$and satisfies the Aubin property. Let the random cost function of player $i$be represented as $J_i(x_i, x_{-i}, \xi)$, which is assumed to be Clarke regular with respect to $x_i$for $\mathbb{P}$-almost every ξ.

A **generalized Nash equilibrium (GNE)** is a strategy profile $x^* = (x_1^*, \dots, x_N^*)$satisfying

$$x_i^* \in \mathbb{E}_{\xi}[J_i(y, x_{-i}^*, \xi)],\ i = 1, \dots, N. \tag{3}$$

The above formulation demonstrates that the given condition is true when each player has an optimal strategy, given the strategies adopted by the other players, under the constraints defined by the strategy correspondence.

When the game admits a potential function $\Phi(x,\xi)$satisfying

$$\partial_{x_i}\Phi(x,\xi) = \partial_{x_i}J_i(x,\xi),\ i = 1,\dots,N,$$

The equilibrium problem becomes equivalent to a non-smooth potential minimization problem. Here, the generalized Nash equilibrium solution is identified with the stationary point of

$$\mathbb{E}_{\xi}[\Phi(x,\xi)]$$

over the joint constraint correspondence. It helps to build a common link between the generalized Nash game model (as expressed in equation 3) and the stochastic nonsmooth optimization problem introduced in Section 3.1.

3.4 A single governing inclusion

The formulations in Sections 3.1 to 3.3 are unified through a definition of zas the decision variable, Z(z)as the feasible constraint correspondence, and $\Psi(z^*,\xi)$as the Clarke regular random potential function. All stationarity conditions can thus be formulated as

$$0 \in \mathbb{E}[\partial_z\Psi(z^*,\xi)] + N_{Z(z^*)}(z^*). \tag{4}$$

Algorithms such as projected stochastic subgradient descent, viability-constrained control, and best response dynamics are perceived as the discretization of the differential inclusion.

$$\dot{z} \in -\partial_z\mathbb{E}[\Psi(z,\xi)] - N_{Z(z)}(z).$$

By selecting $(F,K) = (\mathbb{E},[\Psi]\mathcal{Z})$, the convergence results for the general case can be applied to problems in optimization, control, and games. Consequently, the single projected stochastic subgradient method using Robbins-Monro stepsizes can be applied to all three cases.

4. Theoretical results

Theorem 1 (Existence of Stationary Points)

Let $F(x) = \mathbb{E}[f(x,\xi)]$, where $f(\cdot,\xi)$is Clarke regular and $L(\xi)$-Lipschitz on a compact set Kalmost surely, with $\mathbb{E}[L(\xi)] < \infty$. If the sublevel set

$$\{x \in K : F(x) \le F(x_0)\}$$

is nonempty and lies in the interior of K, then the stationarity condition

$$0 \in \partial F(x^*)$$

admits at least one solution $x^* \in K$.

Proof Sketch:

Under the above assumptions, ∂F(x) is upper semi-continuous and has compact and convex values because of the properties of the Aumann integral. Using the Kakutani-Fan-Glicksberg fixed-point theorem on the normalized sub-gradient mapping ensures that there exists a fixed point such that $0 \in \partial F(x^*)$.

Theorem 2 (Almost-Sure Convergence of Projected Stochastic Subgradient Descent)

Consider the iteration

$$x_{k+1} = \Pi_K(x_k - \alpha_k\zeta_k),$$

where $\zeta_k \in \partial f(x_k,\xi_k)$, $\{\xi_k\}$are i.i.d. samples, and the step sizes satisfy

$$\sum_k \alpha_k = \infty,\ \sum_k \alpha_k^2 < \infty.$$

Afterwards, the interpolated trajectory is guaranteed to converge to an internally chain-transitive set of the projected differential inclusion.

$$\dot{x} \in -\partial F(x) - N_K(x).$$

If $F$is a Lyapunov function for this inclusion, the limiting set becomes

$$\mathcal{S} = \{x \in K : 0 \in \partial F(x) + N_K(x)\}.$$

Proof Sketch:

The process is updated through the Robbins–Monro procedure for stochastic approximation with martingale difference noise. By applying the technique of differential inclusions ODE, the stochastic path almost surely converges to the chain-transitive set of the limit inclusion, which is the stationary set by the Lyapunov criterion.

Proposition 3 (Generalized Nash Equilibrium Existence)

Under the assumptions of the game from Section 3.3 having a potential function Φ, each strategy correspondence Xi (·) is compact, nonempty-valued, and Aubin-continuous, and $\Phi(\cdot\ ⓜ, \xi)$ is Clarke regular and Lipschitz with an integrable bound, a generalized Nash equilibrium exists.

Proof Sketch:

Applying Theorem 1 to the expected potential function $\mathbb{E}[\Phi]$over the joint feasible correspondence

$$\mathcal{X}(x) = \prod_i X_i\,(x_{-i}),$$

ensures the existence of a stationary solution. The potential property

$$\partial_{x_i}\Phi(x, \xi) = \partial_{x_i} J_i(x, \xi)$$

guarantees that this stationary point is in coordinate-wise equilibrium as expressed by equation (3).

4.1 Computational remarks

Theorems 1–2, as well as Proposition 3, do not necessitate the evaluation of the Aumann integral.

$$\partial F(x) = \mathbb{E}[\partial f(x, \xi)].$$

Instead, it is sufficient to sample $\xi_k$and compute a single generalized gradient

$$\zeta_k \in \partial f(x_k, \xi_k).$$

In the case of piecewise-smooth functions that are maxima, minima, or absolute value combinations of smooth functions, the component can be derived analytically or automatically using active smooth selections at x_k.

Hence, the computational complexity of each step in Theorem 2 is identical to stochastic gradient descent. The only distinction lies in the fact that convergence is obtained to the Clarke-stationary set.

$$\mathcal{S} = \{x \in K : 0 \in \partial F(x) + N_K(x)\},$$

instead of the classical critical point of the objective function.

In the case of constrained problems, the projection operator Π_Kis the key computational step if the set Kis nontrivial. In the viable-constrained control and congestion game settings, however, the feasible set is partitioned over states/players, and thus the computation is separable and tractable at the same order of complexity as in the unconstrained setting.

5. Applications

5.1 Piecewise-smooth network training under label noise

A ReLU or max-pooling neural network with loss function $\ell(h_\theta, (x)y)$is Clarke regular with respect to the parameters θalmost everywhere. Under label noise, the observed loss can be represented as a random integrand $f(\theta, \xi)$, where $\xi = (x, y,$corruption$)$. Theorem 1 ensures the existence of a nonempty set of Clarke-stationary solutions for the risk function, whereas Theorem 2 establishes almost sure convergence to the stationary set using the diminishing step size mini-batch stochastic subgradient descent, without making the smoothness assumption that is made in classical SGD.

This property is especially significant when it comes to small-batch learning and highly-corrupted learning, because the stochastic fluctuations

$$\zeta_k - \mathbb{E}[\zeta_k \mid \mathcal{F}_k]$$

may fail to exhibit the properties of a Gaussian distribution. Whereas traditional smooth optimization procedures utilize local linearizations based on differentiability of the critical points under analysis, the martingale-difference approach of Theorem 2 necessitates merely a finite second moment assumption. This assumption is automatically fulfilled if the per-sample generalized subgradient norm L(ξ)is integrable.

5.2 Viability-constrained control of switched stochastic systems

The switched system described in Section 3.2, with state-dependent dynamics and constraint C(t) (such as the safety envelope), satisfies the model structure of this section. The normal cone modification makes the trajectories feasible and a non-smooth-running cost, punishing proximity to constraints, inherits the stationarity characterization from Theorem 1 in a path-wise sense, providing the basis for the principled termination criterion in trajectory optimization under uncertainty. Since $\mathcal{A}$ is really set-valued at switching surfaces, then the state trajectory is not unique off a set of zero measure initial conditions – that is when the Filippov solution concept and the formulation of Theorem 1 in differential inclusion framework are necessary, instead of an ordinary Carathéodory existence theorem; the stationarity characterization (4) holds path-wise for any such Filippov solution, hence the control synthesis problem inherits well-posedness from the static optimization problem

5.3 Equilibrium selection in threshold-toll congestion games

Threshold toll congestion games lead to kinked, non-convex link cost functions where the activation occurs only if the flow on a link surpasses a certain threshold level. If the functional representing the total travel time admits a potential (like the classic Beckmann potential, but generalized non-smoothly through the threshold), then the existence of a Wardrop type of equilibrium is ensured by Proposition 3 despite the fact that the link cost functions are not convex and are non-differentiable at the threshold. Demand random variations in peak/off-peak times are included in the ξ part of the potential function Φ, and hence the existence proof applies to both the classical deterministic threshold toll problem and the random demand case.

5.4 A qualitative numerical illustration

To illustrate the mechanism of Theorem 2, consider the scalar stochastic objective

$$f(x, \xi) = \mid x - \xi \mid + \max(0, x^2 - 1),$$

where ξfollows a two-point label-noise distribution with probability $1 - p$at $\xi = 1$and probability $p$at the corrupted value $\xi = -1$. The expected objective

$$F(x) = \mathbb{E}_\xi[f(x, \xi)]$$

Is Clarke regular at all points? Yes, because Clarke regularity is the sum of regular convexity and regular piecewise quadratics. It is piecewise smooth with non-smooth points at $x \in \{-,11\}$ since ∂F becomes set-valued, otherwise single-valued.

From Theorem 1, there exists a stationary point in any closed interval that contains the required sub-level set of F. For cases where p is less than the critical value when the corrupted solution at x=-1 takes precedence, the stationary point

stays close to the uncorrupted solution near $x = 1$.

Utilizing the predicted stochastic subgradient recursion from Theorem 2 with step size

$$\alpha_k = \frac{1}{k},$$

generates iterates that almost surely converge to the right stationary set. This result is guaranteed by the Lyapunov property of the differential inclusion problem, whereby the trajectory escapes corruption-dominated solutions as Fshrinks outside the stationary set.

The example here illustrates the basic principle at work in the label noise analysis of Section 5.1, where convergence is obtained via Clarke set-valued stationarity and stochastic approximation, without smoothness or optimizer heuristics.

6. Discussion and limitations

The proposed framework captures, as particular cases, the deterministic non-smooth framework and in case of degenerate uncertainty, the smooth stochastic approximation approach of (Robbins & Monro 1951). In case f is differentiable, and the existence result of Rosen (1965) in case the game is convex and deterministic. The price paid is that of the Clarke regularity assumption, which is not always available: functions constructed from max, min, and absolute value operations are always regular, but compositions involving irregular pieces are not, and property (2) can be violated without this condition. One could replace the construction of Theorem 1 with the strictly broader limiting subdifferential concept, at the cost of abandoning convex-valuedness, which is essential in the Kakutani-Fan-Glicksberg step of Theorem 1; this seems to be a promising avenue for future research, together with extending Proposition 3 to the case of continuum of players suitable for congestion and market design applications. A closely related line of recent research investigates the behavior of stochastic subgradient methods through differential inclusion dynamics under a different Lyapunov construction, leading to almost sure convergence results compatible with Theorem 2.

The second constraint is about rate of convergence: Theorem 2 provides an almost-sure qualitative convergence statement, yet unlike the smooth or convex cases, there are no finite-time rates available beyond the smooth or convex case, since the Lyapunov reduction rate utilized in the proof cannot be quantitatively bounded away from zero uniformly over $\mathcal{S}$. Finite-time rates in this setting have been proven thus far assuming additional structure, such as weak convexity with a quantitative convexity modulus Davis & Drusvyatskiy, (2019) or a Kurdyka-Lojasiewicz-type growth condition around the stationary set; integrating such an assumption into the current setting and analyzing its interaction with the set-valued strategy correspondence of Section 3.3 will be addressed in future work.

6.1 Summary of standing assumptions

It is helpful to list, in one place, the assumptions that underlie each result since they all appear in Sections 3–5, in slightly different notation. (i) Measurability: f, ℓ, and Φ are Carathéodory in the sense of Section 2, such that measurable selectors from their subgradients exist. (ii) A.s. Clarke regularity: Each of $f(\cdot,\xi), \ell(\cdot,\xi)$, and $\Phi(\cdot,\xi)$ is Clarke regular $\mathbb{P}$-a.e., which makes the switch (2) and strict descent condition used in the proof of Theorem 2 possible. (iii) Integrable local Lipschitz bound: An integrable L(ξ), with $\mathbb{E}[L(\xi)] < \infty$, dominates the relevant function on the compact domain of interest, such that the Aumann integral is compact and convex and the martingale-difference noise in Theorem 2 has bounded second moment. (iv) Feasibility regularity: The constraint mapping K, C(t), or $\mathcal{X}$(x) is compact-valued and Aubin continuous, which makes it possible to apply the fixed-point technique in (4). None of the above four assumptions are trivial — a piecewise linear or piecewise quadratic construction using max, min, and absolute value functions satisfy (i)–(iii) automatically, and box, ball, simplex, or polyhedral constraints with Slater point satisfy (iv) — however, checking them is necessary prior to applying the theorems in.

7. Conclusion

In bringing stochastic nonsmooth optimization, control, and generalized Nash equilibria within the framework of a common Aumann Clarke subdifferential inclusion, the present paper proves that the existence and convergence results obtained in one of these areas of research apply with little to no change to the other two as well. The three results obtained here — existence of stationary points, almost sure convergence of a projected stochastic subgradient method, and existence of generalized Nash equilibria in nonconvex potential games — are proven under two common

assumptions: Clarke regularity almost surely and Lipschitz condition integrable locally. In finding these as the critical assumptions, we see what can and cannot be proved in non-smooth analysis at the moment.

References

1. Ali, W. (2025). Physics-constrained neural architectures for uncertainty-robust adaptive control in reconfigurable embedded platforms. *Journal of Reconfigurable Hardware Architectures and Embedded Systems*, *2*(3), 48-55.

2. Benaïm, M., Hofbauer, J., & Sorin, S. (2005). Stochastic approximations and differential inclusions. *SIAM Journal on Control and Optimization*, *44*(1), 328-348. https://doi.org/10.1137/S0363012904439301

3. Davis, D., & Drusvyatskiy, D. (2019). Stochastic model-based minimization of weakly convex functions. *SIAM Journal on Optimization*, *29*(1), 207-239. https://doi.org/10.1137/18M1178244

4. Duchi, J. C., & Ruan, F. (2018). Stochastic methods for composite and weakly convex optimization problems. *SIAM Journal on Optimization*, *28*(4), 3229-3259. https://doi.org/10.1137/17M1135086

5. Facchinei, F., & Kanzow, C. (2010). Generalized Nash equilibrium problems. *Annals of Operations Research*, *175*(1), 177-211. https://doi.org/10.1007/s10479-009-0653-x

6. Ferrari, M., Esposito, C., & Lombardi, D. (2025). Uncertainty-Calibrated Deep Cooperative Spectrum Sensing and Channel Access Optimization for Cognitive Radio Networks. *International Academic Journal of Science and Engineering, 12*(3), 259-266. https://doi.org/10.71086/IAJSE/V12I3/IAJSE1263

7. Nisha Milind Shrirao. (2026). Geospatial AI-Driven Soil Moisture Forecasting Model for Climate-Resilient Crop Production Planning. *Journal of Environmental Sustainability, Climate Resilience, and Agro-Ecosystems*, 16–21.

8. Robbins, H., & Monro, S. (1951). A stochastic approximation method. *The annals of mathematical statistics*,22(3), 400-407.

9. Rosen, J. B. (1965). Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica: Journal of the Econometric Society*,33(3), 520-534. https://doi.org/10.2307/1911749

10. Sindhu, S. (2023). The Effects of Interval Uncertainties and Dynamic Analysis of Rotating Systems with Uncertainty. *Association Journal of Interdisciplinary Technics in Engineering Mechanics*, *1*(1), 49-54.