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Robust Optimal Control of Nonlinear Communication Systems under Parametric Uncertainty

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Abstract
Parametric uncertainty and external disturbances, nonlinear dynamics, interference, and changes in system parameters all greatly influence the performance of nonlinear communication systems and classical robust-control techniques with a fixed level of disturbance rejection may be conservative. The present paper proposes a new disturbance attenuation function, $\gamma(x)$which is (globally) bounded by a positive scalar to establish a new and more powerful optimal-control framework for nonlinear communication systems with parametric uncertainties. The proposed formulation introduces a mechanism for state-dependent robustness instead of using a fixed robustness parameter, which is commonly used in existing formulations. The notion of generalized state-dependent $\mathscr{L}_2$-gain condition is given in terms of a dissipation inequality, and a nonlinear minimax differential game is formulated. Based on the Hamilton–Jacobi–Isaacs (HJI) framework, the corresponding optimal control and worst-case disturbance are computed, leading to a modified HJI equation which includes the state-dependent term $1/\gamma^2(x)$ in the robustness. Parametric uncertainty is treated by a normalized bounded uncertainty representation and an augmented disturbance vector, which can be used to treat external disturbances and parametric uncertainty in a unified manner. In addition, a Lyapunov-based robust stability theorem is developed that ensures nominal asymptotic stability and state-dependent disturbance-attenuation performance. Analytical comparison and evaluation with the conventional one with fixed-γ show that the proposed framework is a generalization of the conventional one, provides more robust adaptability and has the potential to reduce the conservatism.

Keywords: Nonlinear communication systems; Robust optimal control; State-dependent disturbance attenuation; H_∞control; L_2-gain; Hamilton–Jacobi–Isaacs equation; Parametric uncertainty; Minimax control; Lyapunov stability.

1. Introduction

1.1 Background and Motivation

Typical circumstances of modern communication systems are those in which nonlinear dynamics and the time-varying channel, external disturbances, interference, parameter deviations, and imperfect models can impact the stability and the communication performance. Variability in system parameters, aside from external

disturbances, can lead to uncertainty, which may result in greater communication errors, control effort, power consumption, and closed-loop instability. To maintain system performance, it is necessary to develop robust control in the presence of these disturbances and uncertainties, which can be achieved based on an established mathematical framework as discussed in Wang et al., 1992, Lee et al., 1994, Wu et al., 2003, and Shaiju et al., 2013.

The $\mathscr{L}_2$-gain and $\mathscr{H}_\infty$-gain theories meet the robustness concept in the form of disturbance attenuation and give mathematical performance guarantees. Nonlinear $\mathscr{L}_2$-gain analysis and state-feedback $\mathscr{H}_\infty$ control have been developed over the past couple of years based upon Hamilton–Jacobi equations and inequalities (van der Schaft, 1992). Nonlinear formulations have also been made robust, and the problem of uncertain nonlinear dynamics (Ito, 1996) has been taken up. Minimax formulations are a type of differential game where the controller aims to minimize the cost of its performance and the disturbances try to work in a counterproductive way (Savkin et al., 1995).

Current robust control, however, typically operates with a single value of the disturbance attenuation parameter over the entire operating range. A continuous demand of this kind can be conservative when a nonlinear communication system encounters varying circumstances of operation. Hence, some works on the state-dependency of scaling and optimal control have been studied for uncertain nonlinear systems (Ito, 1996; Ornelas-Tellez et al., 2014; Wang et al., 2017).

1.2 Problem Statement and Research Gap

The problem addressed in this research is to design a robust optimal controller for nonlinear communication systems under parametric uncertainty, with guarantees of closed-loop stability and robust performance in spite of changes in the operating conditions for disturbance attenuation. This paper proposes robust nonlinear control for state-dependent stability of uncertain systems with minimax $\mathscr{H}_\infty$ performance, which has not been studied in existing literature separately or in related combination (Savkin et al., 1995; Petersen, 2008; Ur Rehman et al., 2014). Robust control studies in the wireless and networked setting have also been addressed in communication contexts (Hassan et al., 2013; Yin et al., 2020; Yang et al., 2023). In this work, these concepts are combined in a nonlinear minimax framework consisting of a bounded state-dependent disturbance attenuation function in the context of parametric uncertainty.

1.3 Research Objectives and Contributions

The objectives include modeling the uncertain nonlinear communication system, formulating the robust optimal-control problem, introducing the state-dependent disturbance attenuation, deriving the HJI equation, stability conditions based on Lyapunov, and analytical evaluation of the state-dependent $\mathscr{L}_2$-gain performance and the comparison of the framework with conventional fixed-γ robust control. The highlight is the fact that the constant attenuation parameter is replaced by a bounded positive function.$\gamma(x)$This introduces "robustness" dependent on the operating conditions and, at the same time, may allow reducing conservatism.

2. Related Work

2.1 Robust Nonlinear Control

Robust control arises when taking uncertainty, disturbance, modelling errors, and parameter variations into account when dealing with nonlinear systems. Wang et al. (1992) proposed the robust control of uncertain nonlinear systems, and Lee et al. (1994) studied robust performance by self-tuning control. van der Schaft (1992) developed a systematic way of attenuating disturbances in the nonlinear $\mathscr{L}_2$-gain analysis as well as in the nonlinear state-feedback $\mathscr{H}_\infty$ control with Hamilton–Jacobi inequalities, and van der Schaft (1991) formulated the nonlinear optimal-control problem with Hamilton–Jacobi equations. Ito (1996) was concerned with nonlinear.$\mathscr{H}_\infty$ control with structured uncertainty; Shaiju et al. (2013) extended robust control to uncertain nonlinear systems, and Wu et al. (2003) were concerned with this extension. However, Petersen (2008) further investigated robust adaptive control with integral quadratic constraints.

2.2 Minimax Optimal Control and HJI Methods

Minimax optimal control is a framework for worst-case disturbance rejection. In the study of Savkin et al. (1995), minimax optimal control was studied using game-type formulations; for the case of uncertain nonlinear systems, minimax control is studied in Ur Rehman et al. (2014). Magni et al. (2001) studied the nonlinear $\mathscr{H}_\infty$ control with receding horizon. Sakamoto et al. (2008) investigated analytical approximations for the stabilizing

Hamilton–Jacobi solutions, and Sakamoto (2013) discussed the stable-manifold techniques for nonlinear optimal control. The minimax approach and the HJI formulation followed here are supported by these studies.

2.3 State-Dependent Control and Parametric Uncertainty

State-dependent control allows the behaviour of the controller to change with every state. Ornelas-Tellez et al. (2014) and Wang et al. (2017) studied state-dependent nonlinear optimal tracking and nonlinear control with state-dependent Riccati equations, respectively. Cong et al. (1993) tackled uncertain nonlinear systems, while Ito (1996) showed the importance of the state-dependent scaling.

2.4 Communication-System Applications and Research Gap

Applications of robust control for wireless and networked systems were presented by Hassan et al. (2013), Yin et al. (2020), and Yang et al. (2023), respectively, and for robust event-triggered control, see Abdelrahim et al. (2017). The work on state-dependent disturbance attenuation, nonlinear minimax control, parametrized uncertainty, and HJI analysis within a single communication-control framework is, however, not covered. The boundedness of such a state-dependent attenuation function and the Lyapunov-based robust performance analysis in this paper fill such a gap.

3. Nonlinear Communication System Model and Problem Formulation

3.1 Nonlinear System Model

This nonlinear communication system of this study is modeled to capture the effect of nonlinear dynamics, control action, external disturbance and uncertain system parameters. Based on the nonlinear-system formulation given by Wang et al. (1992), the nominal system is given by.

$$\dot{x}(t) = f\big(x(t)\big) + g\big(x(t)\big)u(t) + k\big(x(t)\big)d(t), \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system-state vector, $u(t) \in \mathbb{R}^m$ is the control input, $d(t)$ are external disturbances, and $f(x)$, $g(x)$, and $k(x)$ are nonlinear system mappings. This description is in line with strong nonlinear control schemes where the influence of disturbances is modelled as a disturbance itself, and so explicitly included in the system dynamics (Wang et al., 1992). The goal of the control is to control the nonlinear communication-system state, while at the same time minimizing the influence of the disturbances, and to prevent excessive control effort. Therefore, the state and control penalties are represented by the quadratic terms where $Q > 0$ and $R > 0$ are weighting matrices. The first term is used to make control to reduce the deviation of the system state, and the second term is used to prevent excessive control effort.

3.2 Parametric Uncertainty Representation

The parameters of the nonlinear communication system are also uncertain. The parameter vector is stated as:

$$\theta = \theta_0 + \Delta\theta(t), \qquad (2)$$

where $\theta_0$is the nominal parameter vector and $\Delta\theta(t)$are the unknown, but nevertheless bounded, parameter variations. The uncertain nonlinear dynamics can thus be initially represented as:

$$\dot{x} = f(x,\theta) + g(x,\theta)u + k(x)d. \qquad (3)$$

The uncertainty representation reveals that the uncertainty changes the nonlinear system behavior and thereby affects the desired control action. This treatment is based on the robust-control point of view in which the time-varying parameter uncertainty is directly parametrized in the nonlinear system models (Wang et al., 1992). The detailed normalized bounded-uncertainty transformation is presented next for the robustness analysis.

3.3 Robust Minimax Optimal-Control Problem

Conventional H_∞-type performance index for the nominal nonlinear system is formulated as:

$$J(u,d) = \int_0^\infty [x^T Q x + u^T R u - \gamma^2 d^T d]\, dt. \qquad (4)$$

The first two terms stand for performance degradation and control effort, respectively, and the negative disturbance term is for the worst-case disturbance attenuation mechanism. Nonlinear $\mathscr{L}_2$-gain and $\mathscr{H}_\infty$ formulations also relate disturbance energy and system performance (van der Schaft, 1992).

The new model suggested is a state-dependent attenuation parameter which replaces the constant attenuation parameter. The strong optimal-control problem is then christened as a nonlinear minimax differential game.

$$\min_u \max_d J(u, d) \quad (5)$$

The disturbance is the worst-case input that maximises the performance cost, and the controller aims to minimise this cost. This minimax formulation follows that of performing robust optimal control on uncertain systems (Savkin et al. (1995)). Hence, the requirement for disturbance attenuation depends on the instantaneous state, namely through γ(x), and this grounds the state-dependent formulation and its modified HJI derivation in the following sections.

4. Proposed State-Dependent Robust Optimal Control Framework

The framework proposed here allows us to leave behind the idea of a fixed and standard attenuation parameter and rather adopts a bounded state-dependent attenuation parameter, which brings the notions of nonlinear robust control and $\mathscr{L}_2$-gains into the realm of communication systems whose robustness is dependent on their operating conditions. The mathematical focus of the study lies mainly with this state-dependent attenuation mechanism.

4.1 Conventional Fixed-γ Robust Control

In a standard nonlinear $\mathscr{H}_\infty$ control problem, the attenuation level of the disturbance is specified by a fixed $\gamma > 0$ for a control design. The desired output $z(t)$ is supposed to be closed to the standard $\mathscr{L}_2$-gain condition or, equivalently, over a finite time period $[0, T]$. This formulation is to make sure that the controlled output $z(t)$ does not get too much energy from the external disturbance $d(t)$. The connection with nonlinear state-feedback control (NLFBC, van der Schaft, 1992) and Hamilton–Jacobi inequalities and disturbance attenuation is given by nonlinear $\mathscr{L}_2$-gain analysis.

Fixed-γ control, however, forces the same level of attenuation state-wise. However, this might be anti-adaptive and too strict in some areas and not adaptive enough in others for nonlinear communication systems where changing channel conditions, interference, transients, parameter variations, and hardware nonlinearities may render this too strict and/or not adaptive enough. This is the reason why we consider replacing the constant γ by a bounded state-dependent function.

Physically, a lower γ means the disturbance rejection is more stringent, which, in turn, could lead to greater control effort and power consumption; on the other hand, a higher γ makes the disturbance rejection less stringent, but may result in less protection in critical situations. Therefore, fixed-γ control does not directly optimize the trade-off between robustness and the control effort required to achieve it to fit the current nonlinear state.

4.2 State-Dependent Attenuation Mechanism

To address this issue, the proposed framework suggests the following definition of the attenuation level:

subject to

$$0 < \gamma_{\min} \leq \gamma(x) \leq \gamma_{\max}. \quad (6)$$

This boundedness condition ensures that the attenuation function does not become negative and is bounded for any admissible system state. Thus, the underspecified requirement for robustness may be variable and hence does not become physically or mathematically infinite with respect to the operating condition.

The proposes the following specific state-dependent function:

$$\gamma(x) = \gamma_{\min} + (\gamma_{\max} - \gamma_{\min})\left(1 - e^{-a\|x\|^2}\right),\ a > 0. \quad (7)$$

The limiting cases of this function can be evaluated directly, from which the behaviour of the function can be obtained. This mechanism leads to a direct relation between the state of the system and the robustness required.

The more disturbance can be rejected, the smaller is the attenuation value.$\gamma_{\min}$ close to the desired equilibrium. The state is then further off from the equilibrium and $\gamma(x)$ is increasing up to $\gamma_{\max}$, which makes the controller less aggressive in cases of large transients. It is a state-dependent scaling, as has been discussed in the general literature on nonlinear robust control, in which uncertain nonlinear systems have been discussed (Ito et al., 1996).

It is important to note that the robustness requirements will not be lifted in the proposed framework. Instead, it depends on the current system state. This is the basic mathematical step from analyzing traditional fixed-$\gamma\mathscr{H}_\infty$ control to the proposed state-dependent robust optimal-control framework.

4.3 Generalized State-Dependent L_2-Gain Condition

A new change is made in the conventional finite-horizon disturbance attenuation condition in (10) as follows: incorporating the state-dependent attenuation function. The generalized performance condition is:

$$\int_0^T z^T z\, dt \le V(x_0) + \int_0^T \gamma^2\big(x(t)\big) d^T(t) d(t)\, dt, \qquad (8)$$

where $V(x_0)$ is a storage (or Lyapunov) function evaluated at the initial state that is positive definite. The latter expression simply reduces to the disturbance-energy relation controlled by the state-dependent attenuation function when $V(x_0) = 0$.

The dissipation inequality associated with (17) is chosen to be

$$\dot{V}(x) + z^T z - \gamma^2(x) d^T d \le 0. \qquad (9)$$

This provides the proposed generalized state-dependent $\mathscr{L}_2$-gain condition. The point about (8) is that now there's not a single globally universal constant governing robustness. The disturbance contribution is instead weighted by $\gamma^2\big(x(t)\big)$, and therefore, the robustness requirement will change along the trajectory of the nonlinear system. This gives the mathematical link to the modified HJI equation derived in Section 5, where the state-dependent term is incorporated in the form of $1/\gamma^2(x)$. This results in the overall scheme wherein nonlinear $\mathscr{L}_2$-gain analysis, state-dependent attenuation, minimax control, and then HJI-based controller synthesis are combined.The overall structure of the proposed robust optimal control scheme based on state dependency is shown in Fig. 1.

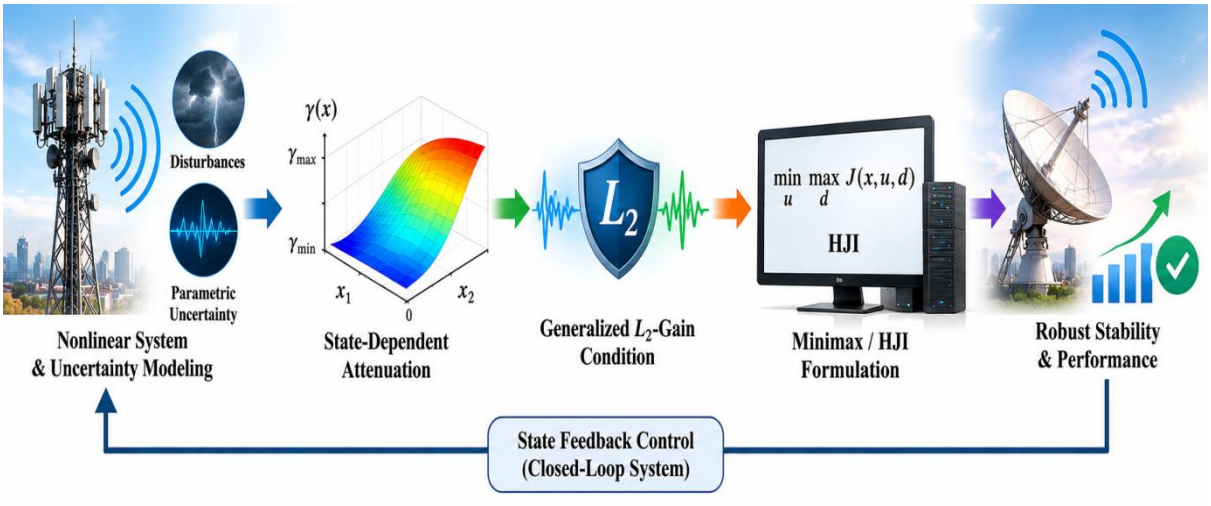


Figure 1. Overall Architecture of the Proposed State-Dependent Robust Optimal Control Framework

5. Modified Hamilton–Jacobi–Isaacs Formulation

The proposed robust optimal-control framework is based on the Hamilton–Jacobi–Isaacs (HJI) formulation. In nonlinear $\mathscr{H}_\infty$ control, it describes the minimax interaction between the controller that minimizes the performance cost and the disturbance that maximizes the performance cost. This renders HJI very well suited for

simultaneous state deviation reduction and disturbance effect reduction, in addition to the control law, with a minimal amount of control effort (van der Schaft, 1992).

In the proposed framework, the constant parameter of the disturbance attenuation is replaced by the bounded state-dependent function $\gamma(x)$. The robustness requirement with the above will then be different depending on the operating condition, and the modified HJI equation is, thus, with a disturbance value that is also state-varying. The proposed robust controller that follows the formulation of the previous paragraph is a mathematical basis.

5.1 Value Function and Hamiltonian

Consider the nominal nonlinear communication system. The state-dependent minimax performance index proposed is

$$J(u,d) = \int_0^\infty [x^T Q x + u^T R u - \gamma^2(x) d^T d]\, dt, \qquad (10)$$

$Q > 0$ and $R > 0$ are positive-definite weighting matrices, and $\gamma(x)$ is a positive-definite disturbance attenuation function. The associated minimax control problem is

$$V(x) = \min_u \max_d J(u,d), \qquad (11)$$

where $V(x)$ denotes the value function. It is the minimum possible worst-case accumulation cost with respect to the starting state x of the system. The gradient of $V(x)$ gives the sensitivity of the optimum cost to changes in the state of the system. Therefore, it dictates the controller action to be taken when minimizing the accumulation cost. The Hamiltonian for the minimax problem is defined as:

$$H(x,u,d) = x^T Q x + u^T R u - \gamma^2(x) d^T d + \nabla V^T [f(x) + g(x)u + k(x)d]. \quad (12)$$

The state penalty, control-effort penalty, disturbance contribution, and system-dynamics contribution are included in the Hamiltonian. The key thing about the conventional formulation is that it recognizes that, in addition to a constant γ, there is a state-dependent term $\gamma^2(x)$.

Therefore, the disturbance penalty will vary depending on the state of the system. The Hamiltonian can therefore be viewed as a state-dependent zero-sum differential game between the controller and the disturbance by respectively aiming for minimization and maximization of H. This is done as per the minimax structure proposed in the framework.

5.2 Optimal Control and Worst-Case Disturbance

The optimal control law demonstrates that the optimal control is calculated based on the gradient of the value function. Thus, from the gradient $\nabla V(x)$the optimal controller can be obtained once the HJI equation is solved.

$$u^*(x) = -\frac{1}{2} R^{-1} g^T(x) \nabla V(x) \quad (13)$$

The argument of the proposed approach is dependent on the state-dependent attenuation function, which makes the worst-case disturbance law a very significant issue. In a traditional, fixed- γ formulation, the scaling factor wouldn't change. This formulation, however, also applies to the case of fixed γ, which is the conventional case.

$$0 = x^T Q x + \nabla V^T f(x) - \frac{1}{4} \nabla V^T g(x) R^{-1} g^T(x) \nabla V + \frac{1}{4\gamma_0^2} \nabla V^T k(x) k^T(x) \nabla V \quad (14)$$

Therefore, the proposed formulation is dependent on the state of the system, thus generalizing the conventional fixed-attenuation formulation without implementing a new control framework for the system. Hence, the mathematical connection between the generalized state-dependent gain condition derived in Section 4 and the robust stability analysis derived in Section 6 is provided via the modified H.J.I. equation. The next section presents sufficient conditions for the resulting nonlinear communication system to be stable and meet the proposed state-dependent disturbance attenuation criteria by using the Lyapunov-based dissipation condition.

6. Robust Stability and Performance Analysis

A state-dependent robust optimal control algorithm is proposed and should be stable with guaranteed performance. Section 5 presents a modified design for the HJI to supply the optimal control and worst-case disturbance design procedures; however, the use of an extra Lyapunov condition is needed so as to infer that the resultant closed-loop communication system of a nonlinear plant is stable while enjoying the desired disturbance attenuation property. There is a natural connection between stability and $\mathscr{L}_2$-gain performance in a nonlinear robust control context using Lyapunov and dissipativity approaches.

6.1 Lyapunov-Based Stability Condition

Consider the nonlinear closed-loop communication system. Let $V(x)$ be a continuously differentiable positive-definite Lyapunov function satisfying:

$$V(0) = 0,\ V(x) > 0,\ x \neq 0. \qquad (15)$$

In the proposed framework, the state-dependent attenuation function should not be allowed to vanish or become negative. This condition is important as it avoids setting the parameter of disturbance attenuation to zero or infinity. Hence, over the operating region, the state-dependent robustness mechanism is clearly defined. The Lyapunov derivative, the controlled output z, and the state-dependent disturbance penalty are mixed together for the proposed robust performance requirement. The necessary dissipation inequality is

$$\dot{V}(x) + z^T z - \gamma^2(x) d^T d \leq -\alpha(\| x \|), \qquad (16)$$

where $\alpha(\| x \|)$is a positive-definite function.

The dissipation inequality serves as the main stability/performance condition of the proposed framework. It does this at the same time as learning three needs. First describes the evolution of the energy of the system/storage function. Second, it is the accumulated Cop (controlled-output performance). Third, a state-dependent allowance which is associated with the disturbance.

Hence, the Lyapunov function will decrease along the nominal closed-loop trajectories. This gives the ground for asymptotic stability of the nominal system. Another connection between stability and disturbance attenuation is through the use of a dissipation inequality. These types of storage-function conditions are commonly applied to guarantee robust input-output performance and $\mathscr{L}_2$- gain-type properties of nonlinear systems.

6.2 Robust Stability Theorem

Theorem: Robust Stability Under State-Dependent Disturbance Attenuation.

Consider the nonlinear uncertain communication system. Assume that a positive definite Lyapunov function V(x)and a bounded state-dependent positive attenuation function exist such that.

If there exists a control law $u(x)$such that

where $\alpha(\| x \|)$is positive definite, then the following properties are guaranteed:

1. The nominal closed-loop system is asymptotically stable when $d(t) = 0$.
2. The system satisfies the proposed state-dependent disturbance attenuation property.
3. The controlled output satisfies the finite-horizon performance bound

$$\int_0^T z^T z\, dt \leq V(x_0) + \int_0^T \gamma^2(x) d^T d\, dt. \qquad (17)$$

The important feature of this theorem is that the disturbance attenuation level is not required to be a single global constant. Instead, $\gamma(x)$is allowed to change with the system state while remaining bounded. Thus, the stability condition and the accomplishment of the performance guarantee are preserved, and the robustness requirement can evolve according to the operating condition.

6.3 Proof and Robust Performance Guarantee

To prove the theorem, start with the dissipation inequality in (16), which is the same as the generalized state-dependent $\mathscr{L}_2$-gain performance condition considered earlier in the proposed framework. Note that $V(x) > 0$ for all $x \neq 0$ and $\dot{V}(x) < 0$ for all x outside an equilibrium under zero disturbance ($d(t) = 0$), which means that the nominal closed-loop system is asymptotically stable. If there are nonzero disturbances, it is no longer necessary that the system has a strictly decreasing Lyapunov function at all times.

This gives the robustness-performance interpretation for the proposed method. For the nominal system, the operating is stable; when it is subject to external disturbances, the value of energy at the output is bounded by the initial Lyapunov energy and the accumulated energy generated by the states due to the disturbance. As such, the mechanism of attenuation can't be too aggressive or too weak. Thus, the proposed controller guarantees the controlled robustness region and provides the disturbance attenuation level depending upon the current state. Lastly, when the bound of the performance reduces to the traditional fixed-γ,

$$\int_0^T z^T z \, dt \leq V(x_0) + \gamma_0^2 \int_0^T d^T d \, dt. \qquad (18)$$

As a result, the traditional robust-control condition is retrieved as a special case. The state-dependent formulation is hence an improvement over the traditional fixed attenuation formulation, and maintains the same simple stability criteria and performance interpretation. The main theoretical guarantee of the proposed framework is given by the analysis and consists of the asymptotic nominal stability and a disturbance-attenuation bound that is different from the standard one and depends on the state instead of only on time. This finding links the modified formulation of HJI in Section 5 with the desirable performance objective and forms the theoretical foundation to assess the proposed controller in the simulation study in the presence of parametric uncertainty and external disturbances.

7. Theoretical Analysis and Comparative Discussion

The generalized state-dependent $\mathscr{L}_2$-gain condition, the representation of bounded parametric uncertainty, the Lyapunov dissipation inequality, and the modified Hamilton–Jacobi–Isaacs (HJI) formulation introduced in the previous sections are used to evaluate the proposed robust optimal-control framework. The main goal is to explore the state-dependent attenuation function and see its effects on the robustness requirement compared with conventional fixed-γ control. The analysis is done using theory and a numerical study of the proposed attenuation mechanism. Even though the numerical evaluation is an abstract illustration that corresponds with the defined attenuation function described in Section 4, it should not be seen as a simulation result in its own right. State-dependent weighting formulation and HJI-based nonlinear robust-control formulations are shown to be consistent with existing nonlinear $\mathscr{H}_\infty$ control formulations.

7.1 Numerical Evaluation of State-Dependent Attenuation

If a numerical evaluation is desired, set $\gamma_{\min} = 0.5$, $\gamma_{\max} = 2.0$, and a$a = 1$. The obtained attenuations show a monotonically increasing tendency of the attenuation value (parameter) as a function of the state magnitude. In this way, the attenuation parameter varies by about 300%, from 0.5 to almost 2.0, across the representative state range. Most importantly, the disturbance weighting$\gamma^2(x)$ varies between 0.25 (at equilibrium), 2.10 (at an intermediate point), and 3.999 (at a larger-scale deviation from equilibrium). This implies that the generalized performance inequality is intrinsically related to the instantaneous operating condition as opposed to one global value for the disturbance contribution.

7.2 Robustness Margin Under Parametric Uncertainty

Parametric uncertainty and disturbance have been represented using the augmented disturbance in Section 6. Thus, both effects are added to the same generalized disturbance measure, which is evaluated under common conditions of robust performance. However, the boundedness of γ(x) ensures that the robustness region that is obtained remains finite and that no more disturbance weighting becomes unbounded. An interpretable robust range for the numerical example is produced. The maximum attenuation of the levels is about four times the minimum attenuation. Thus, the proposed mechanism caters for an attenuation range of up to four, keeping the prescribed range. This limited range of operation is a natural design parameter to control the level of adaptation. Analytical Interpretation of the Proposed State-Dependent Robustness Mechanism is shown in Table 1.

Table 1. Analytical Interpretation of the Proposed State-Dependent Robustness Mechanism

Analysis Aspect	Analytical Observation	Implication for Robust Control
Attenuation range	$\gamma(x)$varies from approximately 0.5 to nearly 2.0	Provides bounded adaptation across operating conditions
Attenuation variation	Approximately 300% increase	Allows robustness requirements to change with nonlinear state
Minimum disturbance weighting	Approximately 0.25 near equilibrium	Lower robustness weighting near the nominal operating region
Intermediate disturbance weighting	Approximately 2.10	Increased robustness under moderate state deviation
Maximum disturbance weighting	Approximately 3.999	Stronger disturbance treatment under larger deviations
Robustness range	Approximately fourfold	Provides a controllable robustness margin
Parametric uncertainty	Combined with external disturbance	Enables unified robust-performance analysis
Stability condition	Lyapunov dissipation inequality	Provides theoretical stability guarantee
HJI formulation	State-varying disturbance weighting	Couples robustness with operating condition
Fixed-γ relationship	Recovered when $\gamma(x)$is constant	Establishes fixed-γ control as a special case

7.3 Comparison with Fixed-γ Control

Conventional fixed-γ control enforces the same single disturbance attenuation requirement over all the state space. However, in the proposed formulation, the weighting $\gamma^2(x)$ is state-dependent, leading to a state-dependent robustness requirement. Table 2 shows the Comparison Between Fixed-γ and Proposed State-Dependent Control.

Table 2. Comparison Between Fixed-γ and Proposed State-Dependent Control

Feature	Fixed-γ Control	Proposed State-Dependent Control
Attenuation parameter	Constant	State-dependent
Robustness requirement	Global	Operating-condition dependent
Adaptation to nonlinear state	No	Yes
Parametric uncertainty	Can be incorporated	Explicitly incorporated
Conservatism	Potentially high	Potentially reduced
HJI formulation	Fixed disturbance weighting	State-varying disturbance weighting
Stability guarantee	Lyapunov/HJI based	Lyapunov/HJI based
Fixed-γ as a special case	—	Yes

The same robustness requirement is imposed at each state with a fixed controller, where the attenuation value is defined at the midpoint. The formulation that is proposed here gives a lower attenuation value close to the equilibrium value and an upper limit for larger deviations. This state sensitivity can then alleviate conservatism, since it no longer needs to account for all operating conditions; a single value of attenuation is needed. The numerical values, however, shown here indicate only analytical evidence of the proposed function behaviour and cannot be claimed as measurable performance improvements.

When $\gamma(x) = \gamma_0$, the proposed formulation exactly reduces to conventional fixed-γ control. The proposed framework is therefore an extension of the conventional formulation, with the traditional approach serving as a special case.

7.4 Computational and Practical Considerations

The solution of an HJI equation is not easy since it is a nonlinear partial differential equation. By introducing$\gamma(x)$, more state dependence is given to the robustness formulation. Thus, the proposed method offers more flexibility in the formulation, which might come with additional computational complexity. Various methods for practical implementation can be considered to approximate the HJI equation, including grid-based, neural network, or offline numerical methods. The computational challenges associated with these aspects of nonlinear $\mathscr{H}_\infty$ control are well known. Thus, the present work offers a theoretical approach and not individual numerical and/or experimental validation. In future studies, further simulation and experimental investigation could be performed to test out the proposed controller in typical nonlinear communication-system scenarios.

8. Conclusion

This work has established a solid optimal-control framework for the non-linear communication systems with parametric uncertainty and external disturbance. The novelty is the disturbance attenuation function $\gamma(x)$ is a variable and is bounded, unlike the fixed attenuation parameter in the classical case; thus, the disturbance attenuation requirement can vary according to the current operational state. A dissipation inequality was used to obtain a generalized state-dependent $\mathscr{L}_2$-gain condition, which was applied in a nonlinear minimax differential-game formulation. The associated optimal control and worst-case disturbance were obtained via the Hamilton–Jacobi–Isaacs (HJI) approach, which led to a modified HJI equation that includes the state-dependent robustness function $\frac{1}{\gamma^2(x)}$. The parametric uncertainty was considered through a bounded uncertainty model together with an additional disturbance vector, which allows considering uncertainty and external perturbations together. Moreover, by using Lyapunov theory, a robust stability result and a proof were developed for nominal asymptotic stability at the same time as a state-dependent disturbance-attenuation performance guarantee. An analytical study in Section 7 revealed that the attenuation mechanism is bounded and depends on operating conditions, and a discussion in comparison revealed that the proposed formulation generalizes the conventional fixed-γ control and can be used to reduce the level of conservatism. More detailed simulation of the present study and its experimental verification would be important future research directions, as the present study is theoretical in nature.

References

1. Shaiju, A. J., & Petersen, I. R. (2013). Discrete-time robust H_∞control of a class of nonlinear uncertain systems. *International Journal of Robust and Nonlinear Control, 23*(15), 1663–1681. https://doi.org/10.1002/rnc.2852

2. Wu, J.-L., & Lee, T.-T. (2003). Robust H_∞control problem for general nonlinear systems with uncertainty. *Asian Journal of Control, 5*(4), 541–550. https://doi.org/10.1111/j.1934-6093.2003.tb00108.x

3. Petersen, I. R. (2008). Robust adaptive H_∞control using integral quadratic constraints. *Automatica, 44*(9), 2382–2388. https://doi.org/10.1016/j.automatica.2008.03.007

4. van der Schaft, A. J. (1992). L_2-Gain analysis of nonlinear systems and nonlinear state feedback H_∞control. *IEEE Transactions on Automatic Control, 37*(6), 770–784. https://doi.org/10.1109/9.256331

5. van der Schaft, A. J. (1991). On a state space approach to nonlinear H_∞control. *Systems & Control Letters, 17*(1), 1–8. https://doi.org/10.1016/0167-6911(91)90022-7

6. Magni, L., Nijmeijer, H., & van der Schaft, A. J. (2001). A receding-horizon approach to the nonlinear H_∞control problem. *Automatica, 37*(3), 429–435. https://doi.org/10.1016/S0005-1098(00)00166-7

7. Sakamoto, N., & van der Schaft, A. J. (2008). Analytical approximation methods for the stabilizing solution of the Hamilton–Jacobi equation. *IEEE Transactions on Automatic Control, 53*(10), 2335–2340. https://doi.org/10.1109/TAC.2008.2006113

8. Wang, X., Yaz, E. E., Schneider, S. C., & Yaz, Y. I. (2017). $H_2 - H_\infty$Control of continuous-time nonlinear systems using the state-dependent Riccati equation approach. *Systems Science & Control Engineering, 5*(1), 232–242. https://doi.org/10.1080/21642583.2017.1310636

9. Ornelas-Téllez, F., Rico, J. J., & Ruiz-Cruz, R. (2014). Optimal tracking for state-dependent coefficient factorized nonlinear systems. *Asian Journal of Control, 16*(6), 1679–1690. https://doi.org/10.1002/asjc.761

10. Ur Rehman, M., Fidan, B., & Petersen, I. R. (2014). Robust control system design for an uncertain nonlinear system using minimax LQG design method. *Asian Journal of Control, 16*(6), 1691–1702. https://doi.org/10.1002/asjc.777

11. Abdelrahim, M., Postoyan, R., Daafouz, J., & Nešić, D. (2017). Robust event-triggered output feedback controllers for nonlinear systems. *Automatica, 82*, 269–277. https://doi.org/10.1016/j.automatica.2016.09.044

12. Yang, Y., Jiang, H., Li, J., & Hua, C. (2023). Robust complete synchronization control design for non-collocated nonlinear master–slave system over delayed communication network. *International Journal of Robust and Nonlinear Control*. https://doi.org/10.1002/rnc.6493

13. Hassan, N. U., Assaad, M., & Tembine, H. (2013). Distributed H_∞-based power control in a dynamic wireless network environment. *IEEE Communications Letters, 17*(5), 1048–1051. https://doi.org/10.1109/LCOMM.2013.042313.130075

14. Yin, Z., Yu, K., & Wang, Y. (2020). Robust control algorithm and simulation of networked control systems. *Computer Communications, 155*, 54–62. https://doi.org/10.1016/j.comcom.2020.04.046

15. Sakamoto, N. (2013). Case studies on the application of the stable manifold approach for nonlinear optimal control design. *Automatica, 49*(2), 341–347. https://doi.org/10.1016/j.automatica.2012.11.032

16. Savkin, A. V., & Petersen, I. R. (1995). Minimax optimal control of uncertain systems with structured uncertainty. *International Journal of Robust and Nonlinear Control, 5*(2), 119–132. https://doi.org/10.1002/rnc.4590050204

17. Ito, H. (1996). Robust control for nonlinear systems with structured L_2- gain-bounded uncertainty. *Systems & Control Letters, 27*(2), 73–81. https://doi.org/10.1016/0167-6911(96)00024-2

18. Lee, T. S., & Chen, B. S. (1994). Robust performance control of uncertain nonlinear systems: A self-tuning approach. *International Journal of Robust and Nonlinear Control, 4*(5), 637–654. https://doi.org/10.1002/rnc.4590040505

19. Wang, Y., Xie, L., & de Souza, C. E. (1992). Robust control of a class of uncertain nonlinear systems. *Systems & Control Letters, 19*(2), 139–149. https://doi.org/10.1016/0167-6911(92)90097-C

20. Cong, S., & Landers, T. L. (1993). Robust control design for nonlinear uncertain systems with incomplete matching conditions. *Transactions of the Institute of Measurement and Control, 15*(1), 3–10. https://doi.org/10.1177/014233129301500107