Minimax Theory and its Applications

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Graph-Theoretic and Game-Theoretic Mathematical Models for Risk Analysis, Organizational Decision-Making, and Sustainable Management

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Abstract

The interlinked risks, unpredictable conditions of operating, conflicting strategic goals, and limited resources are increasingly influencing organizational decision systems. The traditional weighted and deterministic decision models usually maximize nominal performance and average performance and thus could come up with fragile solutions in unfavorable environments. This work creates a mathematical framework of graph-based and game-based decision-making and resource distribution in an organization with robustness. The organization is modeled as a weighted risk graph where nodes indicate decision units and weighted edges define interdependencies and the strength of these risk-transmissions. A strategic game is developed between the decision-maker in the organization and an opposing uncertainty player that creates unfavourable operating conditions. The decision problem obtained is formulated as a minimax optimization formulation selecting resource-allocation strategies by minimizing the optimal loss incurred within a specified set of uncertainty. Risk propagation in graphs is combined with strategic utility, resource constraints and sustainability goals as a single mathematical model. Marrying the numerical scenario analysis under low, moderate, and high uncertainty would analyze the framework in terms of Worst-Case Decision Loss (WDL), Organizational Risk Exposure (ORE), Strategic Objective Achievement (SOA), Resource Utilization Efficiency (RUE) as well as Sustainability Performance Index (SPI). The analysis provides a foundation to consider worst-case robustness and related efficiency-sustainability trade-offs, offering a mathematically-based method of risk-conscious and sustainable organizational decision-making.

*Keywords:*Graph Theory; Game Theory; Minimax Optimization; Organizational Risk Analysis; Strategic Decision-Making; Sustainable Management; Resource Allocation; Robust Optimization.

1. Introduction

Operational units, resources, conflicting strategic goals, and risky uncertainties are increasingly interrelated in organizational decision-making. One unit decision can impact other units through operational dependencies,

flow of information, and interaction of resources to enable exposure of local risks to spread through the organizational network. It is up to decision managers to strike the right balance between minimising risk, strategic success, resource use, and sustainability as they react to unpredictable resource availability and evolving external environments. Graph theory also offers a helpful mathematical model of this interdependence of systems with the organizational units represented as nodes, the dependencies between them as edges, and the strengths of interactions or risks as weighted relationships.

Traditional decision methods are limited in the presence of these dependencies and uncertainties. Weighted decision models have a high reliance on preset importance values whereas deterministic optimization operates under relatively stable parameters and operating conditions. Risk assessment done independently might not capture the spread of risk between interdependent units, but graph-based models can capture structural dependency without taking strategic responses into full account. Game-theoretic models are made to represent interactions among decision entities, but they might overlook the organizational network in which the risks are propagated. On the same note, expected-value optimization focuses on average results and could thus offer inadequate safeguards against extreme but viable bad events.

There is still, however, a research gap in the development of a joint model of organization network structure, strategic interaction, resource allocation, uncertainty and sustainability. Current methods typically focus on these elements independently and it is challenging to assess the risk propagation of a network on the strategic decisions under negative circumstances. As a means of overcoming this shortcoming, this paper constructs a graph-theoretic minimax game model that combines weighted organizational networks, game-theoretic strategic interaction and worst-case optimization. Adverse uncertainty is modelled as a playing opponent, the organizational decision-maker is searching a resource-allocation approach which restricts the possible loss to the maximum. This offers a coherent mathematical process of balancing the robustness, strategic, resource efficiency and sustainability.

The major ones are the creation of a weighted graph model of organizational interdependencies and risk propagation, and a strategic game of modelling decision interactions in case of adverse uncertainty. Minimax optimization mechanism identifies robust resource allocations and collectively considers risk exposure, strategic objectives, resource utilization and sustainability. Scenario-based analysis looks at the framework with varying uncertainty conditions through Worst-Case Decision Loss (WDL), Organizational Risk Exposure (ORE), Strategic Objective Achievement (SOA), Resource Utilization Efficiency (RUE), and Sustainability Performance Index (SPI). The proposed structure thus lays a unifying mathematical foundation to sound, risk sensitive, and sustainable organizational decision making.

2. Related Work

Graph-based techniques offer a good math model of the interrelation of risks, dependency relationships and resource links. Risk-aware allocation and portfolio construction has used weighted network structures [1], network-based analysis of risk has shown the significance of identifying influential nodes and interdependence among risk pathways [6]. Causal network topology: Causal network topology also enables structural characterization of risk dependencies [7] and ecological network analysis builds on these ideas to systems of sustainability [13]. There is also graph-based methods that multi-agents can use to support distributed and adaptive decisions that is facilitated by network structures [4]. These approaches however mostly focus on structural relationship and connectedness of risks as opposed to worst-case strategic allocation of resources.

Game theory is used as a complementary approach to model strategic interaction between decision-making entities with antagonistic interests. It has been applied in environmental management [2], sustainable industrial decision-making [5], risk-averse transportation systems [9] and climate-oriented sustainability decisions [11]. Sustainable transportation decisions have also been studied using integrated game and multi-criteria methods [14], and circular-economy optimization has also been formulated using bargaining [15]. These works determine the topicality of strategic interactions of cooperation and non-cooperation, although network-based risk spreading is not direct in their strategic decision framework.

The process of the resource allocation and sustainability optimization consists of balancing the efficiency, the requirements to conduct operations, and the long-term goals under the constraints. Complex computational systems have been studied in terms of resource-allocation optimization [3], and integrated sustainability frameworks study the relationship between technological and social goals [8]. The same can be said of sustainable supply-chain models, which also take into account resource and environmental needs [10], and energy-based research, which focuses on resource allocation and utilization [12]. However, such formulations

are typically based on nominal solutions or a multi-objective solution or efficiency-oriented solution instead of explicitly reducing the worst loss related to unfavorable uncertainty.

There is a major research gap thus that exists since the graph-based risk propagation, strategic games, minimax optimization, and the sustainable allocation of resources are mostly considered as independent mathematical entities. Current literature deals with network risk [1], strategic environmental decisions [2], resource optimization [3], graph-based adaptive decisions [4], sustainable games [5], network safety [6], causal risk structure [7], integrated sustainability [8], risk-averse games [9], sustainable supply systems [10], climate decision analytics [11], energy efficiency [12], ecological networks [13], multi-criteria games [14], and bargaining-based sustainability [15]. The suggested GT-MGM fills in this gap by combining weighted organizational risk propagation, strategic interaction, limited adverse uncertainty, and minimax resource distribution into a single framework that focuses on minimizing WDL and ORE at the expense of keeping SOA, RUE, and SPI.

3. Proposed Mathematical Framework

3.1 Organizational Graph and Risk Propagation Model.

The organizational model is developed as a weighted risk net whereby the organizational units interact with each other via dependency links that can transmit local disturbances. G (V, E, W, R) models the network, with V being the set of organizational units (v 1, v 2, v n) E being the dependency relationships, W being the normalized dependency strengths (w ij), and R being risks (r i) to nodes. The weighted adjacency model uses the information of dependency and link-risk and the iterative propagation reflects how neighboring units contribute to each node. Fig. 1 demonstrates the entire network representation, comprising of interconnected units and risk-transmission paths. The converged network risks are then converted into an Organizational Risk Exposure (ORE) which is a system level measure whereby the structurally significant units are given more weight.

$$G = (V, E, W, R), V = \{v1, \dots, vn\}, W = [w_{ij}], R = [r_i]. _____ (1)$$

$$A = [a_{ij}], \qquad a_{ij} = w_{ij} r_{ij}. _____ (2)$$

$$R_i^{(t+1)} = (1-\lambda) r_i + \lambda \sum_{j=1}^{n} \tilde{w}_{ji} R_j^{(t)}, \; R_i^{(0)} = r_i, \qquad 0 \le \lambda < 1. _____ (3)$$

$$ORE = \frac{\sum_{i=1}^{n} q_i R_i^*}{\sum_{i=1}^{n} q_i} . _____ (4)$$

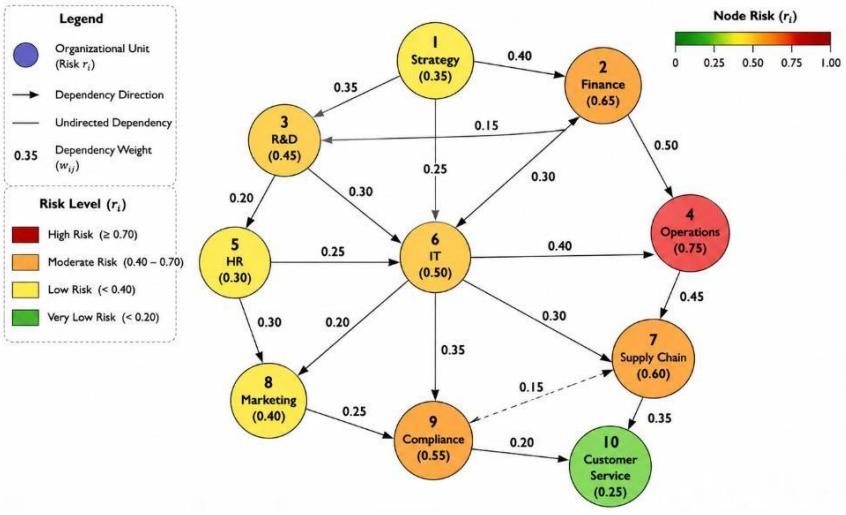


Fig. 1. Graph-Theoretic Representation of Organizational Risk and Interdependency

3.2 Strategic Organizational Game Model

Organizational decision process development is made a strategical game between the decision-making system and unfavorable operating conditions. Organizational player chooses strategies that deal with the redistribution of resources, risk mitigation, resource protection, and sustainability-based investment, and the uncertainty player is the unpleasant changes in organizational risks, dependencies, and available resources.Each strategy is evaluated according to strategic benefit B_i, propagated risk R_i, resource cost C_i, and sustainability contribution S_i. Nonnegative weighting coefficients control the relative importance of these components, thereby linking strategic interaction directly with the graph-based risk state rather than treating network risk and organizational decisions independently.

$$G = \langle N, \{S_i\}_{i\in N}, \{U_i\}_{i\in N} \rangle. \quad (5)$$

$$U_i = \alpha B_i - \beta R_i - \gamma C_i + \delta S_i, \qquad \alpha + \beta + \gamma + \delta = 1. \quad (6)$$

3.3 Minimax Risk-Aware Resource Allocation Model

The central GT-MGM formulation considers an organizational allocation vector $x \in X$ and an adverse uncertainty realization $u \in U$. The feasible decision space restricts nonnegative allocations according to the total normalized resource budget B, while U contains bounded variations in risk, dependency, and resource conditions. An integrated loss is a combination of organizational risk and allocation cost with strategic achievement and sustainability performance. Instead of reducing the expected loss, the model calculates the most pessimistic possible loss of each candidate allocation and picks the allocation with the best worst-case value. This is what constitutes WDL and gives the mathematical process by which the GT-MGM aims at a strong allocation at the same time maintaining SOA, RUE, and SPI.

$$X = \{x \in R_+^n : \sum_{i=1}^{n} x_i \le B\}. \quad (7)$$

$$L(x,u) = \omega_1 R(x,u) + \omega_2 C(x) - \omega_3 A(x,u) - \omega_4 S(x,u), \qquad \sum_{k=1}^{4} \omega_k = 1. \quad (8)$$

$$WDL(x) = \max_{u\in U} L(x,u). \quad (9)$$

$$x^* = arg \min_{x\in X} \max_{u\in U} L(x,u). \quad (10)$$

3.4 Graph-Theoretic Minimax Solution Procedure

The computational solution starts with organizational parameters and builds the weighted dependency graph and then propagates network risks through the network until convergence. Strategic game is then used to analyze the feasible organizational strategies against the bounded uncertainty realizations. In each allocation of candidates, the uncertainty player finds the condition that results in the highest loss and minimax procedure is used to choose the allocation whose WDL is minimum. Fig. 2 shows the entire sequence of organizational parameters to graph construction to risk propagation to strategic game to uncertainty evaluation to worst-case loss to minimax optimization to robust resource allocation to sustainable decision. The convergence is computed based on successive risk status and objective values in such a way that the ultimate solution is a stable worst-case allocation as opposed to a mid-point numerical state.

$$\|R^{(t+1)} - R^{(t)}\|_\infty \le \varepsilon_r. \quad (11)$$

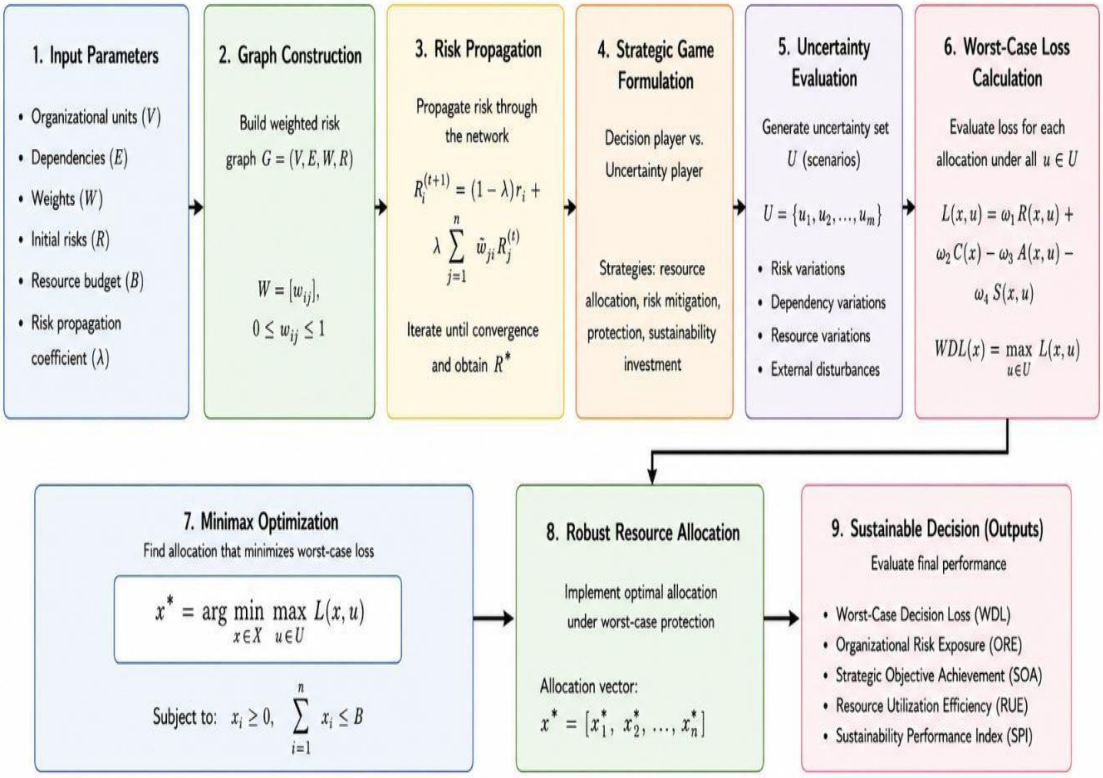


Fig. 2. Proposed Graph-Theoretic Minimax Game Decision Framework

3.5 Mathematical Properties and Minimax Analysis

The theoretical validity of GT-MGM follows from the bounded graph-propagation process and the existence conditions of the minimax problem. Proposition 1: if the normalized dependency matrix satisfies an induced-norm bound and $0 \leq \lambda < 1$, the risk-propagation mapping is contractive, giving a unique bounded fixed point. Proposition 2: when the feasible allocation set X and uncertainty set U are nonempty and compact and $L(x, u)$ is continuous, the worst-case objective attains a minimizer. Theorem 1: if X and U are additionally convex and the loss is convex in x and concave in u, the minimax problem admits the corresponding saddle-point equality. These properties define the propagation of risk, the presence of the robust allocation and the circumstances in which the organizational decision and the adversarial uncertainty have a mathematically consistent saddle-point interpretation.

$$\| T(R) - T(R') \| \leq \lambda \| WT \| \| R - R' \| \leq \lambda \| R - R' \|, \qquad \lambda < 1. \quad (12)$$

$$\min_{x \in X} \max_{u \in U} L(x, u) = \max_{u \in U} \min_{x \in X} L(x, u). \quad (13)$$

4. Numerical Experimental Design

4.1 Numerical Configuration and Uncertainty Scenarios

The controlled numerical organizational networks are considered in the evaluation of the proposed GT-MGM framework to study its mathematical behavior without considering the empirical organizational data. The networks are composed of 20-100 organization units and the density of the network is also differentiated into 0.10 and 0.50 in order to reflect various degrees of structural interdependency. The dependency weights and starting node risk are normalized to the range [0,1], the total resource budget is fixed to 100 normalized units and the risk-propagation coefficient λ was varied in the range [0.1, 0.9]. Every setup is tested on 30 distinct numerical realizations, to minimize reliance on a specific network realization. Three bounded uncertainties are taken into consideration: low uncertainty, which takes into account weak risk perturbations and fixed availability of resources; moderate uncertainty, which takes into account heightened risk and resource perturbations; and high uncertainty, which takes into account heightened disturbance, heightened network-risk propagation, and heightened resource constraints. Table 1 summarizes the entire numerical setup and forms of uncertainty that were applied in the analysis.

Table 1. Numerical Configuration and Uncertainty Scenarios

Parameter	Low Uncertainty	Moderate Uncertainty	High Uncertainty
Organizational nodes, n	20–100	20–100	20–100
Network density	0.10–0.50	0.10–0.50	0.10–0.50
Dependency weight, w_{ij}	0–1	0–1	0–1
Initial node risk, r_i	0–1	0–1	0–1
Resource budget, B	100	100	100
Risk-propagation coefficient, λ	0.1–0.3	0.4–0.6	0.7–0.9
Risk perturbation	±5%	±15%	±30%
Resource variation	±5%	±15%	±25%
Independent runs	30	30	30

4.2 Comparative Models and Performance Evaluation

The suggested Graph-Theoretic Minimax Game Model (GT-MGM) is contrasted with three mathematical counterparts: a Weighted Decision Model (WDM) that models traditional weighted optimization, a Graph-Based Risk Model (GBRM) that models dependencies within an organization and risk propagation, and a Game-Theoretic Decision Model (GDM), which models strategic interactions in the face of uncertainty. This sequential comparison solitary the value that is acquired through graphical expression of risk, strategic modelling, and worst-case of minimax optimization. The performance is measured with regards to five complementary metrics, including Worst-Case Decision Loss (WDL) and Organizational Risk Exposure (ORE) is minimized to show better robustness and less risk on the network level, and Strategic Objective Achievement (SOA), Resource Utilization Efficiency (RUE), and Sustainability Performance Index (SPI) are maximized. Combined, these indicators can show the extent to which GT-MGM is able to constrain the negative organizational performance without achieving robustness at the cost of strategic performance, effective use of resources or the sustainability performance.

5. Results and Discussion

5.1 Worst-Case and Organizational risk performance.

The relative WDL and ORE in Fig. 3 show that there is an upward trend in the decision risk as the uncertainty is progressively becoming worse. For WDL, GT-MGM records 0.22, 0.29, and 0.37 under low, moderate, and high uncertainty, respectively, compared with 0.36, 0.52, and 0.71 for WDM, 0.31, 0.44, and 0.60 for GBRM, and 0.27, 0.38, and 0.51 for GDM. A similar trend is observed for ORE, where GT-MGM maintains the lowest values of 0.19, 0.26, and 0.33, while WDM increases from 0.32 to 0.64, GBRM from 0.26 to 0.49, and GDM from 0.24 to 0.45. In the condition of high uncertainty, consequently, GT-MGM would reduce WDL and ORE by about 47.9 and 48.4 percent respectively, as compared to WDM. Furthermore, its WDL increases with low to high uncertainty only by 0.15, whereas WDM increases by 0.35, and its ORE increases by 0.14, compared to WDM. These findings suggest that risk propagation based on graph theoretical techniques and strategic interaction, coupled with minimax optimization, restrict the worst-case incurred decision loss as well as amplified risk across a network and the benefit of GT-MGM is especially pronounced in situations of severe uncertainty.

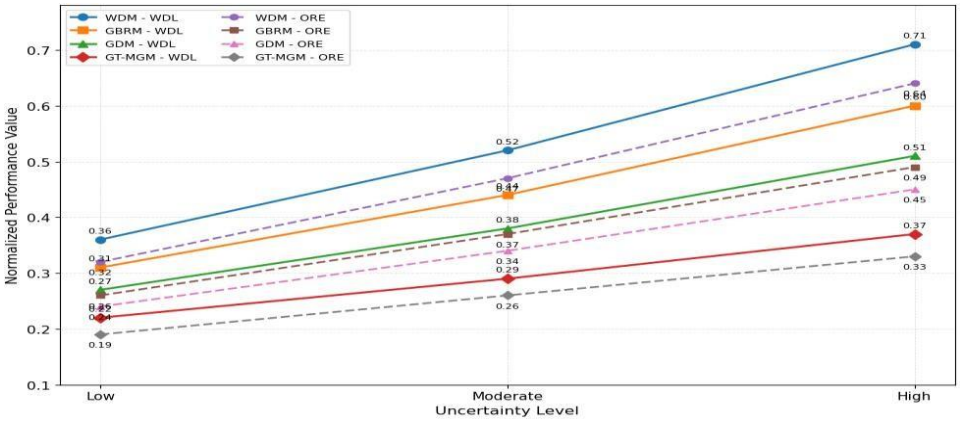


Fig. 3. Worst-Case Decision Loss and Organizational Risk Exposure under Uncertainty

5.2 Strategic, Resource, and Sustainability Performance

Fig. 4 is jointly evaluated in terms of Strategic Objective Achievement (SOA), Resource Utilization Efficiency (RUE) and Sustainability Performance Index (SPI). Under low uncertainty, GT-MGM achieves 92% SOA, 93% RUE, and 91% SPI, compared with 84%, 86%, and 81% for WDM. Under moderate uncertainty, GT-MGM maintains 88%, 89%, and 86%, outperforming GBRM (80%, 82%, 78%) and GDM (83%, 84%, 81%). This is more evident in the case of high uncertainty where GT-MGM has 82% SOA, 84% RUE, and 80% SPI whereas WDM has 65, 68, and 62 respectively. Low to high uncertainty, GT-MGM decreases by 10, 9 and 11 percentage points across the three metrics thus being more stable in unfavorable circumstances. In general, as depicted in Fig. 4, maxim point-based resource allocation becomes more robust and preserves better strategic performance, resource optimization, and sustainability as compared to the comparative formulations.

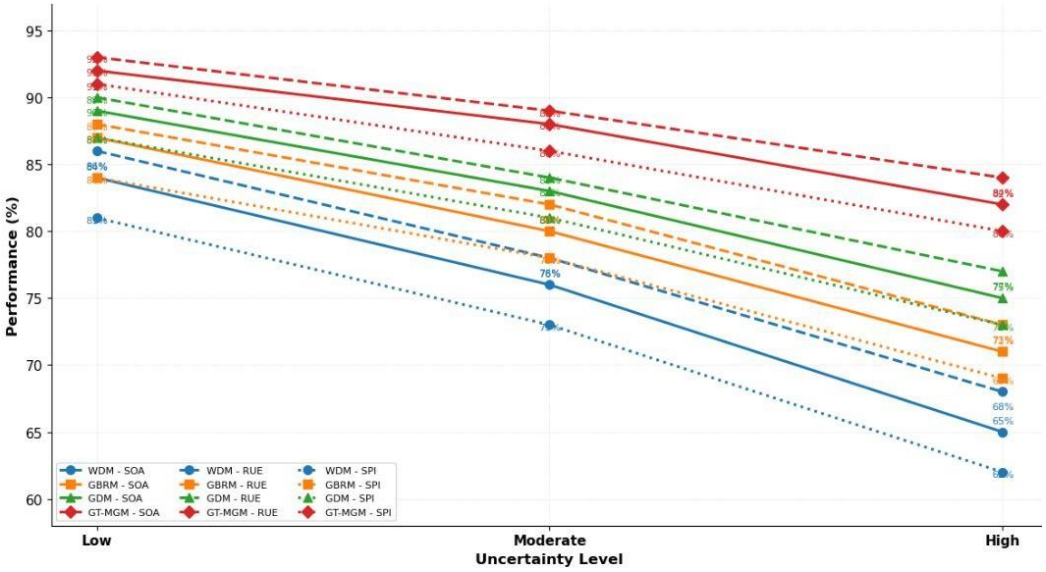


Fig. 4. Strategic, Resource-Utilization, and Sustainability Performance

5.3 Sensitivity and Robustness Analysis

The minimax solution is found to be sensitive to changing the risk-propagation coefficient λ, resource budget B, objective weights and the size of the uncertainty-set. The effect of an augmentation of λ on network-mediated transmission of risks exceeds that of a variation in B on the amount of resources allocated towards protection. Altering the balance between risk, cost, strategic achievement and sustainability is done by modifying objective weights but as the uncertainty-set increases, the model is exposed to worse conditions. A mathematically robust choice of a proposed GT-MGM is only viable when its solution and performance are stable under these parameter variations in such a way that the perceived benefits are not related to a small set of configurations chosen.

6. Conclusion

This paper came up with a Graph-Theoretic Minimax Game Model (GT-MGM) of making sound organizational decisions amid interdependent risks and limited uncertainty. The framework describes the units of the organization and dependencies by a weighted graph and uses a risk-propagation mechanism to measure the exposure across the network. The process of strategic decision-making is developed as a game between strategic decision-maker in an organization and negative uncertainty, and resource allocations are minimized by minimax optimization to reduce the worst-case loss, considering strategic achievement, resource efficiency, and sustainability. Numerical analysis revealed that GT-MGM had a smaller Worst-Case Decision Loss (WDL) and Organizational Risk Exposure (ORE) when uncertainty was high compared to WDM, GBRM and GDM. At high uncertainty, GT-MGM had WDL and ORE of 0.37 and 0.33, respectively, versus WDM of 0.71 and 0.64, indicating much more robust defense against adverse circumstances. The framework also ensured 82% SOA, 84% RUE, and 80% SPI in the face of high uncertainty, indicating that worst-case risk reduction was possible without drastic negative effects on strategic, resource-utilization or sustainability performance. The mathematical analysis also gives the conditions of a limited propagation of risk, the existence of a minimax solution, and saddle-point behaviour with a suitable compactness, continuity and convexity-concavity assumptions. Altogether, the suggested formulation proves the usefulness of the combination of graph-theoretic

spread of risks, strategic games, and minimax optimization of mathematically sound and sustainable resource-allocation choices in uncertain organizational environments.

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