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Python-Based Computational Analysis of Cordial Labeling in Lagoon Step Graphs

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Abstract

Consider the graph $G = \{V, E\}$, which has n vertices and e edges. In this study, we have applied cordial labeling to Four Intersection Opposite Lagoon Step Graph FIO L_nS_m and Four Intersection Mirror Lagoon Step Graph FIM L_nS_m. Python has been used to implement the ideas presented in the paper below.

Keywords: Four Intersection Opposite Lagoon Step Graph, Four Intersection Mirror Lagoon Step Graph, Cordial labeling**.**

Introduction

Numerous fields, including mathematics, operations research, management science, network analysis, computer science, and mobile applications, use graph theory. Cahit [1] was the first to propose the concept of cordial labeling in 1987. Rosa is credited with creating graph labeling [2]. Gallian [3] is cited for a more thorough survey. We adhere to Harary [4] for all terminologies and notations. Driven by the notion of cordial labeling, Sundaram et al. [5] have presented a labeling that resembles cordial labeling but substitutes the product of vertex labels for the absolute difference of vertex labels. In [6], the same authors looked into a few new product cordial graphs. Difference cordial labeling of graphs was established by Ponrajet al. [7]. The most widely used language in the world is Python. On February 20, 1991, Guio Van Rossum made its debut. Python is a well-liked programming language in the field of graph theory that is used for a number of tasks, including graph labeling. The majority of research literature views cordial labeling as an algorithmic problem that can be resolved by computer methods [8]. Here, we employ Python code to generate the Cordial Labeling for FIM L_nS_m and FIO L_nS_m [9][10].

Preliminaries

Definition 1: Consider that $G = (V, E)$ is a graph. Binary vertex labeling is a mapping $f: V \rightarrow (0, 1)$. This induced edge labeling f*: E → (0, 1) for an edge $e = uv$ can be found using the formula f *(e) = $\left| f\ (u\ ) - f\ (v\ ) \right|$. Let V_f (1), V_f (0) represent the number of vertices of G with labels 1 and 0 under f, and $\mathbf{e}_f$ (0) and $\mathbf{e}_f$ (1) represent the number of edges with labels 0 and 1 under f *, respectively.

Definition 2: In graph G, a binary vertex labeling is represented as a cordial labeling if $|\mathbf{e}_f\ (0) - \mathbf{e}_f\ (1\ )| \leq 1$ and $|V_f\ (\ 0) - V_f\ (\ 1)| \leq 1$. If a graph G permits cordial labeling, then it is cordial.

Definition 3: A Lagoon L_n (n ≥ 3& must be an odd) obtained from n-vertices with n-1 edges which is shown below figures.

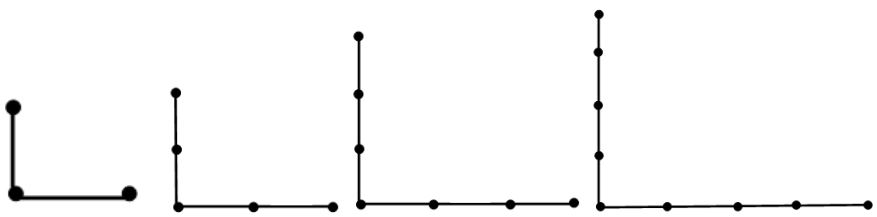


Figure 1. L_3 Figure 2. L_5 Figure 3. L_7 Figure 4. L_9

Definition 4: A Step graph S_m(where m=$\frac{n-1}{2}$& number of steps) is a graph obtained from L_n(n ≥ 3 & must be an odd) which is shown in below figures.

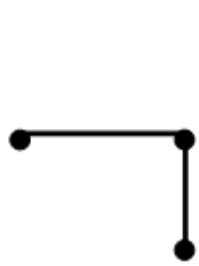


Figure 5. S_1

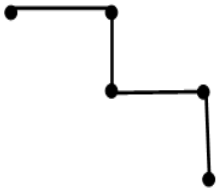


Figure 6. S_2

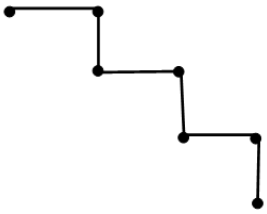


Figure 7. S_3

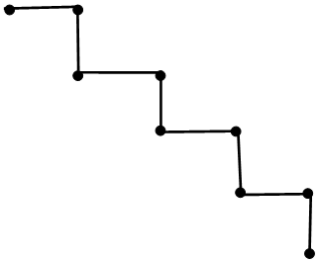


Figure 8. S_4

Definition 5: An illustration of a lagoon step for n vertices (n≥3 must be odd) in the Lagoon Ln (L Shape) graph and step graph S_m (m≥1, m is the number of steps with $m = \frac{n-1}{2}$), L_nS_m is created. The graph has n+2m -1 edges and n +2m -1 vertices, as seen in the figure below.

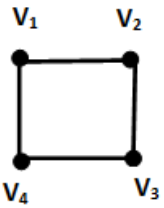


Figure.9. L_3 S_1

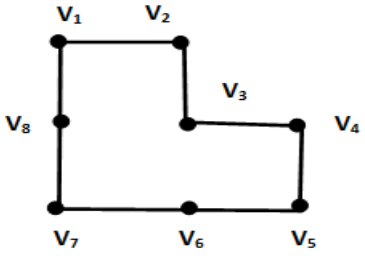


Figure.10. L_5 S_2

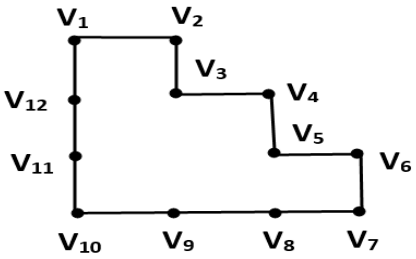


Figure.11. L_7 S_3

Definition 6: Attaching the Lagoon step graph with the mirror image yielded the Intersection Mirror IM L_n S_m. The graphic illustrates a lagoon step graph with 3n +m -3 edges and 3n +m -4 vertices (where m = $\frac{n-1}{2}$, m ≥ 1 if n ≥ 3 must be an odd number), as seen in the figure below.

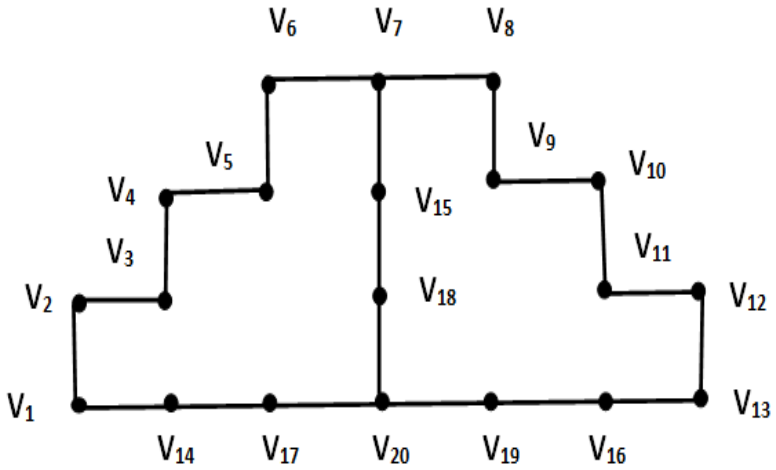


Figure 12. Intersection Mirror Lagoon Step Graph for n = 7 and m = 3

Definition 7: Intersection Opposite IO L_nS_m is the result of attaching a Lagoon step graph in the opposite direction. It has edges of 2n+4m-3 and vertices of 2n+4m-4 (where m = $\frac{n-1}{2}$, m ≥ 1 if n ≥ 3 must be an odd number), as seen in the figure below.

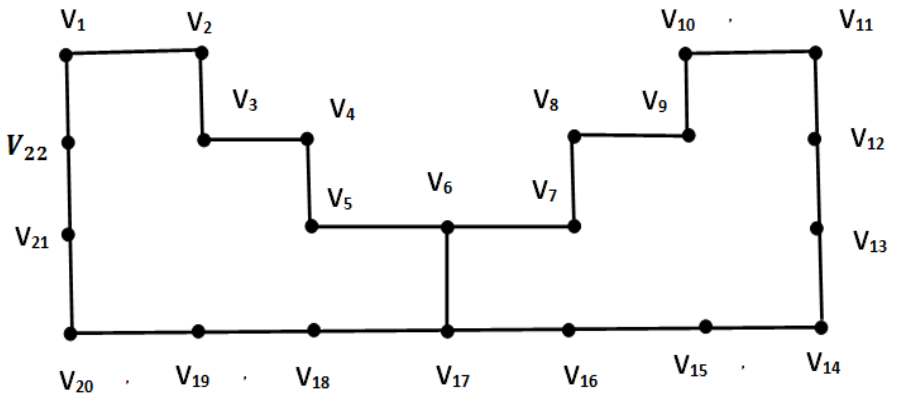


Figure 13. Intersection Opposite Lagoon Step graph n=7, m=3

Definition 8: FIM $L_nS_m$were created by joining two IM $L_nS_m$graphs, as shown in the figure below. These intersections have a total of 3(n -1)+6m edges and 3n +6(m -1) vertices (m = $\frac{n-1}{2}$, m ≥ 1, and m = number of steps if where n ≥ 3 must be an odd number).

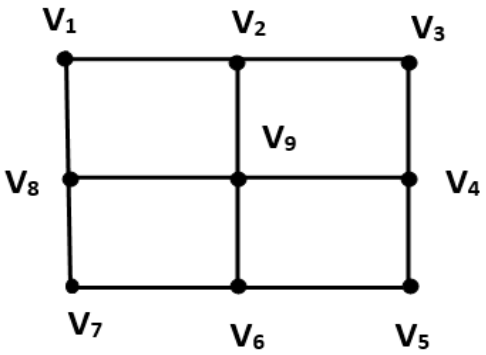


Figure 14. Four Intersection Mirror Lagoon Step Graph for n = 3 and m =1

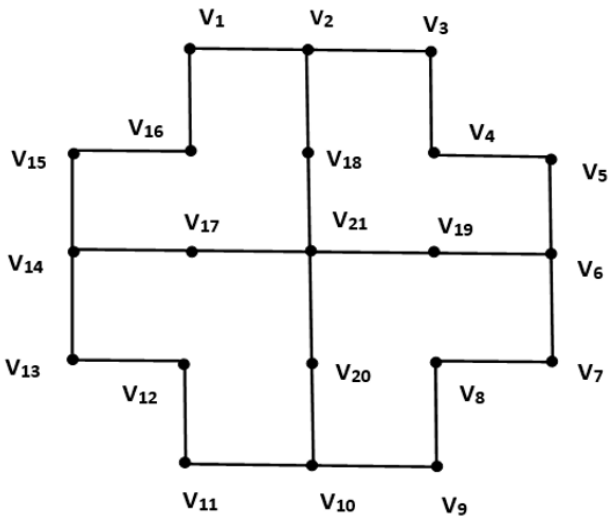


Figure 15. Four Intersection Mirror Lagoon Step Graph for n = 5 and m = 2

Definition 9: In order to acquire the FIO $L_nS_m$graph, two graphs of intersection are attached. These graphs have a total of 7(4m -n)+5 edges and 7(4m -n)+2 vertices (where m = $\frac{n-1}{2}$, m ≥ 1, m = number of steps if n ≥ 3 must be an odd number) as shown below.

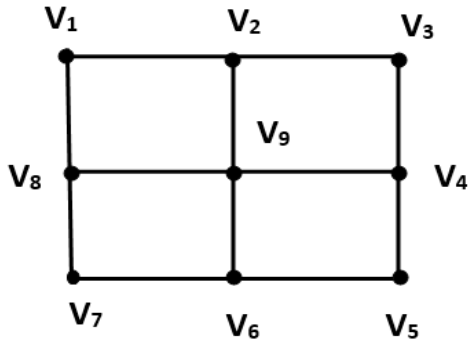


Figure 16. Four Intersection Opposite Lagoon Step Graph for n=3 and m = 1

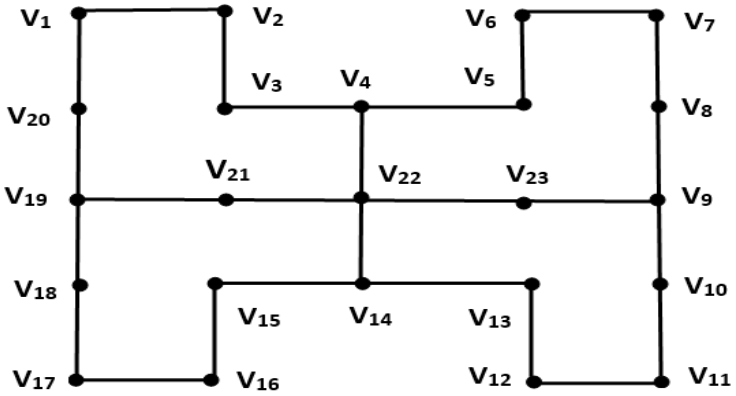


Figure 17. Four Intersection Opposite Lagoon Step Graph for n = 5 and m = 2

Main Results

Theorem 1: FIM $L_n S_m$ allows cordial labeling.

Proof : Consider FIM $L_n S_m$ be a graph with 3(n-1)+6m edges and 3n+6(m-1) vertices. Therefore, m = 1, 2, 3… when n = 3, 5, 7….

Case i : m = 2, 3, 4…when n = 5, 7, 9… respectively.

Let a , b , c , d be the set of all vertices of left (standing line), top (horizontal line), right (standing line) and bottom (horizontal line) respectively. Each set contains exactly three vertices namely a = { a_i}, b = { b_i}, c = { c_i}, d = { d_i} where **i** = 1, 2, 3. Let c be the central vertex. Let h_j and $P_j$be the set of all vertices present in the central horizontal line and vertical line respectively except central vertex c and edges touched the vertices in the sets a, b, c, d, which can be expressed in Table 1.

Table 1. Vertices set Pj and hj

n	m	j	P_j	h_j
5	2	1, 2	P_1, P_2	h_1, h_2
7	3	1, 2, 3, 4	P_1, P_2, P_3, P_4	h_1, h_2, h_3, h_4
9	4	1, 2, 3, 4, 5, 6	P_1, P_2, P_3, P_4, P_5, P_6	h_1, h_2, h_3, h_4, h_5, h_6
. . .	. . .	. . .	. . .	. .

Let Z_i, Y_i, X_i, W_i be the set of all bottom left (B.L) step vertices, bottom right (B.R) step vertices, top right (T.R) step vertices and top left (T.L) step vertices, respectively except the set of all vertices in a, b, c, d, is displayed in Table 2.

Table 2. Vertices set W_i, X_i, Y_i, Z_i

n	m	i	Z_i	W_i	Y_i	X_i
5	2	1	Z_1	W_1	Y_1	X_1
7	3	1, 2, 3	Z_1, Z_2, Z_3	W_1, W_2, W_3	Y_1, Y_2, Y_3	X_1, X_2, X_3
9	4	1, 2, 3, 4, 5	Z_1, Z_2, Z_3, Z_4, Z_5	W_1, W_2, W_3, W_4, W_5	Y_1, Y_2, Y_3, Y_4, Y_5	X_1, X_2, X_3, X_4, X_5
. . .	. . .	. . .	. . .	. . .	. . .	. . .

respectively. W_i, X_i, Y_i, Z_i count starts from T.L to B.L in clockwise direction.

Define a function $f : V(G) \rightarrow (0,1)$ and an induced function f*: $E(G) \rightarrow (0,1)$ as given below $f(a_i) = f(b_i) = f(c_i) = f(d_i) = 0$, when i = 1, 3 and $f(a_i) = f(b_i) = f(c_i) = f(d_i) = f(c) = 1$, when i = 2. Label of vertices h_j and P_j are shown in Table 3.

Table 3. Vertex labeling $f(h_j)$ and $f(P_j)$

n	m	j	$f(h_j)$	$f(P_j)$
5	2	1, 2	$f(h_1) = f(h_2) = 1$	$f(P_j) = 1$
7	3	1, 2, 3, 4	$f(h_1) = f(h_4) = 0$ $f(h_2) = f(h_3) = 1$	$f(P_j) = 1$
9	4	1, 2, 3, 4, 5, 6	$f(h_1) = f(h_6) = 0$ $f(h_j) = 1, j \neq 1, 6$	$f(P_j) = 1$
. . .	. . .	. . .	. . .	. . .
15	7	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	$f(h_1) = f(h_3) = f(h_{12}) = 0$ $f(h_j) = 1, j \neq 1, 3, 12$	$f(P_j) = 1$
17	8	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	$f(h_1) = f(h_3) = f(h_{14}) = 0$ $f(h_j) = 1, j \neq 1, 3, 14$	$f(P_j) = 1$
. . .	. . .	. . .	. . .	. . .

From above $f(h_j)$ observation, last vertex labeling of h_j like $f(h_j) = 0$, when m = 2, 3, 4…., then j = 4, 6, 8…., respectively and according to extended value of n and m for particular $f(h_j)$ as displayed in Table 4.

Table 4. Vertex (particular) labeling $f(h_j)$

n (consider odd only between)	m	j	$f(h_j)$
$15 \leq n \leq 23$	$7 \leq m \leq 11$	3	$f(h_3) = 0$
$25 \leq n \leq 31$	$12 \leq m \leq 15$	3, 5	$f(h_3) = f(h_5) = 0$
. . .	. . .	. . .	. . .

Label of vertices Z_i, W_i, Y_i, X_i are shown in the below Table 5.

Table 5. Vertices labeling of $f(W_i)$, $f(X_i)$, $f(Y_i)$ and $f(Z_i)$

n	m	i	$f(Z_i)$	$f(W_i)$	$f(Y_i)$	$f(X_i)$
5	2	1	$f(Z_1) = 1$	$f(W_1) = 0$	$f(Y_1) = 1$	$f(X_1) = 0$
7	3	1, 2, 3	$f(Z_1) = f(Z_3) = 0$. $f(Z_2) = 1$	$f(W_i) = 0$	$f(Y_i) = 1$	$f(X_i) = 0$
9	4	1, 2, 3, 4, 5	$f(Z_2) = f(Z_4) = 0$. Remaining $f(Z_i) = 1$	$f(W_2) = 1$. Remaining $f(W_i) = 0$	$f(Y_2) = f(Y_4) = 0$. Remaining $f(Y_i) = 1$	$f(Xi) = 0$
11	5	1, 2, 3, 4, 5,	$f(Z_i) = 0$, i = even	$f(W_2) = 1$.	$f(Y_i) = 0$, i = even	$f(X_i) = 0$

		6, 7	$f(Z_i)= 1$, i = odd	Remaining $f(W_i)= 0$	$f(Y_i) = 1$, i = odd	
13	6	1, 2, 3, 4, 5, 6, 7, 8, 9	$f(Z_i) = 0$, i = even $f(Z_i)= 1$, i = odd	$f(W_2) = f(W_4) = 1$. Remaining $f(W_i)= 0$	$f(Y_i) = 0$, i =even $f(Y_i)= 1$, i = odd	$f(X_i) = 0$
15	7	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	$f(Z_i) = 0$, i = even $f(Z_i) = 1$, i = odd	$f(W_2) = f(W_4) = 1$. Remaining $f(W_i)= 0$	$f(Y_i)= 0$, i = even $f(Y_i) = 1$, i = odd	$f(X_i) = 0$
17	8	1, 2, 3, 4, 5, 6, 7, 8, 9, 10,1112,13	$f(Z_i) = 0$, i = even $f(Z_i) = 1$, i = odd	$f(W_2) = f(W_4) = f(W_6)= 1$. Remaining $f(W_i)= 0$	$f(Y_i)= 0$, i = even $f(Y_i) = 1$, i = odd	$f(X_i) = 0$
. . .	. . .	. . .	. . .	. . .	. . .	. . .

respectively.

The following observation for label of vertices and label of edges is shown in Table 6 below.

Table 6. Number of edges and vertices labeled with 0 and 1 in FIM L_nS_m

n	m	$V_f(1)$	$V_f(0)$	$e_f(1)$	$e_f(0)$
odd	even	$\frac{3n+6(m-1)+1}{2}$	$\frac{3n+6(m-1)-1}{2}$	$\frac{3(n-1)+6m}{2}$	$\frac{3(n-1)+6m}{2}$
odd	odd	$\frac{3n+6(m-1)-1}{2}$	$\frac{3n+6(m-1)+1}{2}$	$\frac{3(n-1)+6m}{2}$	$\frac{3(n-1)+6m}{2}$

Above detailed labeling pattern of vertices shown below Figure.18.

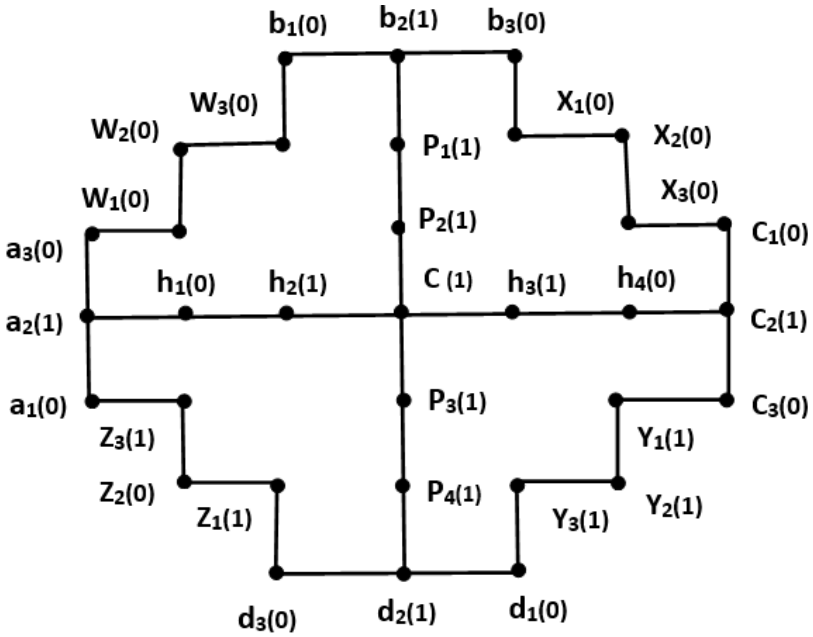


Figure 18. Cordial labeling FIM L_7 S_3

With clarity the graph FIM L_n S_m allows the condition $|e_f(0) - e_f(1)| \leq 1$ and $|V_f(0) - V_f(1)| \leq 1$. Such that FIM L_n S_m is cordial hence m = 2, 3, 4… when n = 5, 7, 9… respectively.

Special case FIM L_3 S_1: Let W_i be the top path vertices, X_i be the middle path vertices and Y_i be the base path vertices. For i=1,2,3, label of vertices are $f(W_1) = f(X_1) = f(W_3) = f(X_3) = f(Y_1) = 1$ and $f(W_2) = f(X_2) = f(Y_2) = f(Y_3) = 0$ as illustrated in Figure 19.

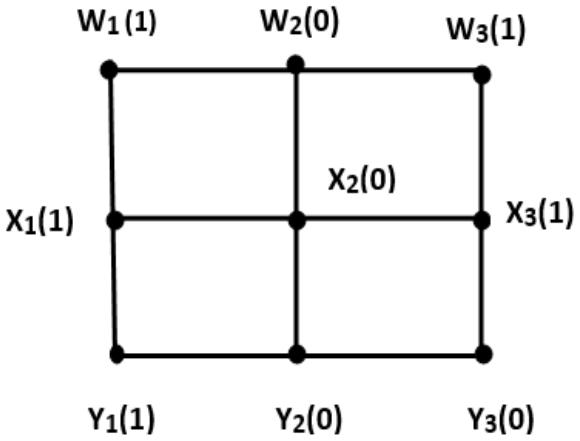


Figure 19. Cordial labeling FIM L_3 S_1

With clarity the graph FIM L_3 S_1 allows the condition $|\mathbf{e}_f(0) - \mathbf{e}_f(1)| \leq 1$ and $|V_f(0) - V_f(1)| \leq 1$. Therefore FIM L_3 S_1 is cordial. Hence FIM L_n S_m allows cordial labeling.

Theorem 2: FIO L_n S_m allows cordial labeling

proof: Consider FIO L_n S_m be a graph with 7(4m-n)+5 edges and 7(4m-n)+2 vertices therefore m = 1, 2, 3… when n = 3, 5, 7… respectively.

case i: m = 2, 3, 4… when n = 5, 7, 9… respectively.

Let W be the left top two corner vertices, X be the right top two corner vertices, Y be the left bottom two corner vertices and Z be the right bottom two corner vertices. So, W and X be first row vertices, Y and Z be base row vertices. Let C be the central vertex and h_i be the central horizontal vertices except central vertex C is illustrated in Table 7.

Table 7. Vertices of h_i

n	m	i	h_i
5	2	1, 2, 3, 4	h_1, h_2, h_3, h_4
7	3	1, 2, 3, 4, 5, 6	$h_1, h_2, h_3, h_4, h_5, h_6$
9	4	1, 2, 3, 4, 5, 6, 7, 8	$h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8$
.	.	.	.
.	.	.	.
.	.	.	.

Between central horizontal vertices and first row vertices, set of all vertices can be represented by row wise R_j. R_j starts from above to central vertices and end with below first row vertices in upward direction. Between central horizontal vertices and base row vertices, set of all vertices represented by row wise Q_j. Q_j starts from below to central horizontal vertices and end with above to base row vertices in downward direction, which can be shown in below Table. 8.

Table 8. Vertices of R_j and Q_j

n	m	j	R_j	Q_j	New row entry
5	2	1	R_1	Q_1	-
7	3	1, 2	R_1, R_2	Q_1, Q_2	R_2& Q_2
9	4	1, 2, 3	R_1, R_2, R_3	Q_1, Q_2, Q_3	R_3& Q_3
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

Where R_1 consists of five vertices in a row namely $R_1 = \{r_1^1, r_2^1, r_3^1, r_4^1, r_5^1\}$ and Q_1 consists of five vertices in a row namely $Q_1 = \{q_1^1, q_2^1, q_3^1, q_4^1, q_5^1$. The remaining R_j' s and Q_j' s consists of six vertices each, which can be shown in Table9 below.

Table 9. Vertex set of R_j and Q_j

j	R_j	vertices	Q_j	vertices
2	R_2	$r_1^{11}, r_2^{11}, r_3^{11}, r_4^{11}, r_5^{11}, r_6^{11}$	Q_2	$q_1^{11}, q_2^{11}, q_3^{11}, q_4^{11}, q_5^{11}, q_6^{11}$
3	R_3	$r_1^{111}, r_2^{111}, r_3^{111}, r_4^{111}, r_5^{111}, r_6^{111}$	Q_3	$q_1^{111}, q_2^{111}, q_3^{111}, q_4^{111}, q_5^{111}, q_6^{111}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

As explained below, define an induced function f*: $E\ (G\ ) \rightarrow (0,1)$ for edge labeling and a vertex labeling function $f: V\ (G\ ) \rightarrow (0,1)$.

f(w) = f(x) = f(y) = f(z) = f(c) = 1, $f(R_1) = \{0, 0, 1, 0, 0\}$, $f(Q_1) = \{1, 0, 0, 0, 1\}$ hence m = 2, 3, 4… when n = 5, 7, 9…….. respectively. The remaining f (R_j)'s and f (Q_j)'s can be shown in Table10 below.

Table 10. Vertex labeling set of f(R_j) and f (Q_j)

n	m	j	f (Q_j)	f (R_j)
7	3	1, 2	f(Q_1), f(Q_2)	f(R_1), f(R_2)
9	4	1, 2, 3	f(Q_1), f(Q_2), f(Q_3)	f(R_1), f(R_2), f(R_3)
11	5	1, 2, 3, 4	f(Q_1), f(Q_2), f(Q_3), f(Q_4)	f(R_1), f(R_2), f(R_3), f(R_4)
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Where f(R_2) = {0, 0, 1, 1, 0, 0}, f(R_3) = {1, 0, 1, 1, 0, 1}, f(R_4) = {0, 1, 0, 0, 1, 0}, f(R_5) = {1, 1, 0, 0, 1, 1}, f(R_6) = {0, 1, 0, 1, 0, 0}, f(R_7) = {1, 1, 0, 1, 0, 1}, f(R_8) = {0, 0, 0, 1, 1, 0}, f(R_9) = {1, 0, 0, 1, 1, 1}, f(R_{10}) = {0, 0, 1, 0, 1, 0}, f(R_{11}) = {1, 0, 1, 0, 1, 1}, f(R_{12}) = {0, 1, 1, 0, 0, 0}, f(R_{13}) = {1, 1, 1, 0, 0, 1}.Same pattern of vertex labeling repeated in same order from f (R_2) to f(R_{13}) for remaining f(R_j) according to FIO $L_nS_m$graph. Now f(Q_2) = {1, 0, 1, 1, 0, 1}, f(Q_3) = {1, 1, 0, 1, 0, 1}, f(Q_4) = {1, 0, 1, 0, 1, 1}, f(Q_5) = {1, 1, 0, 0, 1, 1}, f(Q_6) = {1, 1, 1, 0, 0, 1}, f(Q_7) = {1, 0, 0, 1, 1, 1}. Same pattern of vertex labeling repeated in same order from f(Q_2) to f(Q_7) for remaining f(Q_j) according to FIO $L_nS_m$graph. The following observation for label of vertices and label of edges is displayed in Table 11.

Table 11. Number of vertices and edges labeled with 0 and 1 in FIO L_nS_m

n		m	$V_f(0)$	$V_f(1)$	$\mathbf{e}_f(0)$	$\mathbf{e}_f(1)$
odd		even	$\frac{7(4m\ -n\ )+2-1}{2}$	$\frac{7(4m\ -n\ )+2+1}{2}$	$\frac{7(4m-n)+5}{2}$	$\frac{7(4m-n)+5}{2}$
odd		odd	$\frac{7(4m\ -n\ )+2+1}{2}$	$\frac{7(4m\ -n\ )+2-1}{2}$	$\frac{7(4m-n)+5}{2}$	$\frac{7(4m-n)+5}{2}$

Above detailed labeling pattern of vertices shown in Figure 20.

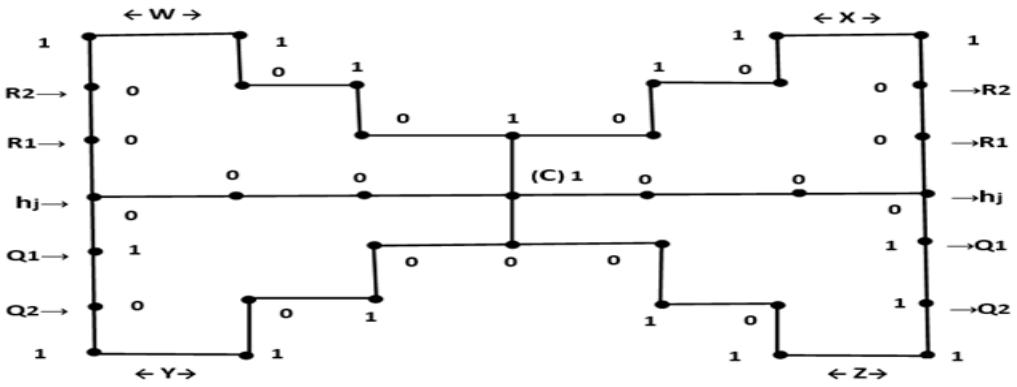


Figure 20. Cordial labeling FIO $L_7\ S_3$

With clarity the FIO $L_n\ S_m$graph allows the condition $|\mathbf{e}_f(0) - \mathbf{e}_f(1)| \leq 1$ and $|V_f(0) - V_f(1)| \leq 1$. Hence FIO $L_nS_m$is cordial such that m = 2, 3, 4… only if when n = 5, 7, 9…… respectively.

Special case FIO L_3 S_1: Let W_i be the top path vertices, X_i be the middle path vertices and Y_i be the base path vertices. For i=1,2,3, label of vertices are $f(W_1) = f(X_1) = f(W_3) = f(X_3) = f(Y_1) = 1$ and $f(W_2) = f(X_2) = f(Y_2) = f(Y_3) = 0$ as displayed in Figure 21.

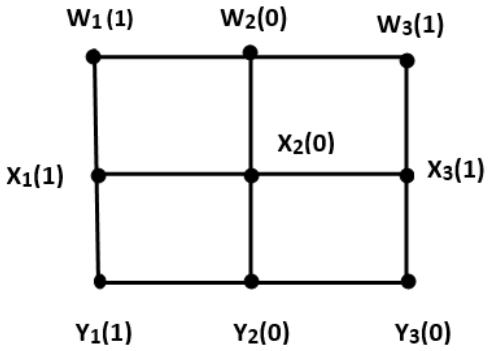


Figure 21. Cordial labeling FIO L_3 S_1

With clarity the graph FIO L_3 S_1 allows the condition $| \mathbf{e}_f(0) - \mathbf{e}_f(1) | \leq 1$ and $| V_f(0) - V_f(1) | \leq 1$. Therefore, FIO L_3 S_1 is cordial. Hence FIO L_n S_m allows cordial labeling.

Checking In To Cordial Using Python Code

Cordial vertex and edge labeling of FIM L_nS_m and FIO L_nS_m implemented using python coding shown below figures

cordial FIMLnSm python.py

```python
def FIMLnSm(n):
    print("Vertex Labeling")  # Heading

    if n % 2 == 0:
        print("Input error: n must be odd.")
        return

    # Step 1: Calculate m, v, and e
    m = (n - 1) // 2
    v = 3 * n + 6 * (m - 1)
    e = 3 * (n - 1) + 6 * m

    # Display calculated vertices and edges
    print(f"Number of vertices (v): {v}")
    print(f"Number of edges (e): {e}")

    induced_edge_labels = {}
    nodes = ["x"] + ["x" + str(k) for k in range(1, v)]
    s = len(nodes)
    if s % 4 == 2:
        induced_edge_labels["x"] = 0
    else:
        induced_edge_labels["x"] = 1

    for k in range(1, s // 2 + 1):
        induced_edge_labels["x" + str(2 * k)] = 1
        induced_edge_labels["x" + str(2 * k - 1)] = 0
```

Figure 22. Verification of the Cordiality of FIM L_5 S_2 using Python Implementation

```
PS C:\Users\Lenovo\Desktop\vas\MOM PHD> python -u "c:\Users\Lenovo\Desktop\vas\MOM PHD\cordial FIMLnSm python.py"
Enter an odd value for n: 5
Vertex Labeling
Number of vertices (v): 21
Number of edges (e): 24
Induced vertex labels: {'x': 1, 'x2': 1, 'x1': 0, 'x4': 1, 'x3': 0, 'x6': 1, 'x5': 0, 'x8': 1, 'x7': 0, 'x10': 1, 'x9': 0, 'x12': 1, 'x11': 0, 'x14': 1, 'x13': 0, 'x16': 1, 'x15': 0,
'x18': 1, 'x17': 0, 'x20': 1, 'x19': 0}
Number of vertices labeled with zeros (Vf(0)): 10
Number of vertices labeled with ones (Vf(1)): 11
Vertex label is cordial.

Edge Labeling
Number of edges labeled with zeros (Ef(0)): 12
Number of edges labeled with ones (Ef(1)): 12
PS C:\Users\Lenovo\Desktop\vas\MOM PHD>
```

Figure 23. Output of vertex and edge labeling FIM L $_5$ S $_2$ using Python Implementation

cordial FIO LnSm python.py

```python
def FIOLnSm(n):
    print("Vertex Labeling")  # Heading

    if n % 2 == 0:
        print("Input error: n must be odd.")
        return

    # Step 1: Calculate m, v, and e
    m = (n - 1) // 2
    v = 7 * (4 * m - n) + 2
    e = 7 * (4 * m - n) + 5

    # Display calculated vertices and edges
    print(f"Number of vertices (v): {v}")
    print(f"Number of edges (e): {e}")

    induced_edge_labels = {}
    nodes = ["x"] + ["x" + str(k) for k in range(1, v)]
    s = len(nodes)
    if s % 4 == 2:
        induced_edge_labels["x"] = 0
    else:
        induced_edge_labels["x"] = 1

    for k in range(1, s // 2 + 1):
        induced_edge_labels["x" + str(2 * k)] = 1
        induced_edge_labels["x" + str(2 * k - 1)] = 0
```

Figure 24. Verification of the Cordiality of FIO L $_7$ S $_3$ using Python Implementation

```
PROBLEMS  OUTPUT  DEBUG CONSOLE  TERMINAL  PORTS                                                    Code

PS C:\Users\Lenovo\Desktop\vas\MOM PHD> python -u "c:\Users\Lenovo\Desktop\vas\MOM PHD\cordial FIO LnSm python.py"
Enter an odd value for n: 7
Vertex Labeling
Number of vertices (v): 37
Number of edges (e): 40
Induced vertex labels: {'x': 1, 'x2': 1, 'x1': 0, 'x4': 1, 'x3': 0, 'x6': 1, 'x5': 0, 'x8': 1, 'x7': 0, 'x10': 1, 'x9': 0, 'x12': 1, 'x11': 0, 'x14': 1, 'x13': 0, 'x16': 1, 'x15': 0,
'x18': 1, 'x17': 0, 'x20': 1, 'x19': 0, 'x22': 1, 'x21': 0, 'x24': 1, 'x23': 0, 'x26': 1, 'x25': 0, 'x28': 1, 'x27': 0, 'x30': 1, 'x29': 0, 'x32': 1, 'x31': 0, 'x34': 1, 'x33': 0, 'x3
6': 1, 'x35': 0}
Number of vertices labeled with ones (Vf(1)): 18
Number of vertices labeled with zeros (Vf(0)): 19
Vertex label is cordial.

Edge Labeling
Number of edges labeled with zeros (Ef(0)): 20
Number of edges labeled with ones (Ef(1)): 20
```

Figure 25. Output of vertex and edge labeling of FIO L $_7$ S $_3$ using Python Implementation

Conclusion

One of the possible research topics is the labeling of discrete structures. FIM L_nS_m and FIO L_nS_m graphs are investigated here for cordial labeling. This study covered both the results of cordial labeling and a computational method for cordial labeling.

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