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Ternary Hesitant Uncertainty Soft Approach of Decision-Making Problems in Medical Diagnosis Aspect

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Abstract
In this paper, we introduce and build the concept Ternary Hesitant Uncertainty Soft Set (THUS) which enhances uncertainty collections, bifuzzy set, hesitant uncertainty set, and multiset of uncertainty as special cases. Then, we analyse of the elementary operations and properties of THUS. Also, we investigate the relationships among the collections mentioned above and then propose an extension principle of THUS's. Additionally, we provide an example to illustrate the applications of THUS in diagnosing kidney failure problems, which is explored in detail.

*Keywords:*Fuzzy set, intuitionistic fuzzy set, dual hesitant fuzzy set, soft set, hesitant fuzzy number. Ternary Fuzzy (hesitant) set, union, and intersection.

Structure and Contribution of this article

We have introduced ternary hesitant uncertainty soft sets, a generalization of uncertainty sets that allows us to focus the situation in which different membership (truth, neutral, falsity) are considered possible. We have established a relationship on ternary hesitant uncertainty collections and have shown that application can be in many real life observations, especially in the medical diagnoses problem.

Novelty

This research offers a new method to reduce sugar level using ternary hesitant uncertainty soft distance measures. In contrast to traditional methods depending on static labels or confidence levels, the new method involves varying levels of hesitation in information's truth, neutral, and falsity aspect.

1. Introduction

The concept of fuzzy set was first proposed by Zadeh in 1965 [15] as an extension of classical notion of sets. In various cases, however, because the membership function is a single-valued function, it cannot be applied to denote both support and objection evidence. The intuitionistic fuzzy set, which is a generalization of Zadeh's uncertainty set, was proposed by K.T. Atanassov [1]. An intuitionistic fuzzy set has both grade of membership and a non-membership function allowing it to better shows the uncertainty character of data than Zadeh's fuzzy set, which only has membership function. In some real life observations, intuitionistic fuzzy set are useful to real life problems connected to the linguistic variable which is function of membership like made only rough. As a final report of intuitionistic fuzzy set flexibility in dealing with uncertainty. We gave an intuitionistic fuzzy approach in selecting specialization [4]. Since there is possible to be hesitation part which represents unknown object [5] and learn recently [10, 11].

Hesitant fuzzy set was proposed by V. Torra [10] which further characterized on element be a set of membership values there by decreasing the loss of information during fuzzification. Hesitant fuzzy set [10] were defined another extension, that is, when defining an element membership, it is not because we have a margin error that makes determining the membership degree difficult or a distribution of possibilities among the availability values, but since we have a range potential values. This situation can arise is multi-criteria decision-

making problem. Distance measure of hesitant fuzzy sets have been studied in [3,13]. Studies on hesitant fuzzy information aggregation techniques and their applications in decision making can be shown in [8]. Zhu. et al. [14] have exposed a hesitant fuzzy geometric Bonferroni mean as an extension of the geometric Bonferroni mean to hesitant fuzzy environment. Rodriguez et al. [9] have introduced hesitant fuzzy linguistic term sets which gives flexibility in applications involving hesitant situation under qualitation settings. Wei [12] have studied the hesitant fuzzy prioritized operators and has illustrated their applications in the hesitant fuzzy multiple attribute decision making problems in which the attributes different priority level. Dual hesitant uncertainty set was first introduced by [2] and their properties like distance and similarity feature exposed by [3].

In this article, we introduce and build the concept Ternary hesitant uncertainty soft set (THUS) which enhance uncertainty collections, bifuzzyset, hesitant uncertainty set and multiset of uncertainty as special cases. Then, we analyse the elementary operations and properties of THUS. Also, we investigate the relationships among the collections mentioned above and then propose an extension principle of THUS's. Additionally, we provide an example to illustrate the applications of THUS in diagnosing kidney failure problems which is explored in details.

2. Preliminaries

The basic concept of this paper is considered as given below.

Definition 2.1. [16] Let X be a universe of discourse. Then a fuzzy set A is an object having the following formulation.

$$A = \{(x, \mu_A(x) \mid x \in X)\}$$

where $\mu_A(x)$ is called the membership degree of x in X.

Example 1. Let $X = \{a, b, c, d\}, be\ a\ set$ Then the fuzzy set is defined as

x	a	b	b
$\mu_A(x)$	0.2	0.3	0.5

Definition 2.2. [1] Let U be a universe. An intuitionistic fuzzy set (IFS) A on U can be expressed as follows;

$$A = \{(x, \mu_A(x), \nu_A(x) \mid x \in X)\}$$

where $\mu_A : U \rightarrow [0,1]\ and\ \nu_A : X \rightarrow [0,1]$ such that $0 \leq \mu_A(x)+\nu_A(x) \leq 1$.

For any $x \in U$, where $(\mu_A(x), \nu_A(x))$ are the degree of membership and degree of non-membership of the element.

Example 2. Let $X = \{l, m, n, p\}, be\ a\ given\ set$. An IFS is explained as follows:

x	l	m	n	p
$\mu_A(x)$	0.3	0.2	0.4	0.6
$\nu_A(x)$	0.5	0.5	0.3	0.2

Definition 2.3. [6] A soft set over U can be represented by the set of ordered pairs

$F_A = \{(x, f_A(x) \mid x \in E, f_A(x) \in P(U))\}$. The collection of all soft sets over U will be denoted by S(U).

Example 3. Let $U = \{m_1, m_2, m_3, m_4, m_5\}$ be a universal set and $E = \{w_1, w_2, w_3, w_4\}$ be a set of parameters.

1. $A = \{w_1, w_2, w_4\} \subseteq E$.
2. $\alpha_A(w_1) = \{0.7/m_2, 0.8/m_4\}$
3. $\alpha_A(w_2) = \{U\}$
4. $\alpha_A(w_4) = \{0.5/m_1, 0.3/m_3, 0.7/m_5\}$

Then the soft set is formulated as

$$F_A = \{(w_4, (0.7/m_2, 0.8/m_4)), (w_2, U), (w_4, (0.5/m_1, 0.3/m_3, 0.7/m_5))\}$$

Definition 2.4. [11] A hesitant fuzzy set HFS is defined as a mapping HFS: $X \rightarrow [0,1]$ and is given by X_a and $X_\mu A = (x, h_A(x) \mid x \epsilon X)$ where $h_A(x)$ called a hesitant fuzzy element (HFE) is a collection of discrete values in [0,1] denoting the membership degree.

Zhu et.al. [15] stated that the dual hesitant fuzzy set (DHS) collections of objects by considering the degree of membership and non-membership simultaneously.

For these HFS, $h,\ h_1,\ h_2,$ the following operations are defined:

1. Lower bound: $h(x) = inf\ h(x)$
2. Upper bound: $h(x) = sup\ h(x)$
3. $h_1 \cup\ h_2 = \{h \epsilon\ h_1 \cup\ h_2 \mid h \geq sup\{h_1, h_2\}\}$
4. $h_1 \cup\ h_2 = \{h \epsilon\ h_1 \cap\ h_2 \mid h \leq min\{h_1, h_2\}\}$

For two dual hesitant fuzzy set (Both, the membership functions occur simultaneously), $D_1\ and\ D_2$, the following operations are defined:

(i) $D_1 \cup\ D_2 = \{h \epsilon\ h_1 \cup\ h_2 \mid h \geq sup\{h_1^-, h_2^-\}\}$
$g \epsilon\ (g_1 \cup\ g_2) \setminus g \leq inf\{g_1^+, g_2^+\}$
(ii) $D_1 \cap\ D_2 = \{h \epsilon\ h_1 \cap h_2 \mid h \leq inf\{h_1^+, h_2^+\}\}$
$g \epsilon\ (g_1 \cap\ g_2) \setminus g \geq sup\{g_1^-, g_2^-\}$

Definition 2.5. Ternary hesitant fuzzy set 'A' is defined as a mapping THUS: $X \rightarrow [0,1]$ and is given by

$$A = \{(w, \alpha_A(w), \beta_A(w), \gamma_A(w) \mid w \epsilon X)\}$$

where $h_A, g_A\ and\ k_A$ represents the discrete set of degrees in [0,1] called truth membership, neutral membership and falsity membership set A is a subset of X and for every element $w \epsilon X, 0 \leq \alpha_A^3(w) + \beta_A^3(w) + \gamma_A^3(w) \leq 1$ then $\pi_A(x) = 1 - (\alpha_A^3(w) + \beta_A^3(w) + \gamma_A^3(w))$ is called the refused membership w in A.

Example 4. Let $X = \{w, 0.3, 0.4, 0.1\}$. Here $0 \leq \alpha_A^3(w) + \beta_A^3(w) + \gamma_A^3(w) \leq 1$ and $0 \leq 0.027 + 0.064 + 0.001 \leq 1, 0 \leq 0.092 \leq 1$ then $\pi_A(w) = 1 - 0.092 = 0.908$ is called the degree of value of refusal membership of r in X.

Definition 2.6. For any THUS's, $\delta_1\ and\ \delta_2$, the union, intersection, equal, subset and complement are defined as follows:

(i) $\delta_1 \subseteq \delta_2 \Leftrightarrow (\forall\ w \epsilon X, \alpha_A(w) \leq \alpha_B(w), \beta_A(w) \leq \beta_B(w)\ and\ \gamma_A(w) \geq \gamma_B(w))$
(ii) $\delta_1 = \delta_2 \Leftrightarrow \delta_1 \subset \delta_2\ and\ \delta_2 \subset \delta_2$
(iii) $\delta_1 \cup\ \delta_2 \Leftrightarrow (w, sup\{\alpha_A(w), \alpha_B(w)\}, inf\{\beta_A(w), \beta_B(w)\}, inf\{\gamma_A(w), \gamma_B(w)\})$
(iv) $\delta_1 \cap\ \delta_2 \Leftrightarrow (w, inf\{\alpha_A(w), \alpha_B(w)\}, sup\{\beta_A(w), \beta_B(w)\}, sup\{\gamma_A(w), \gamma_B(w)\})$
(v) $\delta^c = \{(\gamma_A(w), \beta_A(w), \alpha_A(w))/w \epsilon X\}$

Definition 2.7. The THUS's $O_{THUS}\ and\ I_{THUS}$ in X are defined as follows

(i) $O_{THUS} = \{(w, 0,0,1) \mid W \epsilon X\}$
(ii) $O_{THUS} = \{(w, 0,0,0) \mid W \epsilon X\}$
(iii) $I_{THUS} = \{(w, 1,0,0) \mid W \epsilon X\}$
(iv) $I_{THUS} = \{(w, 0,1,0) \mid W \epsilon X\}$
where I and O denote the constant functions sending all possible elements of X to 1 and 0 respectively.

Proposition 1. For any THUS A, the following exist:

(i) $O_{THUS} \subseteq A, O_{THUS} \subseteq C_{THUS}$

(ii) $A \subseteq I_{THUS,} I_{THUS} \subseteq I_{THUS}$.

Proposition 2. Let $\delta_1\ and\ \delta_2$ be an THUSs in X. Then the following conditions hold

(i) $\delta_1 \subseteq \delta_2 \; and \; \delta_3 \subseteq \delta_4 \; implies \; \delta_1 \cup \delta_3 \subseteq \; \delta_2 \cup \delta_4$
(ii) $\delta_1 \subseteq \delta_2 \, , \; \delta_1 \subseteq \delta_3 \; implies \; \delta_1 \subseteq \; \delta_2 \cap \delta_3$
(iii) $\delta_1 \subseteq \delta_3 \, , \; \delta_2 \subseteq \delta_3 \; implies \; \delta_1 \cup \; \delta_2 \subseteq \delta_3$
(iv) $\delta_1 \subseteq \delta_2 \, , \; \delta_2 \subseteq \delta_3 \; implies \; \delta_1 \subseteq \delta_3$
(v)$\overline{\delta_1 \cup \delta_2} = \overline{\delta_1 \cap \delta_2}$
(vi)$\overline{\delta_1 \cap \; \delta_2} = \overline{\delta_1 \cup \delta_2}$
(vii) $\left(\bar{\bar{\delta}}\right) \; = \; \delta$
(viii) $\overline{\delta_1} \; \subseteq \overline{\delta_2} \; implies \; \overline{\delta_2} \; \subseteq \overline{\delta_1}$
(ix) $I_{THUS,} = \; O_{THUS,}$
(x) $O_{THUS,} = \; \sqrt{2} I_{THUS,}$

Example 5. Let us consider the THUS

$$\delta_1 \; = \{\left(w, \alpha_{\delta_1}(w) \; = \; 0.2, \beta_{\delta_1}(w) \; = \; 0.3, \gamma_{\delta_1}(w) \; = \; 0.5\right)\}$$
$$\delta_2 \; = \{\left(w, \alpha_{\delta_2}(w) \; = \; 0.3, \beta_{\delta_2}(w) \; = \; 0.4, \gamma_{\delta_2}(w) \; = \; 0.3\right)\}$$
$$\delta_3 \; = \{\left(w, \alpha_{\delta_3}(w) \; = \; 0.4, \beta_{\delta_3}(w) \; = \; 0.5, \gamma_{\delta_3}(w) \; = \; 0.4\right)\}$$
$$\delta_4 \; = \{\left(w, \alpha_{\delta_4}(w) \; = \; 0.43, \beta_{\delta_4}(w) \; = \; 0.55, \gamma_{\delta_4}(w) \; = \; 0.1\right)\}$$

We can observe that the above sets $\delta_1, \; \delta_2, \; \delta_3$ and δ_4 satisfy the proposition.

3. some operations in ternary hesitant uncertainty soft set

Bifuzzy set, we use t-norm to consider about the infimum between memberships in intersection operation and use S -norm to deal about supremum between memberships in union operation. So we can utilize t -norm and S-norm it is ternary hesitant uncertainty soft set, where it is helpful to the intersection or the union operations that we have to necessary the infimum and supremum in the same operation but with the hesitant numbers to the truth membership, neutral membership and falsity membership.

$\Delta \; = \; \left(w, \alpha^3(w), \beta^3(w), \gamma^3(w)\right)$; Let $\alpha^3(w) = \beta^3(w) = \gamma^3(w) = \delta(w)$

For intersection, the membership degree can use t-norm and union S-norm. For the problems, we apply I for intersection and U for union.

Consider that $w \, \epsilon \, [0,1]$, we determines

$$w_I^{(n)} \; = \; I\left(w, w, w, \ldots, w = I\left(w_I^{(n-1)}, w\right); \; n \geq 3\right) \qquad (1)$$

$$w_U^{(n)} \; = \; U\left(w, w, w, \ldots, w = U\left(w_U^{(n-1)}, w\right); \; n \geq 3\right)$$

Consider that I and U are a t-norm and S-norm from $[0,1]^n \; to \; [0,1]$.

Accordingly;

(i) A t-norm (and a S-norm respectively) is treated to be continuous if it continues as a function on [0,1].

(ii) A t-norm I (and a S-norm respectively) of THUS $\Delta \; = \; \{\left(w, \alpha^3(w), \beta^3(w), \gamma^3(w)\right)\}$ if say about intersection operators;

$\alpha^3(w)$ tends to inf at ease that there are two numbers for $\alpha^3(w)$ which awakanded Archimedean, $\lim_{n\to\infty}\left(\alpha^3(w)\right) = 0 \; and \; \lim_{n\to\infty}\left(\beta^3(w)\right) = 0 \; and \; \gamma^3(w)$ tends to sup at case there are two numbers for $\gamma^3(w)$ which is treated Archimedean if $\lim_{n\to\infty} \; = 1$, for any $w \epsilon [0,1]$.

Proposition 3. Consider that ϕ is a set of all THUS, I and U represent a t-norm and S-norm. Then

(i) If I and U are Archimedean then $I\left(\alpha^3(w), \alpha^3(w)\right) < \alpha^3(w), I\left(\beta^3(w), \beta^3(w)\right) < \beta^3(w) \; and \; U\left(\gamma^3(w), \gamma^3(w)\right) > \gamma^3(w)$, for all $\Delta \; = \; \{\left(w, \alpha^3(w), \beta^3(w), \gamma^3(w)\right)\}$ for intersection operation.

(ii) If I and U are Archimedean then $U\left(\alpha^3(w), \alpha^3(w)\right) \geq \alpha^3(w), U\left(\beta^3(w), \beta^3(w)\right) \geq \beta^3(w) \; and \; I\left(\gamma^3(w), \gamma^3(w)\right) \; \leq \; \gamma^3(w)$, for all $\Delta \; = \; \{\left(w, \alpha^3(w), \beta^3(w), \gamma^3(w)\right)\}$

Proof: (i) Consider the contrary: to tell that

$$\left(\alpha^3(w)\right)_I^{(3)} = I\left(\alpha^3(w), \alpha^3(w)\right) \geq \alpha^3(w) \text{ and}$$

$$\left(\beta^3(w)\right)_I^{(3)} = I\left(\beta^3(w), \beta^3(w)\right) \geq \beta^3(w) \text{ and}$$

$$\left(\gamma^3(w)\right)_I^{(3)} = U\left(\gamma^3(w), \gamma^3(w)\right) \leq \gamma^3(w) \text{ for some } w \in [0,1]$$

Then we have

$$\left(\alpha^3(w)\right)_I^{(4)} = I\left(I\left(\alpha^3(w), \alpha^3(w), \alpha^3(w)\right)\right) \geq I\left(\alpha^3(w), \alpha^3(w)\right) \geq \alpha^3(w)$$

Similarly, $\left(\beta^3(w)\right)_I^{(4)}$ in the same way/ Also

$$\left(\gamma^3(w)\right)_U^{(4)} = U\left(U\left(\gamma^3(w), \gamma^3(w), \gamma^3(w)\right)\right) \geq U\left(\gamma^3(w), \gamma^3(w)\right) \leq \gamma^3(w)$$

From THUS characteristic by inference we will also have

$$\left(\alpha^3(w)\right)_I^{(n)} = I\left(I\left(\alpha^3(w), \ldots\ldots, \alpha^3(w)\right)\right) \geq \alpha^3(w) .$$

Then $lim\left(\alpha^3(w)\right)_I^{(n)} \neq 0$.

Similarly, $\lim_{n\to\infty}\left(\beta^3(w)\right)_I^{(n)} \neq 0$ and $\left(\gamma^3(w)\right)_U^{(n)} = U\left(\gamma^3(w), \ldots, \gamma^3(w)\right) \leq \gamma^3(w)$

Then $\lim_{n\to\infty}\left(\gamma^3(w)\right)_U^{(n)} \neq 1$.

(ii) Consider the contrary that is to explain that

$$\left(\alpha^3(w)\right)_I^{(3)} = U\left(\alpha^3(w), \alpha^3(w)\right) \leq \alpha^3(w) \text{ and}$$

$$\left(\gamma^3(w)\right)_I^{(3)} = I\left(\gamma^3(w), \gamma^3(w)\right) \geq \gamma^3(w) \text{ for some } w \in [0,1]$$

Then we get

$$\left(\alpha^3(w)\right)_I^{(3)} = U\left(U\left(\alpha^3(w), \alpha^3(w)\right)\right)$$

$$\alpha^3(w) \leq U\left(\alpha^3(w), \alpha^3(w)\right)$$

$$\leq \alpha^3(w)$$

$$\left(\gamma^3(w)\right)_I^{(3)} = I\left(I\left(\gamma^3(w), \gamma^3(w)\right)\right)$$

$$\gamma^3(w) \geq U\left(\gamma^3(w), \gamma^3(w)\right)$$

$$\geq \gamma^3(w)$$

From the THUS characterization and by not finite.

We shall also have;

$$\left(\alpha^3(w)\right)_U^{(n)} = U\left(\alpha^3(w), \ldots, \alpha^3(w)\right) \leq \alpha^3(w).$$

Then $\lim_{n\to\infty}\left(\alpha^3(w)\right)_U^{(n)} \neq 1$ and

$$\left(\gamma^3(w)\right)_U^{(n)} = I\left(\gamma^3(w), \ldots, \gamma^3(w)\right) \geq \gamma^3(w)$$

Then $\lim_{n\to\infty}\left(\gamma^3(w)\right)_U^{(n)} \neq 0$ which is opposition to our assumption.

Proposition 4:

Suppose in case of a continuous function where I, U: $[0,1]^3 \to [0,1]$, these equivalent statements for any THUS as follows;

(i) I represent a continuous Archimedean t-norm and U is a continuous Archimedean S-norm.

(ii) For any THUS, there are a continuous additive generator $\Delta = \left\{\left(w, \alpha^3(w), \beta^3(w)\right)\right\}$ if we discuss about intersection operation $\alpha^3(w) \to inf$ at ease that there are two numbers for $\alpha^3(w), \beta^3(w) \to sup$ at ease that there are two numbers for $\gamma^3(w) \to sup$ that's to tell that there exists a continuous strictly decreasing one.

$I\ [\alpha^3(w)]$: $[0,1] \to [0,\infty], I(1) = 0$ that is uniquely expressed up to a multiple constant such that for all $w_1, w_2[0,1]$, we obtain

$$I\left(\alpha^3(w_1), \alpha^3(w_2)\right) = I^{-1}\left(inf\left\{I\left(\alpha^3(w_1)\right) + I\left(\alpha^3(w_2)\right), I(0)\right\}\right)$$

and strictly increasing

$U\ [\gamma^3(w)]$: $[0,1] \to [0,\infty], U(1) = 0$

that is uniquely explained up to a multiplicative constant such that for all $w_1, w_2 \epsilon\ [0,1]$ we will get

$$U\left(\gamma^3(w_1), \gamma^3(w_2)\right) \geq U^{-1}\left(sup\left\{U\left(\gamma^3(w_1)\right) + U\left(\gamma^3(w_2)\right), U(1)\right\}\right)$$

Proof: Let $I(w_1, w_2) = I^{-1}(inf\{I(w_1) + I(w_2), I(0)\})$

We must show that they are a continuous Archimedean t-norm

$$H_1: I\ (\alpha^3(w), 1)$$
$$= I^{-1}\left(inf\left\{I\left(\alpha^3(w)\right) + I\left(\alpha^3(1)\right), I(0)\right\}\right)$$
$$= I^{-1}\left(inf\left\{I\left(\alpha^3(w)\right) + 0, I(0)\right\}\right)$$
$$= I^{-1}\left(inf\left\{I\left(\alpha^3(w)\right), I(0)\right\}\right)$$
$$= I^{-1}\left(I\left(\alpha^3(w)\right)\right) = \alpha^3$$

which is boundary.

H_2: To show that I is increasing, we obtain

$\alpha^3(w_1) < \alpha^3(m)$ and

$\alpha^3(w_2) < \alpha^3(w_3)$,

$I\left(\alpha^3(w_1)\right) > I\left(\alpha^3(m)\right)$ and

$I\left(\alpha^3(w_2)\right) > I\left(\alpha^3(w_3)\right)$ and

$I\left(\alpha^3(w_1)\right) + I\left(\alpha^3(w_2)\right) \geq I\left(\alpha^3(m)\right) + I\left(\alpha^3(w_3)\right)$

Since I^{-1} is decreasing we obtain;

$I\left(\alpha^3(w_1), \alpha^3(w_2)\right) = I^{-1}\left(inf\left\{I\left(\alpha^3(w_1)\right) + I\left(\alpha^3(w_2)\right), I(0)\right\}\right)$

$$\leq I^{-1}\left(inf\left\{I\left(\alpha^3(m)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right)$$

$$\leq I\left(\alpha^3(m),\alpha^3(w_3)\right)$$

which is monotonicity.

$H_3:\ I\left(\alpha^3(w_1),\alpha^3(w_2)\right)\ =\ \left(\alpha^3(w_2),\alpha^3(w_1)\right)$

is obvious. This is commutative.

$H_4:\ we\ have$

$$I\left(I\left(\alpha^3(w_1),\alpha^3(w_2)\right),\alpha^3(w_3)\right)=\ I^{-1}\left(I^{-1}\left(inf\left\{I\left(\alpha^3(w_1)\right)+I\left(\alpha^3(w_2)\right),I(0)\right\}\right)\right)$$

$$=\ I^{-1}\left(inf\left\{I\left(I^{-1}\left\{inf\left\{I\left(\alpha^3(w_1)\right)+I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right\}\right)\right\}\right)$$

$$\therefore\ I\left(\alpha^3(w_1)+\alpha^3(w_2)\right)<\ I(0)$$

Since I^{-1} is strictly decreasing

We obtain

$$I\left(I\left(\alpha^3(w_1),\alpha^3(w_2)\right),\alpha^3(w_3)\right)=\ I^{-1}\left(inf\ I\left\{I\left(I^{-1}\left(I\left(\alpha^3(w_1)\right)+I\left(\alpha^3(w_2)\right)\right)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right)$$

$$=\ I^{-1}\left(inf\ I\left\{I\left(\alpha^3(w_1)\right)+I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right)$$

$$=\ I^{-1}\left(I\left(\alpha^3(w_1)\right)+I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right)\right)$$

and

$$I\left(\alpha^3(w_1)\right),I\left(\alpha^3(w_2),\alpha^3(w_3)\right)\ =I^{-1}\left(\alpha^3(w_1)\right),I^{-1}\left(inf\left\{I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right)$$

$$=I^{-1}\left(inf\left(\left\{I\left(\alpha^3(w_1)\right)\right\}\right)\right)+I\left(I^{-1}\left(inf\left\{I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right)\right)$$

$$\therefore\ I\left(\alpha^3(w_2)+\alpha^3(w_3)\right)<\ I(m)$$

Since I^{-1} is strictly decreasing

We have

$$I\ \left(\alpha^3(w_1),I\left(\alpha^3(w_2)\right),\left(\alpha^3(w_3)\right)\right)\ =I^{-1}\left\{inf\left\{I\left(\alpha^3(w_1)\right)+I\left(I^{-1}\left(I\left(\alpha^3(w_2)\right)\right)\right)\ +I\left(\alpha^3(w_3)\right)\right\}\right\}$$

$$=I^{-1}\left\{inf\left\{I\left(\alpha^3(w_1)\right)+I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right),I(0)\right\}\right\},\ I(0)\ =1$$

$$=I^{-1}\left\{I\left(\alpha^3(w_1)\right)\ +I\left(\alpha^3(w_2)\right)+I\left(\alpha^3(w_3)\right)\right\}$$

$$\therefore I\left(\ I\left(\alpha^3(w_1),\alpha^3(w_2)\right),\alpha^3(w_3)\right)\ =I\left(\alpha^3(w_1),I\left(\alpha^3(w_2)\right),\alpha^3(w_3)\right)$$

which is Associative. Similarly, we can show to S-norm.

4. Application of Congestive Kidney Failure Problem Under Ternary Hesitant Fuzzy Soft Set

In this section, we analyse the application idea of Kidney failure problem.

Kidney failure happens when the kidney muscle does not pump (create) insulin as well as it should. When this happens, Insulin often backs up and fluid can build up in the pancreas which makes that regulates blood glucose (sugar) levels.

Certain kidney conditions gradually make the kidney too weak or stiff to fill and pump insulin properly. These conditions include narrow arteries in the heart and high blood pressure. Proper treatment may improve the symptoms of kidney failure and may help some people live longer.

Life style changes can develop quality of life. Try to lose weight, exercise use less salt and taking food methods. But kidney failure can be life-threatening; people with kidney failure may have severe symptoms. Some may need a kidney transplant or some other advanced equipped to help the kidney create insulin or diagnosing.

Given the presence of a database an example of a diagnosing kidney problems that is a description of a set of symptoms 'S' and a set of diagnosis 'D' will be assessed. We shall talk about patient condition when they are aware of the outcomes of the testing.

A ternary hesitant uncertainty soft set is used to describe the difficulty, the ternary hesitant fuzzy distances as introduced in the diagnosis technique by S.Ye.X.Chen and Y.Li.

Allow to set of diagnosis to be

$$D = \{Kidney\ failure\ problems\}$$

The Consider set of symptoms is

$$S = \{Urinating\ often, Extreme\ fatigue, Blurry\ vision, Weight\ loss\}$$

For Urinating, for instance, urine up white or pink or Irregular Urine

$$M = \{(0.3,0.4,0.7), (0.7,0.6,0.5), (0.3,0.5,0.7)\}$$

or another symptom extreme fatigue rapid with extreme fatigue

$$M = \{((0.7,0.6,0.4)), (0.4,0.6,0.5), (0.5,0.7,0.4)\}$$

The data is same in reality, but we want to emphasize that our approach need that the valves of both heist parameters, so we also explicitly inclusion of the hesitation margin.

The following three steps are involved:

(i) Identification of the symptoms as indicated.

Table 1: Identification of the symptoms

X	Urinating Often	Extreme Fatigue	Blurry Vision	Weight Loss
John	(*0.3,0.7,0.4*) (*0.2,0.4,0.6*) (*0.2,0.6,0.7*)	(*0.2,0.4,0.7*) (*0.1,0.3,0.5*) (*0.3,0.5,0.6*)	(*0.4,0.7,0.9*) (*0.2,0.3,0.7*) (*0.3,0.7,0.4*)	(*0.1,0.7,0.8*) (*0.2,0.4,0.6*) (*0.1,0.5,0.7*)
Smith	(*0.3,0.7,0.9*) (*0.1,0.5,0.6*) (*0.2,0.3,0.5*)	(*0.1,0.3,0.4*) (*0.6,0.7,0.9*) (*0.3,0.6,0.8*)	(*0.2,0.5,0.7*) (*0.1,0.4,0.8*) (*0.3,0.5,0.9*)	(*0.2,0.7,0.6*) (*0.3,0.4,0.6*) (*0.1,0.4,0.7*)
Jacob	(*0.1,0.7,0.8*) (*0.3,0.4,0.6*) (*0.2,0.5,0.7*)	(*0.3,0.5,0.7*) (*0.1,0.4,0.6*) (*0.2,0.5,0.7*)	(*0.2,0.5,0.9*) (*0.1,0.5,0.7*) (*0.2,0.5,0.7*)	(*0.1,0.2,0.6*) (*0.2,0.5,0.7*) (*0.3,0.5,0.7*)
Jun	(*0.2,0.4,0.6*) (*0.1,0.3,0.5*) (*0.2,0.6,0.7*)	(*0.2,0.5,0.7*) (*0.1,0.4,0.6*) (*0.2,0.5,0.7*)	(*0.3,0.5,0.9*) (*0.2,0.5,0.6*) (*0.7,0.8,0.9*)	(*0.2,0.4,0.6*) (*0.1,0.5,0.7*) (*0.2,0.5,0.4*)

(ii) The simulation medical knowledge through ternary hesitant uncertainty soft collections (Relations).
(iii) Determination of diagnosis using insulin type.

Table 2: Insulin Type

S.No	Insulin Type	Time
1.	Rapid-acting	5-15 min
2.	Short-acting	30-60 min
3.	Intermediate acting	2-4 hrs
4.	Long acting	1-2 hrs

The set of patients under consideration is

B = {John, Smith, Jacob, Jun}.

As before Table.1 lists the patient's distinctive symptoms. All the heist parameters are necessary where exploring each symptom but the facts are the same as in Table.3.

Table.3 presents the collection data. Three hesitations are used to describe each symptom; Truth membership, neutral and falsity membership.

Table 3: Identification of the symptoms

X	Urinating Often	Extreme Fatigue	Blurry Vision	Weight Loss
Kidney	(*0.8,0.7,0.6*)	(*0.3,0.7,0.4*)	(*0.1,0.4,0.7*)	(*0.2,0.3,0.5*)
Failure	(*0.6,0.3,0.1*)	(*0.2,0.5,0.7*)	(*0.6,0.3,0.1*)	(*0.2,0.5,0.7*)
Objects	(*0.4,0.3,0.4*)	(*0.1,0.5,0.3*)	(*0.2,0.4,0.6*)	(*0.1,0.3,0.7*)

It is our responsibility to accurately diagnose every patient B_i, i = 1, 2, 3, 4$. In order to complete the work, we recommend that for each patient B_i we compute the similarity distance measure between the symptoms in Table.3 from a set of symptoms $k = 1, 2, 3, 4$ distinctive for each diagnostic l in Table.3. The lowest distance found indicates an appropriate diagnosis.

In S.Ye.X.Chen and Y.Li, we establish that considering both of the hesitant parameters - truth membership, neutral and falsity membership functions are the sole approximate method fuzzy computing the most popular similarity distances for triad hesitant fuzzy collections. More precisely, this is defined in the normal triad hesitant uncertainty weighted Hausdorff distances for all the symptoms of the i^{th} patient from the l^{th} diagnosis.

Table.3 lists each patient's distances from the considered set of potential diagnostics. The smallest distance indicates the diagnosis kidney failure problem where the number larger than 0.5 refers that this person has affected in this case.

Table 4: Insulin Type

X	Kidyne Failure Problem
John	0.7125*
Smith	0.1024
Jacob	0.5170*
Jun	0.4735

Thus, we conclude uncertainty collections (UCs) in data base; we can express a hesitation w.r.t items under examination. All symptom values taken into account in the diagnosis are problems, which is based on

calculating the similarity distances between a case and all illnesses under consideration. From Table.4, John and Jacob have kidney failure problems.

5. Conclusion

The recommended idea, THUSs is some types of uncertainty generalization of theories, that can be applied in system analysis. We can use in THUS and have explain that application many real life observations.

Future Work

To extend this concept, we may apply in stochastic and probabilistic information systems. Under various medical diagnosis problems.

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