Minimax Theory and its Applications

Volume 11 (2026), No. 2s, 804 to 812

Quantum-Fuzzy Nonlinear Graph Neural Networks for Uncertainty-Aware High-Dimensional Data Representation and Intelligent Decision Making

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Abstract

The representation of data in high dimensions that are nonlinear leads to difficulties with uncertainty, complicated relations between variables and overlapping decision patterns which can lead to a reduction in reliability of the use of traditional learning models. The uncertainty-aware representation and intelligent decision making is proposed by this study, which proposes a Quantum-Fuzzy Nonlinear Graph Neural Network (QF-NGNN). It fuses the fuzzy membership encoding of uncertain observations with nonlinear adaptive graph construction of dependency modeling, graph neural feature extraction, dimensionality-aware fusion, and the quantum-inspired amplitude-phase representation of classification. Different uncertainty levels (0%, 20%) and dimensions (10, 200) are used in the computational experiments with controlled high dimensional nonlinear data. The proposed model is compared against the MLP, GNN and the Fuzzy GNN baselines in terms of Accuracy, Macro-F1, RMSE, representation robustness and uncertainty sensitivity. The QF-NGNN baseline outperforms the other baseline models MLP, GNN and Fuzzy GNN under the aforementioned computational conditions with respect to accuracy, precision, recall and F1-score. The robustness analysis also supports this as improved performance retention with higher uncertainty while at the same time adding a moderate computational cost.This study suggests the feasibility of QF-NGNN for powerful uncertainty-aware high-dimensional representation and intelligent decision-making support.

Keywords: Quantum-inspired learning; fuzzy systems; graph neural networks; nonlinear systems; high-dimensional data; uncertainty-aware learning; intelligent decision making.

1. Introduction

The rise in the use of intelligent systems, scientific computing, cyber-physical systems, industrial monitoring and nonlinear dynamical applications has led to ever larger and increasingly complex data structures. However, the representation of such data can be difficult for a number of reasons, including the non-linear relationships between variables, uncertainty and ambiguity in the data, and overlapping patterns in decision-making. Traditional vector approaches to learning are not effective at capturing the inter-variable relationship between

variables and as dimensionality and uncertainty grow, they become less effective in representing and making decisions about these relationships.

Graph neural networks (GNNs) are a powerful way to model relationships with nodes representing variables or observations, and edges representing the relationships between the nodes. GNNs utilise graph-based message passing to enable them to utilise relational patterns that can be present within the graph and not captured by conventional models. However, conventional GNNs typically rely on the relational geometry and features of graph nodes, which are fixed and can be functions of the relationship.But, typical GNNs assume deterministic geometry and features of the graph, in which case the typical features are functions of the relationship. To overcome this shortcoming, fuzzy representation is introduced by representing uncertain features by grading membership values, and keeping partial overlapping information.

Another way of encoding complex learned features is via quantum-inspired representation. The proposed approach does not need physical quantum hardware and is quantum-inspired. Compact decision representations generated using probability amplitudes, phases and encoding with superposition. Therefore, the proposed Quantum-Fuzzy Nonlinear Graph Neural Network (QF-NGNN) is: high dimensional input —> fuzzy uncertainty encoding —> nonlinear graph construction —> graph neural representation —> dimensionality-aware fusion —> quantum-inspired encoding —> intelligent decision output.

Although existing uncertainty-aware high dimensional learning methodologies based on fuzzy learning, GNN, nonlinear representation and quantum-inspired techniques are widely studied individually, there still remains a lack of integration between all of them. QF-NGNN makes five contributions to this gap: (1) uncertainty-aware fuzzy representation; (2) nonlinear adaptive graph construction for variable dependencies; (3) graph neural learning of relational nonlinear features; (4) uncertainty-aware quantum-inspired representation and decision learning; and (5) uncertainty analysis of the computational robustness under varying dimensions and uncertainties. The framework is then geared towards robust high dimensional representation and intelligent decision making in an integrated computational approach.

2. Literature Review

Most traditional approaches for the representation of high-dimensional data are based on feature selection, dimensionality reduction, deep neural networks, and statistical learning to convert complex observations into low-dimensional representations [1], [11]. Although useful for various classification and prediction applications, these strategies tend to consider observations as vectors of fixed dimension [3]. This means that the relationships between the interacting variables may not be sufficiently preserved in the nonlinear form of the variables [5]. The higher the dimensionality, the more redundant information and noise sensitivity it will have, leading to poorer quality of representation and decisions [4], [9].

Fuzzy systems offer an option for dealing with imprecise, noisy and uncertain, masked in linguistic terms, information using the notion of a "graded membership". The second uncertainty representation, fuzzy neural networks, brings together the membership based uncertainty representation and learning from data, and enable representation of partially overlapping patterns. The conventional fuzzy models, however, focus on feature and/or rule level uncertainty and might offer less ability to explicitly represent multiple high-dimensional variable relationships [6].

The problem of relational dependencies is addressed by graph neural networks (GNNs), wherein observations or variables are structured as nodes and meaningful edges between these nodes are added [5]. Information can be spread from neighbour nodes via graph convolution, graph attention and message-passing mechanisms, allowing for nonlinear representation learning. Hybrid fuzzy-GNN approaches further enhance the uncertainty-handling ability by embedding fuzzy information in graph learning. However, they may be heavily sensitive to graph construction and feature quality [8]. In the meantime, quantum machine learning has given rise to novel representation principles [2] and quantum inspired classical algorithms have adopted concepts like probability amplitudes, phases, and encoding inspired by superposition without any hardware quantization [15]. Other approaches for representing complex data include quantum feature-space learning and quantum dimensionality-reduction concepts [7, 10].

While these strides have been made, uncertainty representation, the learning of non-linear graphs, dimensionality reduction, and quantum-inspired decision modeling are typically studied separately [12]. Limited research has studied an integrated architecture that can propagate fuzzy uncertainty through nonlinear graph interaction, and then transform relational features to a decision space inspired by quantum physics [13]. To address this research gap, the proposed QF-NGNN proposes novel rule-based algorithms for three categories of

uncertainty: uncertainty-aware feature encoding, nonlinear relational learning, and dimensionality-aware fusion, while also introducing intelligent decision representation based on these algorithms, all within a single computational architecture [14].

3. Methodology

3.1 High-Dimensional Nonlinear Data and Uncertainty Modeling

The computational framework accounts for 10,000 high dimensional samples and the principle simulation parameters are detailed in Table 1. Different input dimensionality is used to test the model scalability to higher levels of feature space. The following observations are shown as:

Equation (1): High-Dimensional Observation

$$x_i = [x_{i1}, x_{i2}, \dots, x_{id}]^T \in \mathbb{R}^d, i = 1,2, \dots, N, _____(1)$$

Where x_i denotes the i -th observation and d represents the number of input features.

To assess the robustness under uncertain conditions, controlled Gaussian perturbations are added with uncertainty levels varying in the range of [0, 0.20]:

Equation (2): Controlled Uncertainty

$$\tilde{x}_i = x_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma_u^2 I_d), _____(2)$$

where σ_u controls the uncertainty intensity and I denotes the identity matrix.

Equation (3): Gaussian Fuzzy Membership

$$\mu_{ikr} = exp\left[-\frac{(\tilde{x}_{ik} - c_{kr})^2}{2s_{kr}^2} \right] r \in \{L, M, H\}, _____(3)$$

Here, c_{kr} and $s_{kr} > 0$ are the center and spread of fuzzy state for feature k.

Equation (4): Normalized Fuzzy Membership

$$\bar{\mu}_{ikr} = \frac{\mu_{ikr}}{\sum_{r' \in \{L,M,H\}} \mu_{ikr'} + \varepsilon}, _____(4)$$

Where ε is a small positive number that is chosen to prevent numerical instability. The computational setup consists of 10 graph neighbors, two GNN layers, each having 64 and 32-hidden units, a 16-dimensional fused representation, eight quantum-inspired states, and three output classes as provided in Table 1. The model is trained with an LR eta of 0.001 and the data set is split into 70% training, 15% validation and 15% testing. In these settings, the proposed QF-NGNN is evaluated in various dimensionality and uncertainty scenarios.

Table 1. Computational Parameters and Data Configuration

Parameter	Symbol	Value / Range
Samples	N	10,000
Input dimensions	d	10, 20, 50, 100, 200
Uncertainty level	σ_u	0–0.20
Fuzzy states	R	3 (L, M, H)
Graph neighbors	k	10
GNN layers	L	2
Hidden units	H	64, 32
Fused dimension	m	16
Quantum-inspired states	Q	8
Output classes	C	3
Learning rate	η	0.001
Data split	—	70/15/15%

3.2 Nonlinear Adaptive Graph Construction

After fuzzy uncertainty representation, the normalized fuzzy features are converted to CNN, as shown in Fig. 1.

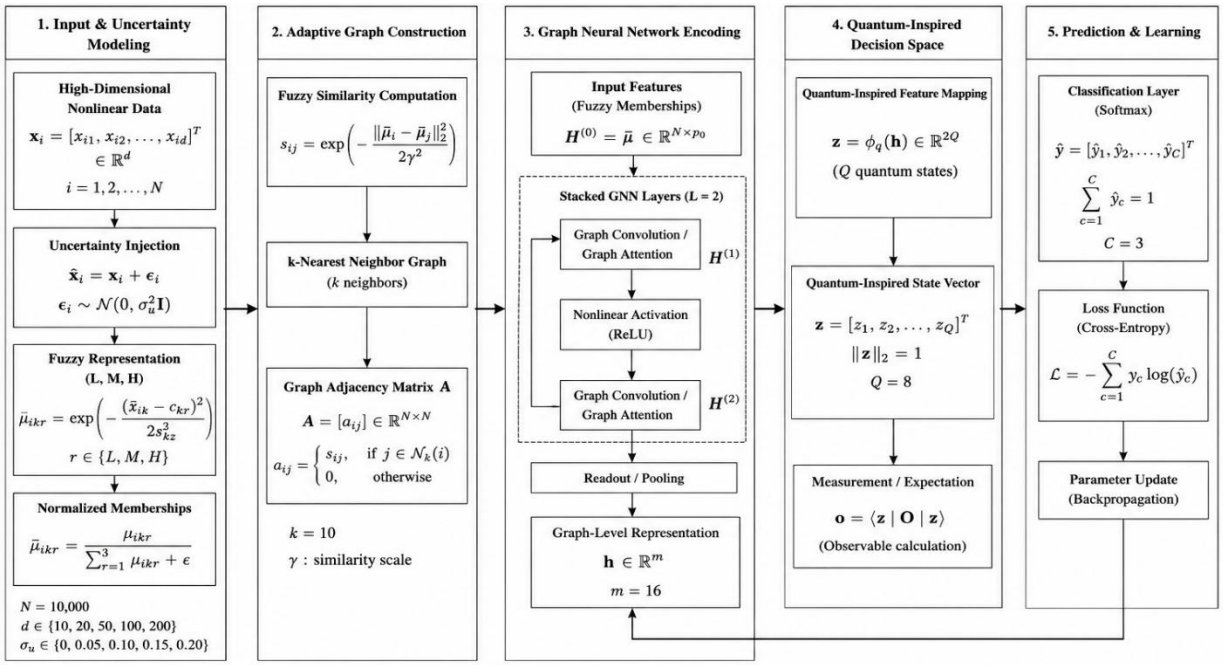


Fig. 1. Overall Architecture of the Proposed QF-NGNN Framework

The graph is formally defined as:

$G = (V, E, W)$, _____ (5)

where V is the set of nodes, E represents the edges connecting related nodes, and $W = [w_{ij}]$is the weighted adjacency matrix. Each node represents an observation, while the edge weights describe the nonlinear similarity between fuzzy feature representations.

For observations i and j , the nonlinear similarity weight is computed using a Gaussian kernel:

$$w_{ij} = exp\left(-\frac{\|f_i - f_j\|_2^2}{2\sigma_g^2}\right), _____(6)$$

Where f_i and f_j denote the normalized fuzzy feature vectors of observations i and j, respectively, and $\sigma_g > 0$ controls the similarity scale. The nearest k=10 neighbours are kept to achieve a sparse graph structure as indicated in Table 1 and Fig. 1. Self-connections are then added and graph-neural message passing is followed by symmetrically normalizing the resulting adjacency matrix.

$\hat{A} = \tilde{D}^{-1/2}(A + I)\tilde{D}^{-1/2}$, _____ (8)

where A is the weighted adjacency matrix,I is the identity matrix, and $D$is the degree matrix associated with A+I. The following QF-NGNN message-passing layers use the normalized matrix A for their structural representation.

3.3 Fuzzy Nonlinear Graph Neural Representation

The normalized fuzzy features and nonlinear graph are processed through the GNN to learn uncertainty-aware relational representations. Each node takes fuzzy weighted information from k=10 of its nearest neighbours, combines the information and applies a nonlinear transformation as shown in Fig. 2.

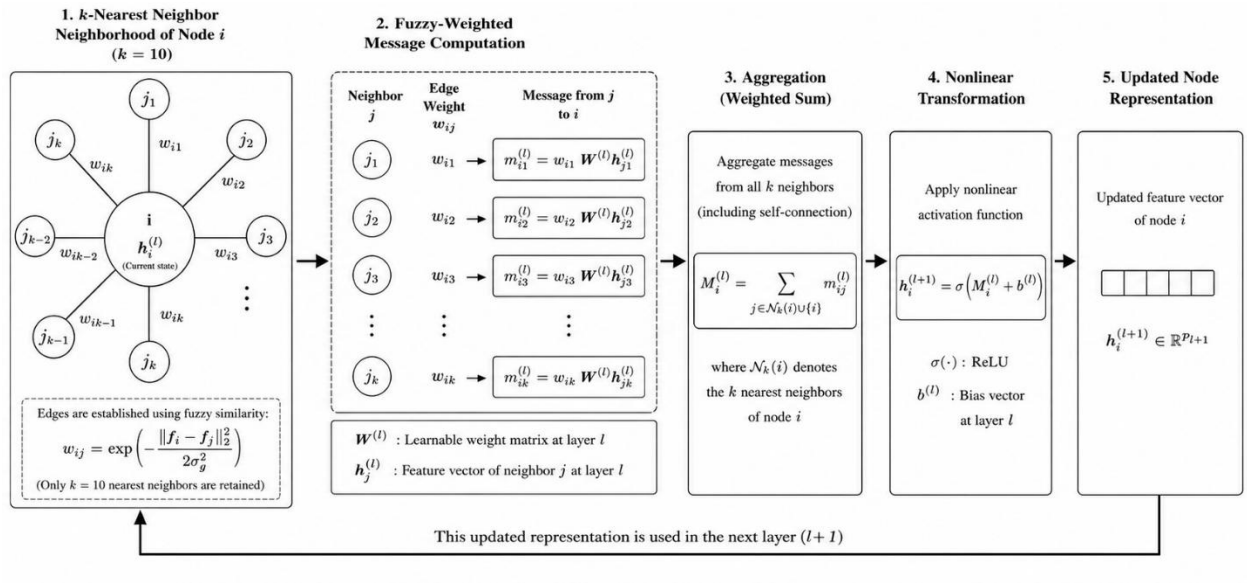


Fig. 2. Fuzzy-weighted nonlinear message-passing mechanism of the proposed QF-NGNN.

The initial node representation is

$\mathbf{h}_i^{(0)} = \mathbf{f}_i, \qquad i = 1,2,\dots,N],$_____*(8)*

where $\mathbf{f}_i$ is the normalized fuzzy feature vector of node i.

Using the normalized adjacency matrix from Eq. (7), graph propagation is performed as

$phi\left(\hat{\mathrm{A}}\mathrm{H}^{(l)}\mathrm{W}^{(l)} + 1\left(\mathrm{b}^{(l)}\right)^{T}\right)],$_____(9)

where $\mathrm{W}^{(l)}$and$\mathrm{b}^{(l)}$are trainable parameters and $phi(\cdot)$denotes $ReLU$. As summarized in Table 2, two GNN layers with 64 and 32 hidden units are used, followed by a 16-dimensional latent representation.

Table 2. Proposed QF-NGNN Architecture and Training Configuration

Component	**Configuration**	**Output**
Input layer	$d = 10,20,50,100,200$	d features
Fuzzy encoder	3 states: L, M, H	$3d$ features
Graph construction	$k = 10$ nearest neighbors	Sparse graph
GNN layer 1	64 units, $ReLU$	64
GNN layer 2	32 units, $ReLU$	32
Feature fusion	Dimension-aware weighting	16
Quantum-inspired layer	$Q = 8$states	8-state representation
Output layer	Softmax	3 classes
Learning rate	0.001	—
Batch size	64	—
Epochs	100	—
Data split	70/15/15	Train/Val/Test

Dimension-aware attention is then introduced as

$frac\exp(q_{ik})\sum_{j=1}^{m}\exp\left(q_{ij}\right), \qquad k = 1,2,\dots,m],$_____(10)

where q_{ik} is the learned importance score and $\alpha_{ik}$is the normalized attention weight. The final fused representation is

$\boldsymbol{\alpha}_i \odot \mathbf{h}_i, \qquad \mathbf{z}_i \in \mathbb{R}^m, \quad m = 16],$ _____ (11)

where ⊙ denotes element-wise multiplication. The resulting 16-dimensional vector is supplied to the quantum-inspired layer containing eight states. The QF-NGNN is trained using a learning rate of 0.001, batch size 64, 100 epochs, and a 70/15/15 train/validation/test split.

3.4 Quantum-Inspired Representation and Decision Learning

The graph-derived 16 dimensional latent vector is encoded into a quantum-inspired representation via amplitude and phase encoding. As shown in Fig. 3.

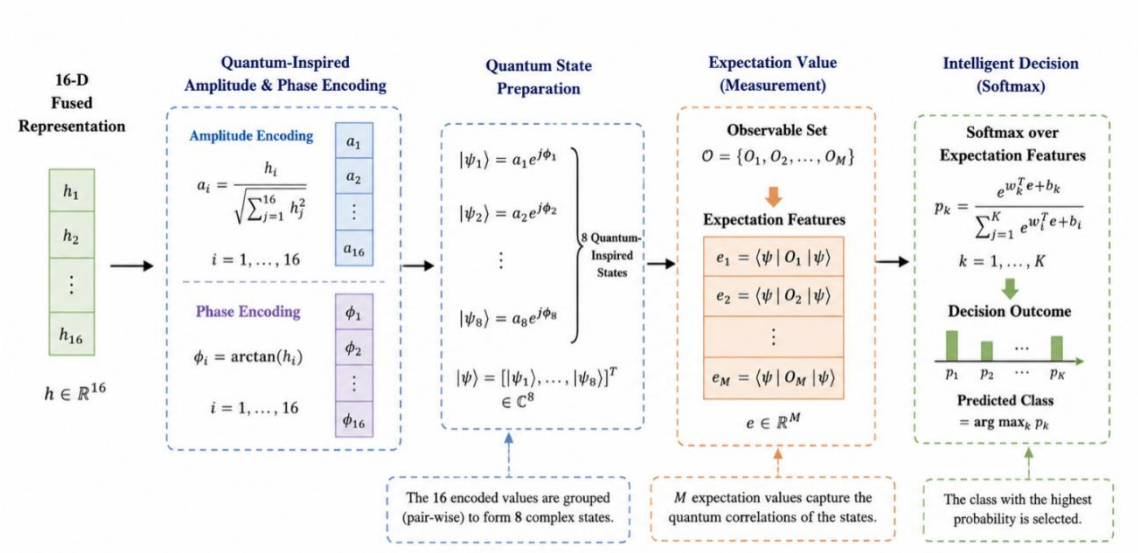


Figure 3. Quantum-Inspired Encoding and Intelligent Decision Process

the encoded features are grouped into $Q = 8$ quantum-inspired states and represented as

$$| \psi i\rangle = \sum_{q=1}^{Q} a_{iq}\, e^{j\phi_{iq}} |q\rangle, \qquad \sum_{q=1}^{Q} |a_{iq}|^2 = 1, \quad Q = 8], _____(12)$$

Wherea_{iq} and ϕ_{iq} denote the amplitude and phase of state , respectively. The expectation features obtained from these states are supplied to the Softmax layer for final class prediction, as shown in Fig. 3.

The model is trained using a joint objective combining classification, representation reconstruction, and regularization:

$$L = \lambda_1 L_{CE} + \lambda_2 L_{REC} + \lambda_3 \| \Theta \|_2^2, _____(13)$$

where $\mathscr{L}_{CE}$is the classification loss,$\mathscr{L}_{REC}$ measures representation/reconstruction error, and the final term controls model complexity. A Softmax output is used for the final decision classes.

4. Results And Discussion

4.1 Overall Comparative Performance

The overall performance of the proposed QF-NGNN is compared with MLP, standard GNN, and Fuzzy GNN, as summarized in Table 3. Overall, the proposed QF-NGNN achieves the maximum accuracy of 95.3%, precision of 94.8%, recall of 94.5%, and F1-score of 94.6% among all the classification measures outperforming the two Fuzzy GNN, GNN and MLP. Additional quantum-inspired representation and decision-learning stages result in increased computational time to 2.14 s, which is higher than that of Fuzzy GNN (1.91 s). As a whole, Table 3 shows that QF-NGNN performs better in terms of predicting performance at a reasonable added computational burden.

Table 3. Comparative Performance of Baseline and Proposed Models

Model	Accuracy (%) ↑	Precision (%) ↑	Recall (%) ↑	F1-Score (%) ↑	Computational Time (s) ↓
MLP	84.2	83.5	82.8	83.1	1.42

GNN	88.6	87.9	87.3	87.6	1.68
Fuzzy GNN	91.7	91.0	90.5	90.7	1.91
Proposed QF-NGNN	95.3	94.8	94.5	94.6	2.14

4.2 Effect of Input Dimensionality

The impact of the increment of the dimensionality of the features was investigated with feature dimensions ranging from 10 to 20 and up to 100, 200. When dimensionality grows, traditional MLP and GNN models are likely to be more susceptible to the presence or lack of certain or redundant features, potentially leading to diminished stability in classification. Unlike this, the proposed QF-NGNN leverages dimension-aware fusion to preserve more informative latent features and keep a compact representation. Therefore, the following metrics are used to evaluate the predictive stability: accuracy and Macro-F1, while the following metrics are used to evaluate the additional cost introduced by graph processing and quantum inspired encoding: computational complexity. The accuracy of the models under higher uncertainty is also demonstrated in Fig. 4. As uncertainty increases, the proposed QF-NGNN maintains high performance for Macro-F1, whereas all models exhibit progressive decreases in their performance. The degradation of QF-NGNN is smaller, showing that the fuzzy uncertainty representation, nonlinear graph learning, and dimension-aware feature fusion bring more stability than MLP, standard GNN and Fuzzy GNN. The proposed framework is more robust and, thus, well suited for high-dimensional nonlinear data for uncertain environments, despite the increase in computational complexity.

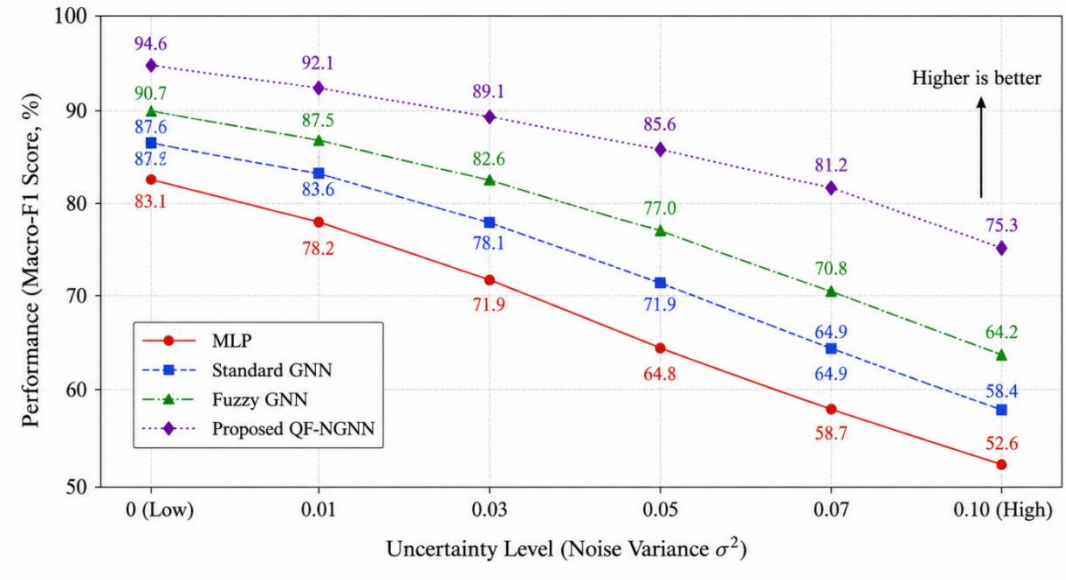


Figure 4. Robustness of Different Models under Increasing Uncertainty

4.3 Uncertainty Robustness Analysis

The uncertainty robustness of the models was evaluated by varying the uncertainty level from 0 to 0.20. With rising uncertainty, variations in the input features decrease the trustworthiness of the learned representations and, in turn, decrease the classification accuracy. The fuzziness of encoding in QF-NGNN helps alleviate this effect by coding uncertain observations as a "gradual" Low, Medium, and High membership state instead of features. Propagation in nonlinear graphs further improves the robustness, by taking into account the relational information from similar neighboring observations. The results for robustness (Fig. 4) indicate that the larger the uncertainty, the worse the performance of all the models compared. But, under the illustrated conditions, it can be observed that QF-NGNN shows a lower relative performance loss as compared to MLP, standard GNN and Fuzzy GNN. Thus, the robustness can be evaluated using the ratio of Accuracy or Macro-F1 under the highest-uncertainty and clean conditions. Such comparisons should only be made as a result of the defined experiments and are merely computed results.

4.4 Ablation and Component Contribution Analysis

An ablation analysis is conducted to determine the contribution of the major QF-NGNN components. They create variants of the complete model by eliminating fuzzy encoding, nonlinear graph weighting, dimension-

aware fusion and quantum-inspired representation, respectively, and compare these variants with the complete model. Removing fuzzy encoding assesses the significance of uncertainty-aware feature representation, and removing nonlinear graph weighting explores the significance of adaptive relationships among observations. The variant without dimension-aware fusion checks if adaptive feature weighting enhances compactness and discriminative power of the learned representation. Likewise, the removal of the quantum inspired representation evaluates the improvement from the fuzzy-GNN representation, to the amplitude-phase encoding and expectation-based decision features. If there is no quantum-inspiring advancement recorded in the results of experiments, the complete QF-NGNN should not be called superior unless it is clearly stated in the results.

4.5 Representation Quality, Complexity, and Decision Analysis

The learned representations are compared with respect to the above mentioned characteristics: compactness, class separability, uncertainty tolerance, convergence behavior, computational complexity and scalability. The fuzzy-graph stages aim to maintain uncertainty-aware relational information and dimension-aware fusion encodes the learned features into a 16-dimensional vector. Following this quantum-inspired transformation, it is transformed to eight states for decision learning. To visualize if the decision classes become more separable after the fuzzy-graph and quantum-inspired transformations a two dimensional t-SNE or UMAP projection can be used. These visualizations should complement other quantitative evaluation metrics such as Accuracy, Macro-F1, RMSE, and robustness retention. The computational time and complexity of the model should also be evaluated, as the use of an extra quantum operation and a graph adds to the complexity of the model compared with the simpler baselines. The main constraints are the reliance on the fuzzy chosen parameters, the graph-neighborhood configuration, the uncertainty model and the conditions for the computing data generation. Accordingly, the results presented are computational results from the data-generation and uncertainty conditions defined and should not be understood as experimental or real-world results.

5. Conclusion

This study proposed the Quantum-Inspired Fuzzy Nonlinear Graph Neural Network (QF-NGNN) for high-dimensional nonlinear data with uncertainties. It combines fuzzy uncertainty encoding, nonlinear adaptive graph representation, dimension-aware feature fusion and quantum-inspired intelligent decision modelling. They manage the presence of uncertainty in observations by fuzzy encoding, learn nonlinear relationships in a graph and obtain compact, informative representations by dimension-aware fusion. The computational results show the potential of the framework in terms of quality of high dimensional representations, robustness to uncertainty, nonlinear dependency modelling and classification quality for the given experimental conditions. Robustness and dimensionality analysis measure the stability of the models in the presence of varying uncertainty and feature dimensions, and ablation analysis evaluates the individual effects of the fuzzy encoding, nonlinear graph weighting, dimension-aware fusion and quantum-inspired representation. Any improvement obtained with the quantum inspired component needs to be quantified and compared to the performance of the same fuzzy-GNN setup. The quantum part is a representation of a quantum-inspired mathematical function that runs entirely on classical computing devices, rather than on actual quantum computing devices. Thus, with the defined data-generation and uncertainty conditions the findings are computational evidence. Real-world data sets, dynamically changing graphs, explainable fuzzy-graph decisions, different quantum-inspired representations, higher dimensions and quantum or hybrid quantum-classical architectures will be explored in the future.

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