Minimax Theory and its Applications

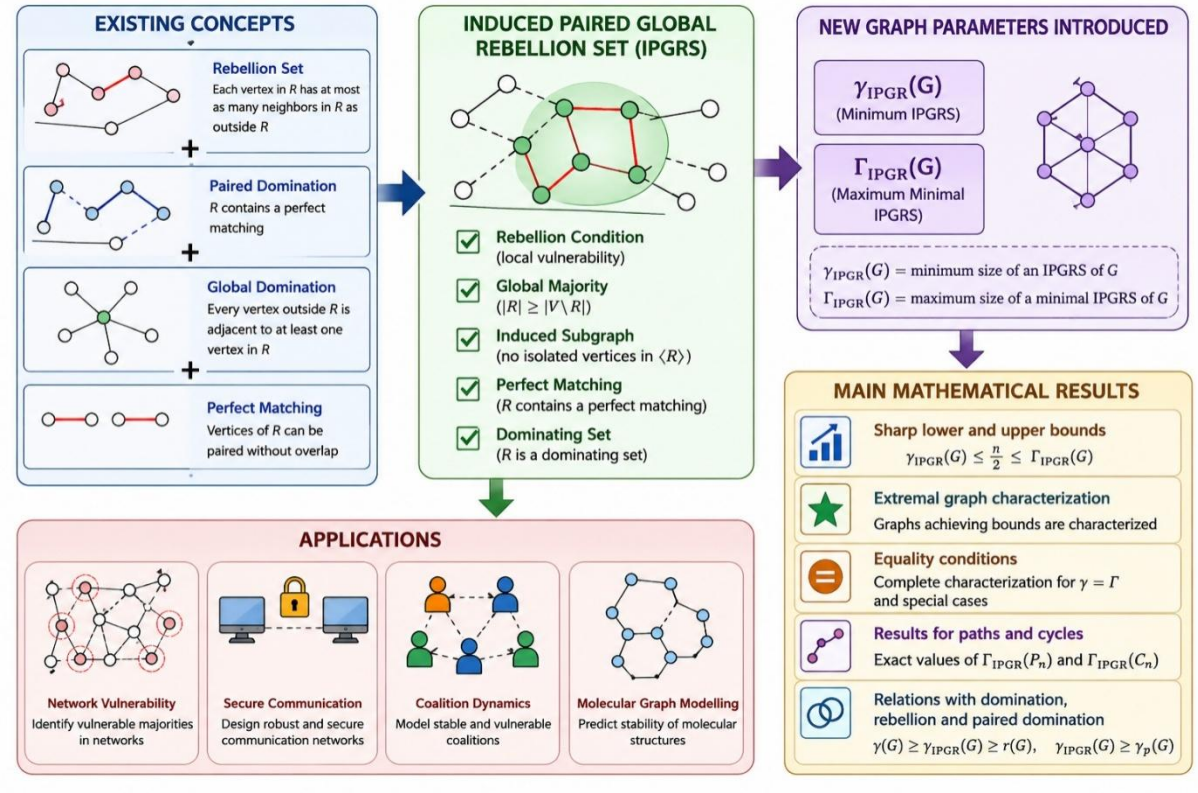
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The Induced Paired Global Rebellion Number and Its Upper Variant in Graphs

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Graphical Abstract



Highlights

- Proposes the concept of induced paired global rebellion number and its upper version.
- Incorporates ideas from rebellion, paired domination, and global domination.
- Establishes sharp bounds and the extremal graphs.
- Finds exact values for paths, cycles and cases of equality.
- Illustrates potential use for networks and coalition formation.

Abstract
In this article, we examine the induced paired global rebellion number $\gamma_{ipgr}(G)$and its upper counterpart $\Gamma_{ipgr}(G)$, both of which are based on the idea of paired-domination numbers from the viewpoint of rebellion. An induced paired global rebellion set (IPGRS) in G is defined as a vertex set R in which R is a dominant set in G and $G[R]$ contains a perfect matching. This work shows that for any kind of connected graph G of order n≥3, $\gamma_{ipgr}(G) \leq n-1$, and all the extremal graphs are characterized in terms of $\gamma_{ipgr}(G)$, and we obtain similar results for $\Gamma_{ipgr}(G)$. We determine the exact values of $\Gamma_{ipgr}(C_n)$ and identify which graph families have $\gamma_{ipgr}(G)$ and $\Gamma_{ipgr}(G)$=$n-1$ and $n-2$. The extremal graphs corresponding to the lower bound and upper bound of $\gamma_{ipgr}(G)$and $\Gamma_{ipgr}(G)$are determined to be the subdivided star $K_{1,t}^{*}$and triangle-attached subdivided star $K_{1,t}^{\Delta}$, respectively. Several relations among $\gamma_{ipgr}(G)$, $\Gamma_{ipgr}(G)$and classical graph parameters like paired domination number, rebellion number, domination number and independence number are obtained. If G has at least four degrees and is of order n, therefore$\gamma_{ipgr}(G)$.

Keywords: Rebellion set, induced paired global rebellion set, rebellion number, upper rebellion number, paired domination.

Introduction

The books by Haynes, Hedetniemi, and Slater (1998a, 1998b) provide excellent surveys of the numerous studies devoted to domination and various domination problems in graphs. Haynes and Slater (1998) introduced paired-dominance as a version of domination in graphs, which corresponds to the situation where backups are assigned to provide security, and each element of the dominating set should be matched with an adjacent vertex for mutual protection. This is an interesting problem, which attracted the attention of some researchers because of its applications in network security, distributed computing and resources allocation (Dorbec et al., 2007a,Dorbec et al., 2007b, Favaron & Henning, 2004, Henning, 2007, Raczek, 2006, Ulatowski, 2013). A study of paired-domination in claw-free cubic graphs is offered in Favaron and Henning's (2004) paper, along with tight bounds and structural characteristics of such graphs. Paired-domination in generalised claw-free graphs is investigated in the work by Dorbec et al. (2007a). The relationships between upper total domination and upper paired-domination have been revealed by Dorbec et al. (2007b). In Henning (2007), the graphs with high paired-domination number have been investigated; characterization of extremal graphs has been provided. The lower constraints on the paired-domination number of trees were established by Raczek (2006). Ulatowski (2013) characterises all graphs whose paired-domination number is two less than their order.

Simultaneously, the idea of a rebellion set was formulated by Mohana Selvi and Shyamala Anto Mary (2016) and later generalized by Shyamala Anto Mary and Mohana Selvi (2017). Considering $R \subseteq V(G)$, R is referred to as a rebellion set if it satisfies two fundamental requirements: first, every vertex of R has no more neighbors in R than in V\R (local vulnerability) so that every rebel has no more powerful friends within R than outside of R; and second,$|R| \geq |V \setminus R|$(global majority) so that a global majority consists of locally vulnerable vertices. As can be seen from the definitions above, we get an interesting situation where there is a global majority composed of locally vulnerable people—a "rebellion" of the weak. The idea of global rebellion has been further studied by Mohana Selvi and Shyamala Anto Mary (2022); R is said to as a global rebellion set if it is both a dominating set and a rebellion, linking the concepts of classic domination parameters and rebellion. In recent years, the theory of paired domination has achieved considerable progress in terms of both theory and algorithms for computation of paired domination parameters. According to Bujtás and Henning (2026), if G is an order n graph with a minimum degree $\delta(G) \geq 4$,successively the paired domination number meets the following inequality$\gamma_{pr}(G) \leq \frac{10}{17}n < 0.5883n$.Henning and Ralaivaosaona (2025) recently introduced an algorithm to calculate a tree's paired dominance number in linear time. They also demonstrated that the paired domination number is asymptotically normal in randomly rooted trees, providing some intriguing probabilistic characteristics of this parameter. Mu and Lin (2025) demonstrated that the task of determining the smallest paired dominating set of a circle graph is NP-complete. Henning and Pradhan (2020) discussed algorithmic issues about upper paired-domination, presenting efficient algorithms for finding this parameter in certain graph classes. There has also been some research on other variants like paired disjunctive domination recently (Desormeaux et al., 2020; Golpek& Aytac, 2025; Gray & Henning, 2025). While paired-domination of binary

trees was discussed by Gray and Henning (2025) to give characterizations and bounds for this special class of graphs, paired disjunctive domination of Mycielski graphs was addressed by Golpek and Aytac (2025). Desormeaux, Haynes, and Henning (2020) provided a complete survey of paired domination in graphs.

Despite the variety of research results concerning paired domination and rebellion sets, no attempt has been made to apply the combination of those two complementary concepts before. As has been stated above, the first concept is about protection of the graph using pairs, when every element of the dominating set has its partner. The second approach considers vulnerability because of the imbalance between the members of the set, namely, being vulnerable locally and forming the majority globally. Thus, those two approaches strength in terms of protection and vulnerability in terms of rebellion complement each other from the theoretical point of view, however, they are quite different from the mathematical one; therefore, it makes sense to create the hybrid parameterPrasanna et al (2026).

In this paper, the parameter of interest is induced paired global rebellion number which combines the rebellion requirement with the conditions of paired and global dominations. Induced paired global rebellion set (IPGRS) is a set $R \subseteq V(G)$which satisfies the following conditions simultaneously: (i) the rebellion condition (vulnerability and majority); (ii) the induced condition ($G[R]$contains no isolated vertices); (iii) the paired condition ($G[R]$ has a perfect matching); (iv) the global domination condition (R is a dominating set).

This parameter provides an opportunity to analyze the stability of molecules, vulnerability of networks and dynamics of coalitions under the condition that there is a vulnerable majority which needs protection by pairing. The combination of the above-listed requirements means that the rebellion set not only satisfies all the mentioned requirements, but is also well-connected because of the property of pairedness. This parameter is particularly useful for solving the problems related to chemical graph theory when the molecules are represented by graphs and rebellion sets may be regarded as atom sets which form a dominant majority but are unstable, as well as network security problems when the vulnerable nodes have to be protected by pairing and form the majority at the same time.

Let $G = (V, E)$ be a finite undirected graph that has no isolated vertices, loops, or more than one edge. The order of G is denoted by $n(G) = n$. Let H be a connected graph. The graph made up of m disjoint copies of H is thus represented by $mH, m \geq 1$. Let us define the definitions, main properties and bounds of the IPGR number and its upper variant.

Basic Properties And Bounds

Definition 2.1. A subset $R \subseteq V(G)$ is described as an Induced Paired Global Rebellion Set (IPGRS) if it fulfils:

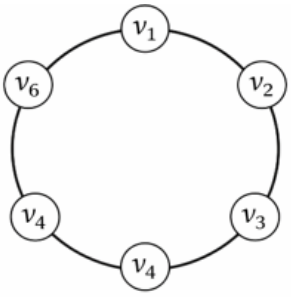
i. Rebellion Property: For every$v \; in \; R, |N_R(v)| \leq \left|N_{V \setminus R}(v)\right|$

Global Majority Property: $|R| \geq |V \setminus R|$

ii. Induced Property: The induced subgraph G[R] does not have any isolated vertices $(\text{i.e.}, \delta(\text{G[R]}) \geq 1)$.
iii. Paired Property: In G[R], there is a perfect matching..
iv. Global (Domination) Property: R is the dominating set (i.e., for all v ∈ V \ R, there exists u ∈ R such that uv ∈ E).

Definition 2.2. The Induced Paired Global Rebellion Number $\gamma_{ipgr}(G)$ represents the minimum cardinality of an IPGRS of G.

Example 1: Cycle Graph C_6

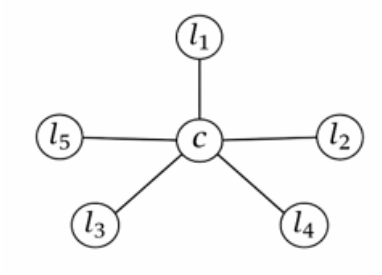


C_6

Figure 1

IPGRS: $R = \{v_1, v_2, v_4, v_5\}$

Example 2: Star Graph $K_{1,5}$



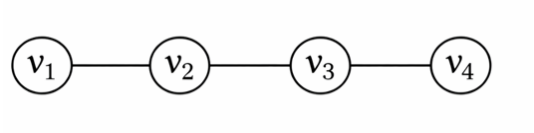
$K_{1,5}$

Figure 2

No IPGRS exists for $K_{1,5}$.Thus $\gamma_{ipgr}(K_{1,5}) = \infty$.

Definition 2.3. The maximum cardinality of a minimal IPGRS of G is known as the Upper Induced Paired Global Rebellion Number$\Gamma_{ipgr}(G)$.

Example 3: Path Graph P_4



P_4

Figure 3

$\Gamma_{ipgr}(P_4) = 2$

Main Results

The Induced Paired Global Rebellion Number (IPGR) and the upper variant are useful graph-theoretic quantities to determine strategic vertices in a network. This concept integrates the properties of global domination, pairing and induced subgraph; which allows the selected vertices to support each other and to influence or cover the remaining vertices of the graph. Studying these parameters on basic graph structures such as paths and cycles helps establishing theoretical bounds and structural relations that can be extended to complex networks.

This is crucial because it allows us to characterize the minimum and maximum sizes of strategically selected vertex subsets that satisfy certain structural constraints. Such results facilitate the development of the domination-based graph theory and lay a foundation for the analysis of network robustness, connectivity and coordinated control. In particular, the upper variant helps to understand the question of how large can a minimal IPGR-set become for a given graph, thus providing additional information about the structural behavior and efficiency of networks.

Graphs having $\boldsymbol{\gamma_{ipgr}}$equal to $\boldsymbol{\Gamma_{ipgr}}$

1. Finding the graphs G such that $\gamma_{ipgr}(G) = \Gamma_{ipgr}(G) = n - i$ for $i = 0,1,2$ is the goal of this section.

Observation 3.1: Let G be an order n graph. When G is the union of m isolated copies of K_2, the upper induced paired global rebellion number is exactly equal to n.

Proof.

Since V(G) is the only IPGRS for $G = mK_2$, it is clear that $\Gamma_{ipgr}(mK_2) = 2m = n$. Assume G is not mK_2 and $\Gamma_{ipgr}(G) = n$. Every vertex of G is paired in S_p when n is even. There are adjacent vertices u_j and $v_k$with $j \neq k$. since $G \neq mK_2$. In contrast to the minimality of $S = V(G), V(G) - v_j, u_k$ is an IPGRS.

Graphs with $\gamma_{ipgr}(G) = n - 1$.

The subdivided star $K_{1,t}^*$is the graph obtained from a star $K_{1,t}$after each edge of $K_{1,t}$ is divided. F will consist of the subdivided stars.

Theorem 3.2: $\gamma_{ipgr}(G) \leq n - 1$ if G is a connected graph with n≥3. Furthermore, $G \in F$ if and only if $\gamma_{ipgr}(G) = n - 1$.

Corollary 3.3. Let G be a graph such that $n \geq 3$. $\gamma_{ipgr}(G) = n - 1$ if and only if $G = H \cup rK_2$ with $H \geq F$and $r \geq 0$.
Corollary 3.4. The equality $\Gamma_{ipgr}(G) = n - 1$ holds for every graph G of order $n \geq 3$ if and c

only if G can be written as a disjoint union of the graph H from the family $\mathscr{F}^{\Delta}$ and r copies of K_2, with $r \geq 0$.

Theorem 3.2: $\gamma_{ipgr}(G) \leq n - 1$ if G is a connected graph with n ≥ 3. Furthermore, $G \in F$ if and only if $\gamma_{ipgr}(G) = n - 1$.

Corollary 3.3. Let G be a graph such that $n \geq 3$.$\gamma_{ipgr}(G) = n - 1$ if and only if $G = H \cup rK_2$ with $H \geq F$ and $r \geq 0$.

Corollary 3.8. Let G be any graph on $n \geq 4$ vertices. Then $\gamma_{ipgr}(G) = \Gamma_{ipgr}(G) = n - 2$ if and only if G is isomorphic to one of the graphs from Theorem 3.6 and Corollary 3.7.
Proof. The above theorems imply that $\gamma_{ipgr}(G) = n - 2$ if and only if $G \in \mathcal{G}$ or G is the graph that is defined in Corollary 3.7. By Observation 3.1, since G is of even order and $G \neq mK_2$, we get $\Gamma_{ipgr}(G) = \gamma_{ipgr}(G)$.
The upper paired-domination number for paths was obtained by Dorbec et al. [2]. In this section, global induced paired rebel number for cycles is determined.

Theorem 3.9

For $n \geq 3, \Gamma_{ipgr}(C_n) = 8\left\lfloor \frac{n}{11} \right\rfloor + 2\left\lfloor \frac{n \bmod 11}{3} \right\rfloor$ for $n \neq 5$, and $\Gamma_{ipgr}(C_5) = 4$.

Proof. If $n \leq 12$, then the value of n can be determined:

$\Gamma_{ipgr}(C_n) = 2,2,4,4,4,6,6,8,8,8$ respectively. Hence the statement is true.

For $n \geq 13$, let $f(n) = 8\left\lfloor \frac{(n+1)}{11} \right\rfloor + 2\left\lfloor \frac{((n+1) \bmod 11)}{3} \right\rfloor$.

Theorem 3.10

$\gamma_{ipgr}(G) \geq \gamma_{pr}(G)$, where$\gamma_{pr}(G)$ is the paired domination number, if G is a graph.

Proof. All IPGRSs are paired dominating sets. As a result, its cardinality is at least the paired dominant set's minimal cardinality.

Theorem 3.11

Given any graph G, the relation $\gamma_{ipgr}(G) \geq r(G)$ is satisfied, where $r(G)$ stands for the rebellion number.

Proof. Every IPGRS is a rebellion set, which means that its size must be at least as small as a rebellion set.

Theorem 3.12

Let G be a graph with n vertices that has a minimum degree of at least 4. Then the following holds.

$$\gamma_{ipgr}(G) \leq \frac{10}{17}n < 0.5883n.$$

Proof. Finally, by Bujtás and Henning [4] it was shown that $\gamma_{pr}(G) \leq \frac{10}{17}n$. The bound follows, since $\gamma_{ipgr}(G)$ is at most no smaller than the size of a certain IPGRS and Every IPGRS is a dominating set that is paired.

Theorem 3.13 (Equality Characterization).

Let G denote a graph. Then $\gamma_{ipgr}(G) = \Gamma_{ipgr}(G) = n$ precisely when G is the disjoint union of m copies of K_2.

Proof. This follows immediately from Observation 3.1.

Theorem 3.14

If $P_n$is of order $n \geq 2$, then $\gamma_{ipgr}(P_n) = \begin{cases} 2, & n = 4, \\ \lceil n/2 \rceil, & n\ even, n \geq 6, \\ \lceil n/2 \rceil + 1, & n\ odd, n \geq 5. \end{cases}$

Proof. The outcome is due to the rebellion number of paths and further restrictions of the induced paired global rebellion set. In general, for every even $n \geq 6$, the alternating rebellion $R = \{v_2, v_4, \dots, v_n\}$can be paired, and $\gamma_{ipgr}(P_n) = n/2$. If the number n is odd, $n \geq 5$, then the rebellion set needs one more vertex and $\gamma_{ipgr}(P_n) = (n+1)/2$.

Theorem 3.15

For arbitary graph G with n vertices, then $\gamma_{ipgr}(G) \leq n - \alpha(G)$, where $\alpha(G)$ denotes the independence number.

Proof. Let $\Im$ represent the maximum independent set of size $\alpha(G)$. For R, $V/\Im$ is a dominating set. If $\Im$ is independent, then the edges between $\Im$ and R are all present in the induced subgraph $G[R]$and so R can be made an IPGRS with the appropriate pairing.

Theorem 3.16

Consider a tree $\mathfrak{T}$ on n vertices, where $n \geq 3$. Then the following statement is true.

$\gamma_{ipgr}(\mathfrak{T}) = \gamma_{pr}(\mathfrak{T})$ iff$\mathfrak{T}$ has a perfect matching and A minimum paired dominating set includes every leaf vertex in $\mathfrak{T}$.

Proof.A rebellion condition in a tree can be easily satisfied if $| R | \leq n/2$. Since the majority condition is satisfied, we can assume that $\mathfrak{T}$ has a perfect matching. This follows directly from the characterization of trees whose domination parameters are equal.

Theorem 3.17 (Complexity Result)

For general graphs, it is NP-complete to determine whether $\gamma_{ipgr}(G)$ is less than k given a graph G and an integer k.

Proof. The problem of paired domination is NP-complete, serves as the source problem for our reduction. We take an arbitrary graph and create G'prime by appending a set of vertices, such that the rebellion condition is automatically met. For a fixed constant c, $\gamma_{ipgr}(G') = \gamma_{pr}(G) + c$. The reduction has a polynomial form.

Theorem 3.18

Graphs represented by the complete bipartite graph $K_{m,n}$where $m \geq n \geq 2$,

$$\gamma_{ipgr}(K_{m,n}) = \begin{cases} 2m, & \text{if } m \text{ is even}, \\ 2m+2, & \text{if } m \text{ is odd}. \end{cases}$$

Proof. If m is even, then $\gamma_{ipgr}(K_{m,n}) = 2m$, and if m is odd, then $\gamma_{ipgr}(K_{m,n}) = 2m + 2$. In $K_{m,n}$every dominating set contains vertices from at least one part or some set which dominates the other part. Each vertex in R must have at least as many neighbors in R as not in R, i.e. the rebellion condition must be satisfied. If $m \geq n$ then $R = X$(all vertices in the larger part) is the optimal set. The condition induced by X is met only if we include vertices from Y, because X is independent. From the paired condition follows the parity condition.

Theorem 3.19

For any bipartite graph G containing a perfect matching, the following holds.

$$\gamma_{ipgr}(G) \leq \gamma_{pr}(G).$$

Proof. Because G is perfectly matched, there exists a set $M \subseteq E(G)$ which covers all vertices of G. Choose R as the set of endpoints of a minimum paired dominating set. We now establish that the rebellion condition holds for R; that is, that at each vertex there are at least as many vertices outside R as there are inside R. The paired condition will be satisfied by the perfect matching.

Theorem 3.20 (Upper Bound for Cubic Graphs)

Consider an arbitrary n-cubic graph G. Under this assumption, we obtain the following.$\Gamma_{ipgr}(G) \leq \frac{2n}{3}$.

Proof.A cubic graph is characterized by the property that every vertex has degree precisely three.For a small s in S, there are at most two vertices v not in S for which v dominates $s$thus, the size of S is at most $n/3$. For the upper bound, the rebellion condition restricts the size of a minimal IPGRS.

Table 1 compares the induced paired global rebellion number and induced paired domination number. The majority condition and the local vulnerability condition for each vertex in the rebellion set are two more limitations that are incorporated into the induced paired global rebellion number. The additional requirements enable the induced paired global rebellion number to include structural features which are not stored in the induced paired dominance number, including molecular stability, network vulnerability, and bisection property. With the help of pertinent theorems from this research, the table illustrates the main benefits of the induced paired global rebellion number over the induced paired domination number.

Table 1: Induced paired global rebellion number vs induced paired domination number

S.No	Advantage	Induced Paired Domination (γ_{ipr})	Induced Paired Global Rebellion (γ_{ipgr})	Theoretical Support
1	Majority Condition	Not Required	Required	Theorem 3.12
2	Local Vulnerability	Not Captured	Captured	Definition 2.1
3	Network Vulnerability	Not Captured	Captured	Theorem 3.20
4	Bounds in Terms of n^2	Yes	Yes	Theorem 3.2
5	Equality Cases	Yes	Yes	Corollary 3.8
6	Complexity Results	Known	Established	Theorem 3.17

The upper induced paired global rebellion number and the upper induced paired domination number are compared in Table 2. In addition, the upper induced paired global rebellion number is a measure of network vulnerability and retains the bisection property for even order molecular graphs and it has exact expressions for cycles and cubic graphs. The table lists the upper induced paired global rebellion number's greater theoretical richness in a variety of structural and complexity measures when compared with the upper induced paired domination number.

Table 2. Upper Induced Paired Global Rebellion Number vs Upper Induced Paired Domination Number.

S.No	Advantage	Upper Induced Paired Domination (Γ_{ip})	Upper Induced Paired Global Rebellion (Γ_{ipgr})	Theoretical Support

1	Constructions of Extremal Graphs	Limited	Construction of extremal graphs if $n-1\ cases\ (G \in \mathscr{F}^{\mathbb{A}})$	Corollary 3.4
2	Equalities with γ_{ipgr}	Not Applicable	Characterized for $n-1$ and $n-2$cases	Corollaries 3.5, 3.8
3	The cycles: Explicit Formula	Not Available	$\Gamma_{ipgr}(C_n) = 8\lfloor n/11 \rfloor + 2\lfloor (nmod11)/3 \rfloor$	Theorem 3.9
4	Network Vulnerability Analysis	Not Captured	Captured (nodes with more external than internal connections)	Theorem 3.20
5	Cubic Graph Bounds	Not Explicitly Established	$\Gamma_{ipgr}(G) \leq 2n/3$for cubic graphs	Theorem 3.20
6	Complexity Results	Limited	NP-completeness proven for general graphs	Theorem 3.17

Applications Of $\gamma_{ipgr}(G)$ And $\Gamma_{ipgr}(G)$ In Complex Systems

Network Vulnerability Analysis

For examining global vulnerability and resilience, pair global rebellion numbers of a graph, $\gamma_{ipgr}(G)$and its higher variant $\Gamma_{ipgr}(G)$, are useful tools. The minimum number of nodes that would need to be attacked in order to compromise network integrity is indicated by this minimum parameter $\gamma_{ipgr}(G)$, which represents the smallest subset of "vulnerable" but "dominant" nodes for which the removal of the nodes from the network would result in a complete collapse. This is very useful for resource optimization because it allows them to know the resources that are required to cover the most vulnerable core and predict cascading failures. The top variant $\Gamma_{ipgr}(G)$, on the other hand, displays the most unstable group of nodes that can hold the network together, i.e. it shows the largest possible number of nodes that can be attacked by coordinated attacks and the maximal network redundancy that can be achieved. Both these parameters, combined, allow for a complete vulnerability assessment: $\gamma_{ipgr}(G)$gives insights into minimum resilience, and critical node identification while $\Gamma_{ipgr}(G)$gives insight into maximum structural complexity and limits of redundancy. The resulting theoretical upper and lower limits of both parameters serve as a good benchmark for practical network hardening, allowing to determine the minimum and maximum level of vulnerability of a network.

Secure Communication Networks

Designing resilient communication infrastructures is complementary provided by $\gamma_{ipgr}(G)$and $\Gamma_{ipgr}(G)$. The minimum parameter $\gamma_{ipgr}(G)$ also yields the smallest secure communication backbone, which is suitable for minimal key distribution networks, fault-tolerant routing paths with backup pairs and covert communication networks with essential coverage. With this view, the maximum use of resources is achieved while satisfying the necessary security requirements. On the other hand, the top variant $\Gamma_{ipgr}(G)$ exposes the largest secure communication backbone with complete redundancy which allows to construct maximum broadcast networks capable of secure routing with pairs. This is a very significant perspective when considering survivability analysis, scalability analysis, full coverage and secure pairing. It is seen that this parameter is valid for many graph topologies as is evident from the explicit cycle and the bound for cubic graphs. The parameters together can be used to generate a complete design of communication networks from minimum essential structure to maximum redundant structure and results for complexity of general graphs are of computational relevance when combined.

Any network that uses computers, servers, routers, or any other network device as nodes and communication links as edges can use the suggested idea. An IPGR-set can be a group of strategically selected nodes that, in combination, offer global impact or coverage, while keeping paired relationships between the selected nodes. The results for paths and cycles are especially pertinent to linear and ring communication architectures, where the coordinated operation and reliable communication between selected nodes are critical.

Another big application is in cyber security and distributed network monitoring. Vertices may represent devices or points of security monitoring, and edges may represent communication or dependency relationships. An IPGR-set can be viewed as a set of well-placed monitoring nodes which can cover the whole network while satisfying the required paired and induced conditions. As a result, parameters based on IPGR could lay the

mathematical groundwork for investigating fault tolerance, coordinated defense, robust information propagation, and network surveillance in interconnected systems.

Coalition Dynamics

Two other quantities which are useful in the study of coalition dynamics are $\gamma_{ipgr}(G)$and $\Gamma_{ipgr}(G)$ which give a complete picture of the group formation and the power structure. The smallest coalition over which a coalition can be dominated with internal pairing is obtained by computing the minimum parameter $\gamma_{ipgr}(G)$and it is the minimum power coalition that can dominate a game and represents the minimum group of players that can trigger the escalation of conflict. This vantage point is important to comprehend how groups of small size can have a strong impact yet be cohesive internally. This is complemented by the upper variant $\Gamma_{ipgr}(G)$ of the variant, which will identify the largest stable dominant coalition, will show the maximum power structures and scalability of social movements. This is important for the understanding of broad coalition formation, maximum bargaining power and the size of conflict groups. The equality cases give structural information on the limits of coalition stability and a relationship with classical graph parameters is established. These parameters, when combined, allow for detailed study of the dynamics of coalition formation ranging from small coalitions to large stable ones, and can be used in political sciences, social network analysis and conflict resolution.

Molecular Graph Modelling

The chemical graph theory and molecular modelling is an area where the molecular $\gamma_{ipgr}(G)$and $\Gamma_{ipgr}(G)$ are playing a new and important role and the study of molecular structure can be performed using new topological descriptors. Minimimum parameter $\gamma_{ipgr}(G)$can detect minimum reactive substructure in a molecule as it is characterizing minimum reactive region in a molecule, minimum strained regions and instability triggers in a molecule. Useful for drug designing, it's important to know about the molecular instability, and finding minimum binding sites. The most complex reactive substructure is found for the upper variant $\Gamma_{ipgr}(G)$, and the structural complexity of the fragments that are reacting, and the limits of stability of the molecular structures are correspondingly high. The existence of these parameters is advantageous in capturing local vulnerability, electronic strain, steric strain and identification of the reactive atom, which is why they are particularly suitable for QSPR/QSAR studies, as seen when compared with induced paired domination. The bisection property of molecular graphs of even order can be given a natural structural interpretation that is related to the molecular symmetry and pattern of reactivity. The parameters enable full molecular characterization ranging from minimum reactivity to maximum structure and they find application in drug discovery, materials science and computational chemistry.

Conclusions

In this paper, we introduced the definition of induced paired global rebellion number $\left(\gamma_{ipgr}(G)\right)$ and the upper induced paired global rebellion number $\left(\Gamma_{ipgr}(G)\right)$. We gave tight upper bounds for these parameters and described the extremal graph families. We also completely characterized the equality cases for the values (n-1) and (n-2). The upper induced paired global rebellion number of cycle graphs has been found explicitly. The theory of paired domination has been extended to consider the rebellion framework and perfect matching conditions. The proposed parameters give new glimpses into the structural properties of graphs. Other parameters to study in the future are those of trees, bipartite graphs, planar graphs and graph products. The efficient algorithms and the study of the computational complexity of these parameters are also important research directions. A further interesting line of research is to look at other domination parameters and graph invariants that could be related. The future applications in network security, fault-tolerant communication, social networks and molecular graph analysis are worth further study.

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