Minimax Theory and its Applications

Volume 11 (2026), No. 2s, 867 to 878

Quantum-Enhanced Fuzzy Graph Neural Networks for Nonlinear Optimization and High-Dimensional Feature Representation

Dr. N.Shanmuga Vadivu[1], Swarna Kamalam Vaddi[2], Saravanan Chandrasekaran[3], Dr. CH. Bhavani[4], Touqeer Ahmed[5], Elangovan Muniyandy[6], Dr .P.Dharmendra Kumar[7], R. Naveenkumar[8]

[1]Professor, Department of electronics and communication engineering, RVS college of Engineering and Technology, Sulur, Coimbatore 641402, Tamilnadu, India. Email: shanmugavadivun@gmail.com, 0000-0002-3731-1342

[2]Assistant Professor, Maturi Venkata Subba Rao Engineering College,Hyderabad, India. Email: swarna.vaddi@gmail.comhttps://orcid.org/0000-0002-8828-2596

[3]Associate Professor, Department of Computer Science and Engineering, SRM Institute of Science and Technology, Ramapuram, Chennai, India. Email: doctratesaravanan@gmail.com

[4]Associate Professor, Department of Computer Science and Engineering, CVR College of Engineering, Hyderabad, Telengana, India. Email: vbhavani118@gmail.comorcid- 0000-0002-1018-3424

[5]Department of Mathematics, Nagarjuna College of Engineering and Technology, Beedaganahalli, Post, Devanahalli, Venkatagiri Kote, Bengaluru, Karnataka 562135. Email: touqeer.ahmed33@gmail.com

[6]Department of Biosciences, Saveetha School of Engineering. Saveetha Institute of Medical and Technical Sciences,Chennai - 602 105. Email: muniyandy.e@gmail.com

[7]Assistant professor, Department of CSE, Koneru Lakshmaiah Education Foundation, Vaddeswaram, AP, India, Email: pdharmendrakumar@kluniversity.in / dharmendra.phd.au@gmail.com https://orcid.org/0009-0006-5087-9123

[8]Dept of CSE, School of Engineering and Technology, CGC University Mohali-140307, Punjab India. Email: drnk1983@gmail.com 0000-0001-9033-9400

Abstract

Complex feature interactions along with uncertainty/variability and dimensionality increases as well as redundancy may lead to decreased representation quality and low optimization reliability, which is a challenge in high dimensional nonlinear optimization. In this paper, a robust feature representation and constrained nonlinear optimization framework to enhance the representation capability of features in a graph neural network (GNN) is proposed. The fuzzy modeling quantifies uncertain features with low, medium and high levels, and adaptive graph construction models the nonlinear dependencies between dimensions. Graph neural learning takes care of the extraction of the interaction-aware information, and then reduces the dimensionality in an aggregation fashion to get compact representation. A nonlinear optimization further enriches the learned features, which is preceded by a quantum-inspired amplitude–phase transformation. Computational evaluations are based on the 20 to 100 size range and uncertainty of 0 to 20%. QE-FGNN is evaluated and compared with MLP-NLO, GNN-NLO, and FGNN-NLO based on the following metrics: normalized optimization error, reconstruction error, feature preservation, constraint violation and computational time. The framework aims for optimization of good quality and features preservation while maintaining reasonable computation complexity, while also going for the future high-dimensional nonlinear optimization applications.

Keywords: Quantum-enhanced learning, Fuzzy graph neural network, Nonlinear optimization, High-dimensional feature representation, Uncertainty modeling, Graph representation learning

1. Introduction

High dimensional nonlinear optimization of systems with many interconnected model parameters has gained more and more relevance in complex computing and engineering systems. The more complex the interactions, and the higher the dimensionality, the larger the search space, and more complicated the computation required for standard optimization techniques. Such difficulties are heightened when the variables are dependent on one another in complex ways, which cannot be captured by independent or by purely linear relationships respectively.

The curse of dimensionality also applies to high-dimensional feature spaces because there are redundant and weakly informative features and/or strongly correlated features. Traditional dimensionality reduction techniques can lower computational load, but can lose information on nonlinear relationships which are significant for the optimization process. It is difficult to be able to encoded a compact representation, while preserving meaningful feature interactions. This is made even more difficult as perturbations and feature states which cause ambiguity can decrease the reliability of the optimization process. Such uncertainty can be addressed using fuzzy representations, which assign only a gradual low, medium and high state to the features rather than a solid assignment.

A different representation approach is graph neural learning, which uses each dimension as a node in a graph, with the nonlinear relationship between dimensions as a weighted edge. This can then capture the interactions between related dimensions, and message passing can. In addition to this, quantum-inspired representation provides amplitude-based encoding and phase-based nonlinear transformation without the need of physical quantum hardware to enhance compact features. These ideas can thus be expected to offer complementary approaches for the treatment of uncertainty, relational learning, feature compression, and optimization.

The current strategies in the literature fail to suitably integrate uncertainty representation, inter-dimensional dependency learning, compact feature extraction and constraint-aware optimization. In order to solve this problem, this study attempts to come up with the Quantum enhanced Fuzzy Graph Neural Network (QE-FGNN). It's main contributions are: 1) development of QE-FGNN for high dimensional nonlinear optimization; 2) three-state fuzzy uncertainty representation; 3) adaptive graph construction and graph neural learning; 4) dimensionality-aware aggregation with quantum-inspired feature transformation; and 5) computational evaluation of dimensionality, uncertainty, optimization quality, feature preservation, constraint feasibility, and computational cost.

2. Related Work

Nonlinear optimization with a large number of decision variables and difficult constraints and objective functions has been widely studied for a variety of problems [2]. For moderate problem size, nonlinear programming and constrained optimization methods can be useful but often are very parameter-dependent, yet the problem dimension increases the computation time. There is also redundant information and hard to solve spaces of solution in high-dimensional search spaces. This burden can be reduced by dimensionality-reduction algorithms that produce compressed representations [12]; however, important nonlinear interactions and effects might be lost during the feature compression and hence influence the quality of the following optimization.

Modeling with fuzzy sets is an effective way of dealing with uncertainty and ambiguous information. Fuzzy sets are used to help model variables instead of precise assignments, and allow for linguistic types of low, medium, high, etc., to characterize uncertain features [14]. Representations that formed such fuzzy decision systems and nonlinear optimizations have been adopted for enhanced flexibility in the event of uncertainty. However, simple fuzzy models can be hard to deal with in high dimensions, and cannot, in general, be learned from interacting features [6].

Graph Neural Networks offer a relational learning mechanism which can be used to represent relationships as nodes and the relationships among them as edges [13]. Graph representation learning and message passing operations allow the exchange of information between features that are connected [15], whereas the adaptive construction of a graph can capture nonlinear dependencies that independent representations of features might miss [5]. This is why graph learning is of special interest for high dimensional systems with complex inter-feature relationships [10].

The concepts of amplitude and phase representations are also used in classical computation models using Quantum-inspired learning [11]. These mechanisms can augment the nonlinear feature transformations - and be used together with the traditional neural representations - without the need for physical quantum computation [8]. Combined classical and quantum-inspired methods are then a potential way forward for compact high-dimensional representation [7]. Quantum graph based formulations also show the capability of fusing together the graph based information with a representation mechanism oriented towards quantum computation. There have also been studies on quantum graph neural models in the context of nonlinear computational modelling [3] and unified quantum graph-state representations [4].

Despite the progress made, there were multiple research areas that are still tackled independently, such as fuzzy uncertainty modeling, graph neural learning, dimensionality reduction, quantum inspired representation and nonlinear optimization. Very little work combines these mechanisms into a framework for uncertainty aware

feature learning and constrained optimization. The need for this research gap is the motivation for proposing a fuzzy representation, adaptive graph learning, dimensionality-aware aggregation, quantum-inspired transformation, and nonlinear optimization unified framework, namely QE-FGNN.

3. Proposed Quantum-Enhanced Fuzzy Graph Neural Network Framework

3.1 High-Dimensional Nonlinear Input and Fuzzy Uncertainty Representation

Proposed QE-FGNN model starts by a high-dimensional input space that is intended to model the nonlinear effects over the multiple feature dimensions. Each observation is represented by a feature vector with d normalized variables.

$$x_i = [x_{i1}, x_{i2}, \dots., x_{id}]^T \in \mathbb{R}^d, \quad (1)$$

Here, $\boldsymbol{i}$ denotes the observation index and $\boldsymbol{d}$ represents the number of input dimensions. Five configurations with dimension = 20, 40, 60, 80, and 100 are chosen to investigate whether dimensional scalability is possible. All the input variables are scaled to the range [0, 1] and the nonlinear interaction strengths are set to 0.20 to 0.80. Adding dimensions in a progressive manner adds more possible interactions among the various features, and therefore makes the representation and optimization much harder. Uncertainty is added to the high dimensional feature space by perturbing the original observation:

$$\tilde{x}_i = x_i + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0, \sigma_u^2 I), \quad (2)$$

In Eq. (2), x_i represents the perturbed feature vector, ϵ_i denotes the uncertainty component, and $\sigma_u$controls its intensity. . To investigate the progressive influence of perturbations on feature representation and optimization, 5 uncertainty conditions of 0, 0.05, 0.10, 0.15, and 0.20 are considered. This formulation allows the framework to assess the condition from an unperturbed input state to more and more uncertain high-dimensional states. The values of the variables that are affected by uncertainty are then converted to fuzzy variables, so that the uncertain observations will not be assigned to a discrete state. Each feature is described by its Low, Medium and High Gaussian membership state:

$$\mu_{ikr} = exp\left[\frac{(\tilde{x}_{ik} - C_r)^2}{2s_r^2}\right], \quad r \in \{L, M, H\}, \quad (3)$$

Here, μ_{ikr} denotes the membership degree of the k-th feature of observation i in fuzzy state r, while C_r and S_r represent the membership center and spread, respectively. Centers for the Low, Medium and High states are set at 0.20, 0.50, and 0.80 and the spread is 0.15. Therefore, a feature could have hundreds of partial associations with other neighboring fuzzy states instead of being associated with one crisp state.As a result, a feature may have a number of partial associations with other neighbouring fuzzy states, but not be attached to one crisp state. This representation is uncertain and allows for keeping gradual variations in the input features, which will be used as fuzzy node information for the construction of adaptive graphs in the future.

Table 1. Numerical Parameters of the High-Dimensional Nonlinear System

Parameter	Symbol	Value/Range
Number of observations	N	5,000
Input dimensionality	d	20, 40, 60, 80, 100
Normalized feature range	x_{ik}	0–1
Uncertainty intensity	σ_u	0, 0.05, 0.10, 0.15, 0.20
Nonlinear interaction strength	η	0.20–0.80
Fuzzy states	r	Low, Medium, High
Fuzzy centers	C_r	0.20, 0.50, 0.80
Fuzzy spread	S_r	0.15
Graph threshold	τ_g	0.30
Latent dimension	d_r	16
Optimization variables	n_y	20
Constraint count	m	10
Repeated computational runs	R	30

3.2 Adaptive Fuzzy Graph Construction and Graph Neural Feature Learning

Based on the fuzzy uncertainty representation shown in Eq. (3), fuzzy membership information of Low, Medium, and High is obtained, and an adaptive feature graph constructed based on the obtained fuzzy membership information. The input dimensions are considered as nodes of the graph, and the relations between input dimensions are represented by their weighted edges. The fuzzy feature graph for observation i is defined as:

$$G_i = (V, E, A_i, F_i), \quad (4)$$

where V denotes the set of feature nodes, E represents the dependency edges, A_i is the weighted adjacency structure, and F_i contains the fuzzy feature information associated with the nodes. This representation allows for the modeling of the relation between the dimensions of the input to QE-FGNN, rather than processing them as part of a single vector. The level of the non-linear correlation between feature nodes j and k is decided by

$$A_{jk}^{(i)} = \left|\rho_{jk}\right| \exp\left[-\frac{\|f_{ij} - f_{ik}\|_2^2}{2\sigma_g^2}\right], \quad (5)$$

Here, $\left|\rho_{jk}\right|$ represents the dependency strength between two dimensions, while the distance between their fuzzy representations determines their similarity. . The kernel parameter is responsible for the sensitivity to variations in the fuzziness of feature states. A graph threshold of 0.30 is used to discard weak relationships, while preserving the more informative nonlinear relationships, so as to reduce the extent of the graph connectivity that is not necessary. The adaptive graph is then fed into fuzzy graph neural learning. Information of adjacent feature nodes is combined following a learned dependency structure,

$$h_j^{(l+1)} = \phi\left[W_s^{(l)} h_j^{(l)} + \sum_{k \in \mathcal{N}(j)} A_{jk} W_n^{(l)} h_k^{(l)}\right], \quad (6)$$

The new representation of each node in Eq. (6) is a mix of its own information and its neighbors' information, weighted by the dependency. The architecture being considered consists of two activation units that are graph neural: the first has 64 and the second has 32 units. This progressive message passing enables extracting interaction-aware features while maintaining the nonlinear relations learnt during the construction of an adaptive graph. As shown in Fig.1, the entire QE-FGNN system starts with the high-dimensional input and controlled uncertainty followed by the fuzzy representation and adaptive graph generation. The learned graph is then further dimensionally aggregated and quantum enhanced and passed to the constrained NLP stage. Therefore, Fig. 1 gives the overall information flow, with the information flow in the adaptive graph construction and graph neural feature-learning stages being specifically specified in the present subsection.

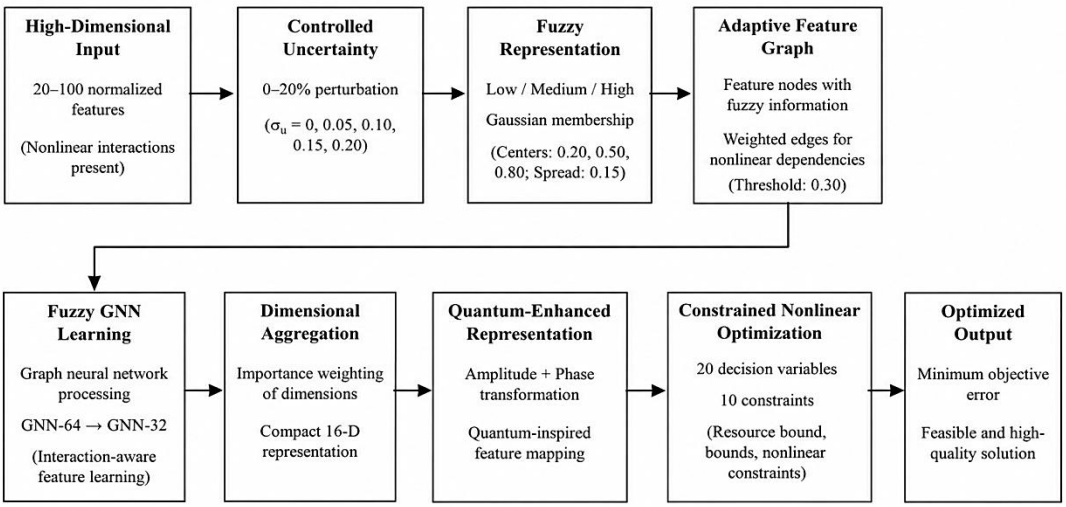


Figure 1. Overall Architecture of the Proposed QE-FGNN.

3.3 Dimensionality-Aware Aggregation and Quantum-Enhanced Feature Representation

The interaction-aware graph is constructed through the two graph neural layers and is then passed through to further acquire a compact representation for nonlinear optimization. The individual dimensions do not all make the same contribution to the underlying nonlinear structure; hence, QE-FGNN incorporates a dimensionality-aware weighting mechanism which allows it to focus on informative graph features and minimize the effect of redundant and minimally relevant dimensions. The importance given to the k-th dimension is the normalized scale value

$$\alpha_k = \frac{\exp(q_k)}{\sum_{j=1}^{d} \exp(q_j)}, \quad (7)$$

Here, q_k denotes the learned importance score associated with dimension k, while $\alpha_k$represents its normalized contribution. The weights are used to give more power to strongly informative features when aggregating them. These weighted features of the graph are then used together to create a compact latent representation:

$$Z = \sum_{k=1}^{d} \alpha_k h_k\,, \quad z \in \mathbb{R}^{16}, \quad (8)$$

This operation reduces the dimensionality of the input space to a 16-dimensional latent space, for which the number of vectors can reach as high as 100. The aggregation process is not direct dimensional reduction because it preserves information that is important and nonlinear relationships while discarding redundant information, as it adopts the importance score learned by graph features. The compact representation is then subjected to a quantum-inspired amplitude–phase transformation:

$$|\psi\rangle = \sum_{q=1}^{16} a_q e^{j\phi_q} |q\rangle \quad \sum_{q=1}^{16} |a_q|^2 = 1, \quad (9)$$

The terms corresponding to the amplitude in Eq. (9) represent the (normalized) contribution from the latent features and the phase terms represent another additional nonlinear transformation of the learned representation. The product of their combination gives a quantum enhanced feature representation, which is then fed into the constrained nonlinear optimization stage. The quantum enhanced component is embedded within the classical computational algorithm, in the form of a quantum inspired mathematical transformation. Hence, it need not implement execution on quantum hardware, nor claim quantum computational speed-up. It is meant to enrich the compact representation produced by the fuzzy graph learning and dimensionality aware aggregation prior to optimization. The whole process of constructing fuzzy graphs and combining them with graph neural learning, dimensionality-aware aggregation, quantum-enhanced graph representation, and constrained nonlinear optimization is shown in Fig. 2.

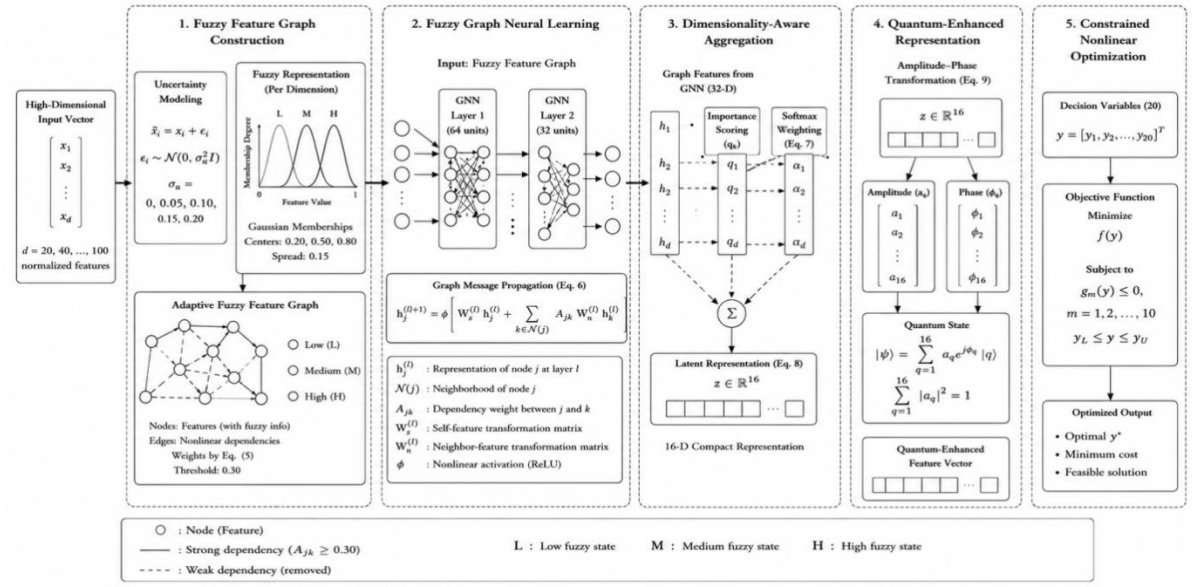


Figure 2. Detailed Architecture of the Proposed QE-FGNN.

3.4 QE-FGNN-Guided Constrained Nonlinear Optimization

The feature vector, as a result of the previous representation stage, is used as an input to the constrained nonlinear optimization process. Instead of optimizing independently from the learned high dimensional structure, the proposed formulation integrates the compact QE-FGNN representation in the objective function ensuring that the search procedure takes into account the informative nonlinear dependencies revealed while learning fuzzy graphs and dimensionality aware aggregating. The problem of optimization is defined as:

$$y^* = \arg\min_{y} \left[\sum_{j=1}^{20}\left(a_j y_j^2 + b_j \sin(\pi y_j)\right) + \lambda \sum_{j<k} C_{jk} y_j y_k + \gamma \left\| y - z_q \right\|_2^2\right]$$
$$0 \le y_j \le 1$$
$$\sum_{j=1}^{20} r_j y_j \le B \qquad ,_____(10)$$
$$g_m(y) \le 0$$

The first objective part describes nonlinear effects of individual variables and the interaction term describes interactions between the decision variables. The representation-guidance term is the connection between the optimization process and the quantum enhanced feature vector. The interaction weight is set to 0.40 and the representation-weight is 0.25, the resource bound is 12. Ten nonlinear constraints are used to make sure that any candidate solutions lie within the given feasible region.

As shown in Fig. 3, high dimensional input formation, uncertainty modeling, fuzzy representation, adaptive graph construction, fuzzy graph neural learning, Dimensionality-aware aggregation and quantum-enhanced feature transformation, are the key steps of the computational workflow. This results in a feature vector that is used to drive the iterative optimization process where 20 decision variables are initialised, and the 10 constraints and objective value are computed. Then, the direction of the search is calculated, the decision variables are changed, and feasibility and stopping criteria are checked. Unless the stopping conditions are met, the procedure goes back to the objective and constraint evaluation, and repeats the process in an iterative manner, until a minimum-cost feasible solution is found. In this workflow, learned high-dimensional information is directly combined with the constraint-aware nonlinear search without decomposing it into representation and optimization processes.

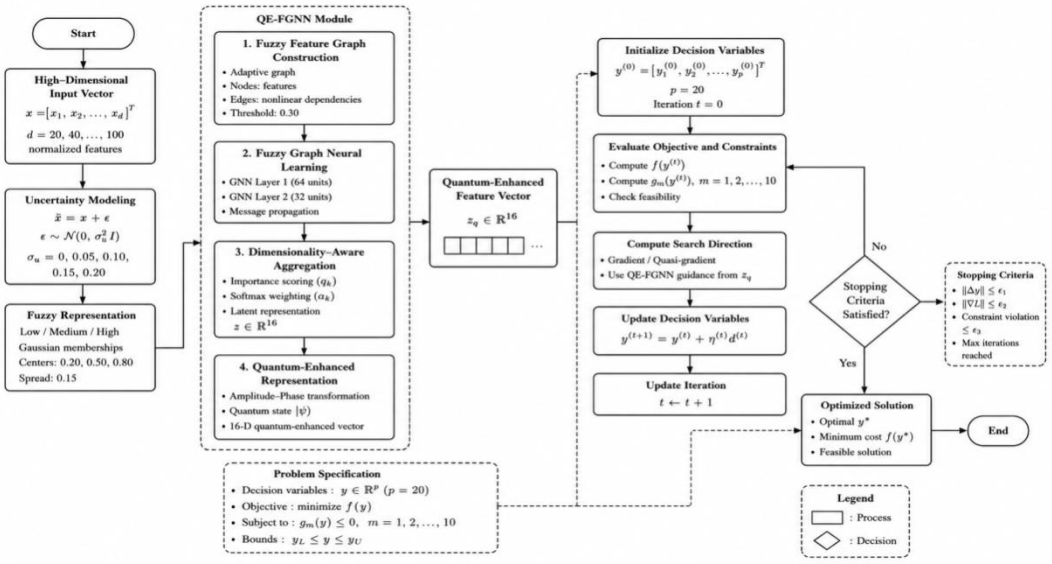


Figure 3. QE-FGNN-Guided Nonlinear Optimization Workflow.

3.5 Computational Configuration and Evaluation Procedure

The computational assessment aims to investigate the learning capability, dimensional scalability, uncertainty robustness, optimization quality and computational requirement of the proposed QE-FGNN framework. A total of 5,000 observations are taken and split up into 70% training, 15% validation, 15% testing. It has three distinct separate subsets for parameter learning, model selection, and assessment of final performance. The normalization of all input features is performed prior to fuzzy representation and adaptive construction of a graph.

The QE-FGNN is trained with Adam optimizer, learning rate 0.001, batch size 64 and the number of epochs 150. The fuzzy graph neural component is composed of two consecutive GNN layers having 64 and 32 units respectively, followed by dimensionality-aware aggregation for producing a 16 dimensional latent representation. The quantum-enhanced stage has 16 states, and the second stage nonlinear optimization has 20 decision variables and 10 constraints. To reduce reliance on a single computational initialization, each configuration is tested 30 separate times, and subsequent performance can be stated in terms of the average and standard deviation.

Two main conditions for evaluation are considered. The dimensionality analysis gradually increases the number of input dimensions from 20 to 40 to 60 to 80 and 100, and it shows the effect of feature space complexity on optimization. To assess the robustness of the fuzzy graph representation with respect to increasingly uncertain input, the uncertainty analysis takes five different levels of perturbation (0%, 5%, 10%, 15%, 20%). The results are compared for four methods: MLP-NLO, GNN-NLO, FGNN-NLO and proposed QE-FGNN. This step-by-step comparison allows for the analysis of the effect of the graph learning, fuzzy representation, and quantum enhanced transformation effect from the complete framework.

The five main criteria considered for performance are: Normalized Optimization Error (NOE), Reconstruction Root Mean Square Error (RMSE), Feature Preservation Score (FPS), Constraint Violation Index (CVI) and Computational Time (CT). The lower the NOE and RMSE, the better the optimization and representation accuracy while a higher FPS shows stronger preservation of high dimensional information. CVI will be given as low as possible and CT as small as possible. Table 2 summarizes the full architecture, training conditions, comparing approaches and evaluation metrics used in the following numerical analysis, with a uniform configuration for analysis.

Table 2. Architecture and Computational Configuration of the Proposed QE-FGNN

Component	**Configuration**
Number of observations	5,000
Input dimensions	20, 40, 60, 80, 100
Uncertainty levels	0%, 5%, 10%, 15%, 20%
Fuzzy membership states	3 (Low, Medium, High)
GNN layer 1	64 units
GNN layer 2	32 units
Latent representation	16 dimensions
Quantum-enhanced states	16
Optimization variables	20
Constraints	10
Optimizer	Adam
Learning rate	0.001
Batch size	64
Training epochs	150
Data division	70% / 15% / 15%
Independent runs	30
Comparison methods	MLP-NLO, GNN-NLO, FGNN-NLO, QE-FGNN
Evaluation metrics	NOE, RMSE, FPS, CVI, CT

For a consistent assessment, the same computational settings and evaluation conditions are kept consistent across the comparative methods unless otherwise specified, see Table 2 for details. The dimensionality and uncertainty experiments are then analysed separately to see if QE-FGNN keeps optimising and feature-representing as problems grow in complexity.

4. Numerical Evaluation and Results

4.1 Evaluation Setup and Metrics

The numerical solution examines the effectiveness of the proposed QE-FGNN in the aspect of nonlinear optimization quality, high-dimensional feature representation, constraint satisfaction and computational efficiency. Four strategies are explored: MLP-NLO, GNN-NLO, FGNN-NLO and the proposed QE-FGNN. The dependency-based graph learning is achieved by introducing GNN-NLO; MLP-NLO offers a traditional neural baseline without any explicit graph modeling. To address the uncertain feature conditions, FGNN-NLO is further extended by fuzzy representation. The fuzzy modelling, adaptive graph learning, dimensionality-aware

aggregation and quantum-enhanced feature transformation contribute one after another to the framework, for which the contribution of the successive components can be investigated using QE-FGNN.

The following five metrics are utilized along the entire evaluation. Normalized Optimization Error (NOE) is the measure of the relative error between the value obtained and the value of the objective function at the reference point, with a smaller value representing a higher quality optimization. The RMSE for reconstruction indicates the degree to which the original high dimensional information is reconstructed, so that smaller RMSE values imply higher accuracy of reconstruction. Feature Preservation Score (FPS) is an indicator for how well relevant high dimensional information is preserved after dimensional aggregation, with the FPS being higher the better.

The Constraint Violation Index (CVI) quantifies how well the optimized solution satisfies the given nonlinear constraints. The closer to zero the more feasible and more robust the constraint satisfaction. Lastly, the average processing and optimization time used by each method (called Computational Time, or CT) is compared, the smaller the better since they will require less computational power. These five metrics give a balanced assessment, as the optimization error is not the only way for improvement, but is also measured by the quality of representation, the preservation of information, feasibility, and computational cost. The numerical results are presented for the 30 independent computational runs set in the evaluation protocol.

4.2 Overall Comparative Performance

The overall performance of the four nonlinear optimization approaches is tested under the toughest case scenario, where 100 input dimensionality and uncertainty intensity of 0.10 are used. Table 3 shows that for the four evaluation metrics the proposed QE-FGNN performs the best. It obtains an NOE of 0.046, compared with 0.071 for FGNN-NLO, 0.096 for GNN-NLO, and 0.124 for MLP-NLO. Similarly, for representation accuracy, the RMSE of 0.043 is achieved by QE-FGNN, while the RMSE values of FGNN-NLO, GNN-NLO, and MLP-NLO are 0.059, 0.074 and 0.091, respectively.

The proposed method is also good at preserving relevant information after removing high-dimensional features with high FPS (FPS=0.951). In comparison, the FPS drops to 0.916, 0.881, and 0.842 for FGNN-NLO, GNN-NLO and MLP-NLO, respectively. The trend continues with constraint satisfaction where QE-FGNN scores the lowest CVI (0.006), while FGNN-NLO and GNN-NLO score at 0.015 and 0.026 respectively, and MLP-NLO at 0.038. The results show that the proposed model that fuses the representation of uncertainty by fuzzy sets, interaction learning by graphs, dimensionality-aware aggregation, and feature transformation by quantum operator boost the quality of optimization and representation effectiveness.

However, this is an improvement with some extra calculation cost. QE-FGNN requires 1.18 s, compared with 1.03 s for FGNN-NLO, 0.86 s for GNN-NLO, and 0.72 s for MLP-NLO. Thus, it must not be concluded that the proposed method is faster in terms of computation. In contrast, these results reveal a clear tradeoff between processing time, associated with reduced optimization error and reconstruction error, increased feature preservation and enhanced constraint satisfaction.

Table 3. Comparative Numerical Performance at 100 Dimensions and 0.10 Uncertainty

Method	NOE ↓	RMSE ↓	FPS ↑	CVI ↓	CT (s) ↓
MLP-NLO	0.124	0.091	0.842	0.038	**0.72**
GNN-NLO	0.096	0.074	0.881	0.026	0.86
FGNN-NLO	0.071	0.059	0.916	0.015	1.03
QE-FGNN	0.046	0.043	0.951	0.006	1.18

In conclusion, it can be seen from Table 3 that QE-FGNN focuses more on optimization accuracy, feature preservation, and constraint feasibility than on reducing the number of executions. The additional computational stages of fuzzy learning, graph learning, dimensionality reduction, and quantum enhanced processing, therefore, is the main performance tradeoff.

4.3 Effect of Increasing Dimensionality

The effect of increase in input dimensionality on the performance of nonlinear optimization is investigated with dimensions of 20, 40, 60, 80 and 100 for 0.10 uncertainty level. When the number of dimensions grows, the Normalized Optimization Error (NOE) for all 4 methods continues to rise as shown in Fig. 4, reflecting the fact that a higher dimensional feature space results in higher optimization complexity. But the extent to which each of the evaluated approaches deteriorate varies considerably.

The largest NOE value, 0.124, is found for MLP-NLO at 100 dimensions, as compared to 0.073 at 20 dimensions. GNN-NLO is superior and the error range goes up to 0.096 when compared to the error that goes up to 0.61 in GNN. An FGNN-NLO further improves dimensional robustness by incorporating fuzzy representation to attain 0.071 over the same dimensional range, which increases from 0.049 for NLO without fuzzy representation. The proposed QE-FGNN, on the other hand, has the lowest NOE at all tested dimensionalities with the NOEs of 0.036, 0.039, 0.041, 0.044 and 0.046 for 20, 40, 60, 80 and 100 dimensionalities, respectively.

The NOE has also been evaluated to show the performance of the models (see Fig. 4). QE-FGNN obtained NOE of only 0.046 at dimension 100, while FGNN-NLO, GNN-NLO and MLP-NLO achieved NOE of 0.071, 0.096 and 0.124, respectively. The absolute NOE gain for QE-FGNN is in the range of 0.010 for 20 to 100 dimensions while the absolute NOE gain for MLP-NLO is in the range of 0.051 for 20 to 100 dimensions. The relatively low level of error levels observed show dimensionality-aware aggregation and quantum enhancement of feature transformation to ensure that nonlinear relationships remain informative as the dimensionality of the feature space increases. Overall, the results showed that QE-FGNN is not very sensitive to increasing dimensionality when compared to the three comparison methods.

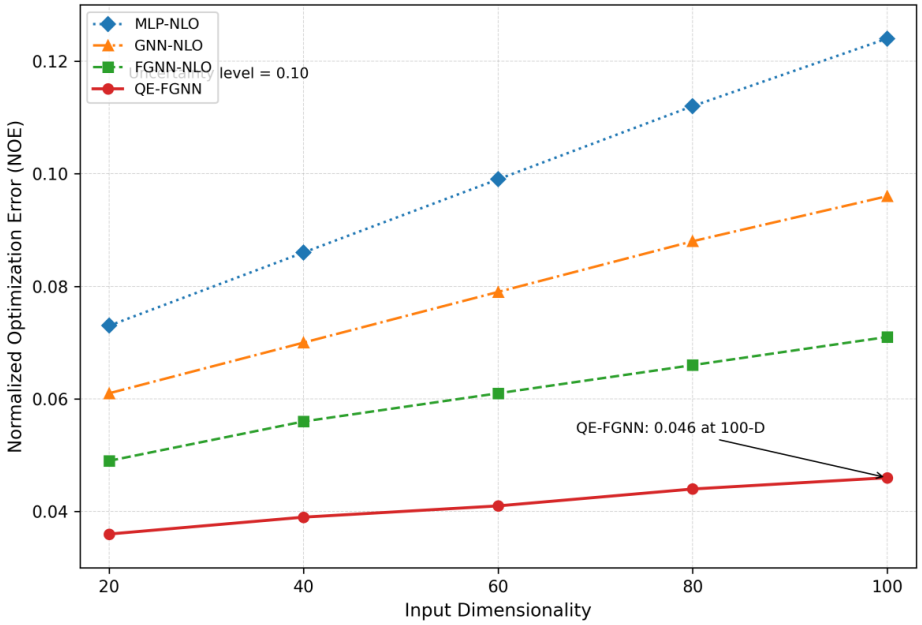


Figure 4. NOE Performance under Increasing Input Dimensionality.

4.4 Robustness under Increasing Uncertainty

To inspect the robustness of the tested methods the uncertainty intensity is then raised from 0 to 0.20, but without the dimensional optimization being changed. As uncertainty gets larger, the Normalized Optimization Error (NOE) gets larger for all approaches, which shows that the harder the perturbation, the more difficult the feature representation and nonlinear optimization. The proposed QE-FGNN however shows the least degradation for the entire uncertainty range.

MPL-NLO shows the highest drop in NOE, going from 0.091 at 0 uncertainty to 0.183 at 0.20. GNN-NLO superiorly exhibits robustness, up to NOE (Noise Output Error) of 0.143 (up from 0.073), and FGNN-NLO has even lower sensitivity to perturbations (or NOE), increasing from 0.055 to 0.104. QE-FGNN, on the other hand, not only has the lowest NOE across the whole range of evaluations, but it also keeps an increasing uncertainty level from 0 to 0.20 largely unchanged at low values.

The gradual rise found for QE-FGNN shows that the fuzzy representation is directly related to uncertainty tolerance, since the features are not necessarily assigned only to one of the three states (low, medium, high), but can rather be partially membership in all three of them. This abstracted representation has a gradual effect by minimizing the perturbation before the construction of the abstracted graph and graph neural learning. In a later step, the informative structure is retained under uncertain conditions with two further steps of dimensionality-aware and quantum-enhanced stages. robustness to increase uncertainty in the input was observed, with QE-FGNN being more robust than MLP-NLO, GNN-NLO and FGNN-NLO.

4.5 Ablation Analysis

The individual contribution of each component, namely the fuzzy representation, graph-learning, and quantum-enhanced representation comes into play in the ablation analysis, which is carried out to determine the performance of the QE-FGNN. Four configurations are taken into account: without fuzzy representation, without graph learning, without quantum-enhanced representation, and frequency quantum-enhanced fuzzy graph neural network (QE-FGNN). The fuzzy representation enhances the NOE and RMSE values from 0.046 to 0.067 and 0.043 to 0.061 respectively, with a decrease of FPS from 0.951 to 0.907, showing that the fuzzy representation is helpful to maintain the quality of the representation under uncertain feature conditions. With the removal of graph learning, the highest degradation is achieved with its NOE of 0.078, RMSE of 0.069, and FPS of 0.891. This finding is significant as it shows the value of addressing nonlinear relationships between high-dimensional features. Without the enhancement in representation, NOE rises to 0.059, RMSE rises to 0.052 and FPS falls to 0.928, suggesting a similar contribution of the amplitude–phase transformation to the overall performance of the algorithm.

The degradation of performance is shown relative to the complete QE-FGNN to give a common comparison scale for the three performances. After deletion of fuzzy representation, 45.7% increase in NOE, 41.9% increase and 4.6% FPS reduction. When the graph learning is removed, the degradation is the highest, which is evident by 69.6% more NOE, 60.5% more RMSE and 6.3% less FPS. When removing the quantum-enhanced part, degradations are relatively smaller at 28.3%, 20.9% and 2.4%, respectively. The completely QE-FGNN is shown as the zero degradation reference in Fig. 5. Overall, ablation results demonstrate that all three key components of the proposed framework bring value to the framework, with the graph learning component making the highest individual contribution in the considered setting.

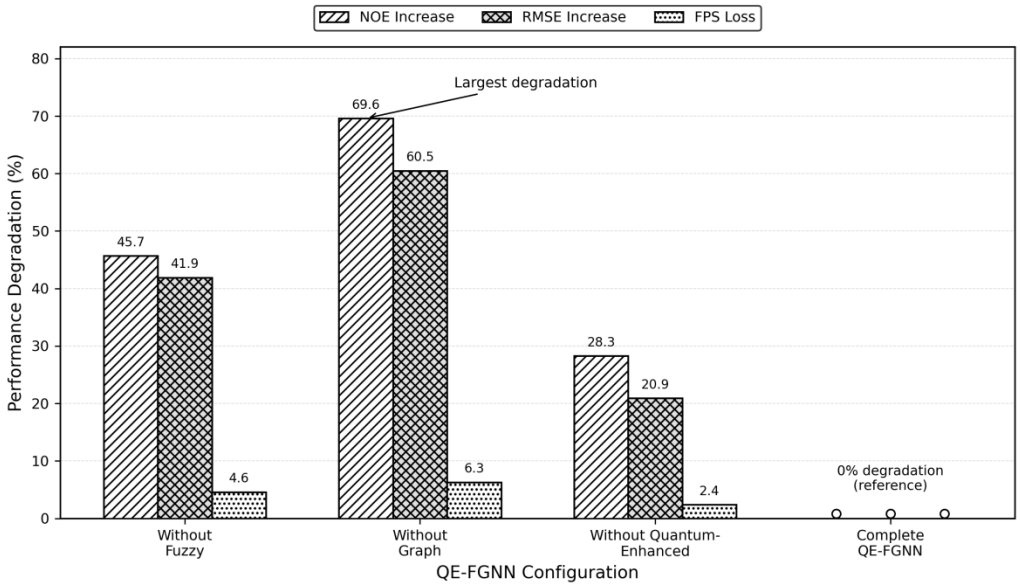


Figure 5. Component Ablation Analysis of QE-FGNN.

5. Discussion

The numerical results demonstrate that optimization performance of QE-FGNN is fairly stable with the increase of dimensions from 20 to 100. Graph modeling takes into account nonlinear relationships between features, and dimensionality-aware aggregation focuses on information-rich features and discards redundant data. As a result, the optimization error of QE-FGNN grows smaller than that of MLP-NLO, GNN-NLO, and FGNN-NLO, which results in better high dimensional representation.

Additionally, under conditions of uncertainty, a fuzzy representation is applied to further enhance robustness. The membership of perturbed features may have gradual memberships for the Low, Medium and High membership states, instead of a hard cutoff. This offers more stable information for adaptive graph building, and helps to reduce the growth in the optimization error with increasing uncertainty.

The quantum-enhanced stage enhances the compact representation by performing amplitude–phase transformation. The ablation results show that the ablation of this component substantially worsens the optimization and reconstruction errors and decreases the feature preservation. But this mechanism is quantum inspired, but classically implemented, and does not necessarily mean faster than classical execution or quantum supremacy, or even execution on physical quantum hardware.

QE-FGNN results in lower optimization error, constraint violation and higher feature preservation than comparison methods on overall. However, additional computational time is needed for the above improvements due to fuzzy processing, graph construction, propagation using graph neural networks (GNNs), dimensional aggregation and a quantum-enhanced transformation. So, the proposed plan is based on a clear complexity–performance relationship.

The formulation of the study is only computational and the uncertainty ranges are predetermined, which is not a strong point, as well as using graph and fuzzy parameters, and the greater complexity of calculation. Also, the quantum enhanced component is still quantum inspired. It should then be interesting for future evaluation to include application-specific high-dimensional optimization problems, more general uncertainty conditions, time-varying graph structures, and possible quantum-hardware implementations.

6. Conclusion

This study proposed the Quantum-Enhanced Fuzzy Graph Neural Network (QE-FGNN) framework on the high-dimensional nonlinear optimization problem with uncertainty. It builds an adaptive graph structure, performs fuzzy representation and dimensionality-aware aggregation on the graph, applies graph neural learning, utilizes quantum-inspired feature transformation, and establishes constraint nonlinear optimization, and all these are performed in a single computational structure. The feature conditions of low, medium and high represent fuzzy states and the adaptive feature graph captures nonlinear inter-dimensional dependencies. The GNN layers then learn the interaction-aware information, while the dimensionality-aware aggregation compresses the information learned by the GNN layers into a compact 16-dimensional representation. This representation further gains richness via an amplitude–phase transformation, prior to being fed into the constraint-aware nonlinear optimization process.

The Quantitative Evaluation under increasing dimensionality and uncertainty shows that QE-FGNN performs better on the whole than MLP-NLO, GNN-NLO and FGNN-NLO. The proposed framework has reduced values of normalized optimization error, reconstruction RMSE, constraint violation, and increased value of feature preservation in particular. The dimensionality and uncertainty analyses also exhibit relatively constant performance as the complexity of the problem is raised. According to the results of the ablation analysis, each of the fuzzy representation, graph learning and quantum-enhanced transformation contributes to the performance. The improvements come with a computational cost though, and there is a tradeoff between optimization quality and computational complexity.

Future research will explore application-specific high dimensional optimization problems beyond the scope of 100 dimensions, dynamic and time-varying graph structures, and other dimensions. There are also alternative fuzzy membership functions, adaptive graph thresholds, multi-objective nonlinear optimization and explainable feature-importance mechanisms that will be discussed. Developing further, a component of the project could involve quantum-inspired simulation using quantum circuits, and potentially quantum hardware implementation, of actual parameterized quantum circuits to determine if the quantum-inspired representation can be extended to practical hybrid quantum-classical optimization.

References

1. Arvinth, N. (2025). AI-Driven Predictive Control and Optimization Framework for Renewable-Integrated Smart Power Grids. *National Journal of Intelligent Power Systems and Technology*, 34-41.

2. Beyes, J. O., & Chakma, S. (2025). Computational Optimization of Acoustic Metamaterials for Advanced Sound Manipulation. *Advanced Computational Acoustics Engineering*, *3*(1), 1-7.

3. Collis, B., Patel, S., Koch, D., Cutugno, M., Wessing, L., & Alsing, P. M. (2023). Physics simulation via quantum graph neural network. *AVS Quantum Science*, *5*(2).

4. Daskin, A. (2025). A unifying primary framework for QGNNs from quantum graph states. *The European Physical Journal Special Topics*, *234*(20), 6279-6288.

5. Geetha, K. (2025). Autonomous IoT Clustering in Ubiquitous Environments Using Multi-Agent Graph Reinforcement Learning. *National Journal of Ubiquitous Computing and Intelligent Environments*, 29-36.

6. Guo, X., Tian, B., & Tian, X. (2022). Hfgnn-proto: Hesitant fuzzy graph neural network-based prototypical network for few-shot text classification. *Electronics*, *11*(15), 2423.

7. Havlíček, V., Córcoles, A. D., Temme, K., Harrow, A. W., Kandala, A., Chow, J. M., & Gambetta, J. M. (2019). Supervised learning with quantum-enhanced feature spaces. *Nature*, *567*(7747), 209-212.

8. Hussai, M., & Getachew, B. (2025). Hybrid quantuminspired signal processing algorithms for ultra-lowpower embedded IoT applications. *National Journal of Signal and Image Processing*, *1*(2), 10-18.

9. Mernyei, P., Meichanetzidis, K., & Ceylan, I. I. (2022, June). Equivariant quantum graph circuits. In *International conference on machine learning* (pp. 15401-15420). PMLR.

10. Ryu, J. Y., Elala, E., & Rhee, J. K. K. (2023). Quantum graph neural network models for materials search. *Materials*, *16*(12), 4300.

11. Schuld, M., & Killoran, N. (2019). Quantum machine learning in feature Hilbert spaces. *Physical review letters*, *122*(4), 040504.

12. Wexford, A. (2024). Dimensionality Reduction Biases in Embedding Feature Spaces. *Journal of Artificial Intelligence in Fluid Dynamics*, *3*(2), 1-7.

13. Wu, Z., Pan, S., Chen, F., Long, G., Zhang, C., & Yu, P. S. (2020). A comprehensive survey on graph neural networks. *IEEE transactions on neural networks and learning systems*, *32*(1), 4-24.

14. Zhang, C. Y., Lin, Y. N., Chen, C. P., Yao, H. Y., Cai, H. C., & Fang, W. P. (2023). Fuzzy representation learning on graph. *IEEE Transactions on Fuzzy Systems*, *31*(10), 3358-3370.

15. Zhou, J., Cui, G., Hu, S., Zhang, Z., Yang, C., Liu, Z., ... & Sun, M. (2020). Graph neural networks: A review of methods and applications. *AI open*, *1*, 57-81.