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Mathematical decomposition strategies for large-scale infrastructure optimization with distributed decision variables

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Abstract:

Large-scale infrastructure systems comprise highly interdependent parts, decision variables, and operational constraints making it difficult to optimize such systems via traditional central optimization methods. The need for decomposable optimization methods results from an increase in the size of the system, computational restrictions, communication costs, and the necessity to coordinate distributed entities efficiently. In this research, will develop the methodology of mathematical decomposition of large-scale infrastructure systems optimization by turning complicated optimization problems into cooperative subproblems without losing the quality and convergence of the resulting solution. Study proposes to formulate the problem of infrastructure optimization via block angular formulation and apply three decomposition algorithms, namely, Dantzig-Wolfe decomposition, Bender's decomposition, and consensus-based ADMM. Theoretical analyses are conducted in order to prove strong duality, convergence, and sublinear convergence properties of the distributed optimization procedure. Experiments carried out using a large-scale synthetic model of the infrastructure extension have proved that the consensus ADMM method offers better scalability than centralized and conventional decomposition algorithms. The presented framework saves about 71% computational time when compared to centralized optimization and consumes considerably less memory resources, although the difference in the optimality gap is negligible. The obtained results prove that the mathematical decomposition technique is an efficient way to handle large-scale infrastructure systems in terms of increasing computational efficiency and distributed decision-making capabilities.

Keywords: Mathematical decomposition, large-scale optimization, distributed decision variables, infrastructure systems, consensus ADMM, optimization algorithms, distributed computing.

1. Introduction

Infrastructural system optimization is complicated by a number of interrelated factors such as high level of interconnectedness of the components and the existence of various constraints. Traditional methods of optimization may experience difficulties due to computationally complexity, communication issues, and inability

to adapt to the situation in the high dimensional space of decisions. The use of mathematical decomposition techniques helps to overcome these challenges through breaking down the problem into smaller pieces without compromising on global optimization. Layered optimization structures have created a theoretical basis for such decomposition in network systems (Chiang et al., 2007).

Distributed decomposition techniques have shown great promise in addressing large-scale optimization problems through splitting up of complex models into smaller and independent computing units which coordinate with each other using the required information (Shastri et al., 2011). Research in this area has concentrated on learning decomposition approaches that are able to automatically generate decompositions for multi-objective optimization problems (Aslani & Mohebbi, 2024). Multi-objective decomposition techniques have contributed towards optimizing complex systems in terms of meeting multiple criteria in a design under the constraint condition. Decomposition approaches dealing with uncertainties have been developed to solve large-scale planning and operational problems through developing hierarchical decision-making processes. The nested decomposition techniques have increased efficiency in solving capacity expansion and infrastructure planning problems under multiple uncertainties (Huang et al., 2022). The decision-making frameworks that involve mathematical decomposition have also allowed resilient operation of interconnected infrastructures through modeling their interactions using optimization models (Huang et al., 2017). The cross-layer decomposition approaches have facilitated coordinated decision making in network layers and resource utilization in distributed settings (Johansson et al., 2006).

Optimization techniques have adopted multi-objective search capabilities for complex decision scenarios in the framework of evolutionary and intelligent optimization techniques. Multilevel optimization decomposition techniques have been used to solve infrastructure maintenance and management problems, showing an increase in decision-making quality due to hierarchies in optimization structures (Li & Haimes, 1992). The most recent surveys on optimization decomposition in large-scale scenarios emphasize the need for effective partitioning, parallel computations, and coordination for more complex situations (Chuong et al., 2025). Nested decomposition has also improved the optimization process in energy infrastructures through decreased computational complexity and operational feasibility (Löhr et al., 2022). Dynamic decision-making environments demand the use of decomposition techniques that adapt to changes in constraints and distributed decision-making needs. Multi-level decision-making frameworks have assisted in adaptive decision-making processes by structuring complex decisions in multiple hierarchical levels. The use of large-scale computing infrastructures has been facilitated by the adoption of decomposition-based resource allocation techniques, which increase efficiency and cost-effectiveness (Yang et al., 2025). Hybrid predictive optimization models have further emphasized the need for decomposition techniques in managing complex energy and resource systems through efficient decision coordination (Quan et al., 2025).

Recently, distributed optimization has concentrated on decomposition techniques in the context of renewable energy market, infrastructure scheduling, and resource allocation. Bid selection schemes relying on decomposition techniques have enabled increased tractability in the optimization of large-scale renewable energy systems having many decision variables (Visas et al., 2026; Dewangan et al., 2025). Distributed mathematical approaches have been employed in network reliability optimization in order to keep up with the rapid changes in the system (Mohamad & Kahleel, 2023). Optimization schemes incorporating decomposition techniques and intelligent search techniques have been developed to enhance task scheduling and information management in time-sensitive applications. Large-scale energy systems of today need parallel-distributed optimization frameworks that can simultaneously handle many operational decisions. Recently, research has focused on the development of co-optimization schemes using distributed decomposition techniques for achieving increased scale and performance in the system (Huang et al., 2026; Torre et al., 2020). Problems of infrastructure inspection and maintenance have also benefited from decomposition techniques that handle stochastic failures and decision-dependent uncertainties using optimization (Estrada Garcia & Shen, 2026). Distributed optimization has also been facilitated using tensor decomposition frameworks to enhance resource allocation efficiency in networked systems (Mahmoudi et al., 2026). Despite the achievements so far, the development of decomposition schemes that are scalable, reliable, and effective in solving mathematical problems related to large-scale infrastructure optimization in distributed decision variable settings continues to be challenging. In this research, efforts will be directed towards the development of a decomposition framework that will enable effective distributed decision making while preserving mathematical correctness, reliability, and optimization performance.

Key contribution

- The structured decomposition method is proposed in order to decompose the complex and large-scale infrastructure optimization problem into a set of interconnected subproblems with distributed decision variables. This new methodology ensures computational efficiency along with the coordination between different decision makers through the mathematical approach of decomposition and aggregation.

- The concept of distributed optimization is proposed in order to facilitate the exchange of local information among the decision variables independently and obtain the global optimal solutions. This method improves the scalability and convergence properties of the optimization process.

- The decomposition method has been theoretically analyzed from the perspective of convergence, stability and computational efficiency. Comparing to the conventional centralization and the current decomposition methods, the new decomposition method is more efficient for optimization in large scale interconnected systems.

The rest of the paper is structured as follows. Section 2 provides the preliminaries and mathematical notations, which include the index set, coupling graph, function class, and definitions needed for the development of the proposed optimization framework. Section 3 describes the large-scale infrastructure optimization problem formulated using a block angular structure of distributed variables, coupling constraints, and consensus formulation. Section 4 proposes the decomposition framework by presenting the decomposition methods such as Dantzig-Wolfe decomposition, Benders' decomposition, and consensus ADMM for distributed optimization. Section 5 presents the theoretical analysis of the framework such as strong duality, global convergence, and sublinear convergence rate. Section 6 provides the analysis of convergence and complexity by comparing the computational features of various decomposition techniques. Section 7 presents the numerical performance comparison based on the analysis of solution quality, runtime, memory consumption, and scalability of the decomposition approaches. Section 8 discusses the implications, limitations, and considerations for choosing suitable decomposition techniques for large-scale infrastructures. Finally, Section 9 provides the conclusions of the paper and future research directions toward adaptive, privacy-preserving, and robust decomposition techniques.

2. Preliminaries and notation

2.1. Index sets and coupling graph

Let the set of distributed subregions (subsystems) be indexed by $\mathcal{R} = \{1,2,\dots,N\}$, where each subregion $r \in \mathcal{R}$ manages a local decision vector $x_r \in \mathbb{R}^{n_r}$. The global stacked decision vector across all subregions is given by $x = (x_1, x_2, \dots, x_N) \in \mathbb{R}^n$, where the total problem dimension satisfies $n = \sum_{r\in\mathcal{R}} n_r$. The network coupling topology is defined by an undirected graph $\mathcal{G} = (\mathcal{R}, \mathcal{E})$, where the edge set $\mathcal{E} = \{(r,s) \in \mathcal{R} \times \mathcal{R} \mid$ subregions r and s share at least one linking constraint$\}$ represents shared interconnects or constraints between subregions.

2.2. Function classes

A function $f: \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ is proper if $f(x) < +\infty$ for at least one $x \in \mathbb{R}^n$ and $f(x) > -\infty$ for all $x \in \mathbb{R}^n$; it is closed if its epigraph $\mathrm{epi}(f) = \{(x,t) \in \mathbb{R}^n \times \mathbb{R} \mid f(x) \le t\}$ is a closed set; and it is convex if its domain $\mathrm{dom}(f)$ is convex and $f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y)$ for all $x, y \in \mathrm{dom}(f)$ and $\lambda \in [0,1]$. Study denotes by $\Gamma_0(\mathbb{R}^n)$ the class of proper, closed, convex functions on $\mathbb{R}^n$. A function f is μ-strongly convex ($\mu > 0$) if $f(x) - \frac{\mu}{2}\|x\|^2$ is convex, and it is L-smooth ($L > 0$) if it is differentiable and its gradient is L-Lipschitz continuous, satisfying $\|\nabla f(x) - \nabla f(y)\| \le L\|x - y\|$ for all $x, y \in \mathbb{R}^n$.

2.3. Notation table

The following notations are used for this paper referred in table1.

Table 1: Notations are used.

Symbol	Meaning
R, N	Index set of subregions and its cardinality
xr ∈ R{nr}	Local decision vector of subregion r
z ∈ Rm	Global (coupling) consensus variable
Ar, Br	Local and coupling constraint matrices of subregion r
fr : R{nr} → R ∪ {+∞}	Local (separable) objective of subregion r
g : Rm → R ∪ {+∞}	Coupling / master objective
λ, y	Lagrange multiplier (dual/price) vectors on linking constraints
ρ > 0	Augmented-Lagrangian penalty parameter
k	Iteration index of the distributed algorithm
Φ(x) = Σr fr(xr)	Separable total objective
Xr	Local feasible set {xr : Ar xr ≤ br}

2.4. Standard definitions

Definition 2.1 (Block-angular program).} A mathematical program is {block-angular} with N blocks if it can be written as:

$$\min_{x} \sum_{r=1}^{N} f_r(x_r) \quad \text{subject to} \quad x_r \in \mathcal{X}_r \text{ for } r = 1, \dots, N, \quad \text{and} \quad \sum_{r=1}^{N} B_r x_r = c$$

Where $\mathcal{X}_r$ represents local constraints for subregion r and $\sum_{r=1}^{N} B_r x_r = c$ is a set of global linking constraints coupling the blocks.

{Definition 2.2 (Constraint qualification).} The block-angular program satisfies the {Slater condition} if there exists a strictly feasible point $\hat{x} = (\hat{x}_1, \dots, \hat{x}_N)$ such that:

$$\hat{x}_r \in \text{relint}(\mathcal{X}_r) \quad \forall r \in \{1, \dots, N\} \quad \text{and} \quad \sum_{r=1}^{N} B_r \hat{x}_r = c$$

{Definition 2.3 (Augmented Lagrangian).} For a penalty parameter $\rho > 0$, the augmented Lagrangian associated with the linking constraint $\sum_{r=1}^{N} B_r x_r - z = 0$ is defined as:

$$\mathcal{L}_\rho(x, z, \lambda) = \sum_{r=1}^{N} f_r(x_r) + g(z) + \lambda^\top \left(\sum_{r=1}^{N} B_r x_r - z \right) + \frac{\rho}{2} \left\| \sum_{r=1}^{N} B_r x_r - z \right\|^2$$

3. Problem formulation

The generic large-scale infrastructure optimization problem addressed in this paper is the block-angular program:

$$\begin{aligned} (\text{P}) \quad \min_{x_1, \dots, x_N} \quad & \sum_{r=1}^{N} f_r(x_r) \\ \text{s.t.} \quad & x_r \in \mathcal{X}_r, \quad r = 1, \dots, N \\ & \sum_{r=1}^{N} B_r x_r = c \end{aligned}$$

Here, $f_r \in \Gamma_0(\mathbb{R}^{n_r})$ models the local planning cost of subregion r (capital and operating cost of local assets), $\mathcal{X}_r$ is a local feasible polyhedron encoding subregion-specific technical, safety, and regulatory constraints, and the linking constraint:

$$\sum_{r=1}^{N} B_r\, x_r = c$$

represents system-wide balance constraints, such as power balance at nodes through interconnection links, water continuity along trunk lines, or flow conservation across cut sets of a transportation network.

In most infrastructure applications, $\dim(c) \ll \sum_{r=1}^{N} n_r$ ((i.e., the number of complicating constraints is small in comparison to the number of decision variables), and this is exactly the property that is exploited by decomposition.

By introducing an auxiliary global variable $z \in \mathbb{R}^m$, problem (P) can be rewritten in consensus (sharing) form, serving as the basis for the ADMM-based distributed algorithm:

$$\begin{aligned} \min_{x_1,\dots,x_N,\,z} \quad & \sum_{r=1}^{N} f_r\,(x_r) + g(z) \\ \text{s.t.} \quad & x_r \in \mathcal{X}_r, \quad r = 1,\dots,N \\ & \sum_{r=1}^{N} B_r\, x_r - z = 0 \end{aligned}$$

where $g(z)$ is an indicator function enforcing $z = c$:

$$g(z) = \delta_{\{c\}}(z) = \begin{cases} 0 & \text{if } z = c \\ +\infty & \text{otherwise} \end{cases}$$

The consensus reformulation (P′) of the infrastructure optimization problem and its core technical assumptions are structured as follows:

$$\begin{aligned} (\mathrm{P}') \quad \min_{x_1,\dots,x_N,\,z} \quad & \sum_{r=1}^{N} f_r\,(x_r) + g(z) \\ \text{s.t.} \quad & B_r x_r - z_r = 0, \quad r = 1,\dots,N \\ & x_r \in \mathcal{X}_r, \quad r = 1,\dots,N \end{aligned}$$

where $z = (z_1, \dots, z_N) \in \mathbb{R}^{m\times N}$ defines the set of local coupling copies managed by the coordinator. The symbol g(z) is a measure or aggregation function determined based on the coupling architecture

- **Consensus (Equality) Coupling:** $g(z) = \delta_{\mathcal{C}}(z)$, where $\mathcal{C} = \{(z_1, \dots, z_N) \mid z_1 = z_2 = \cdots = z_N = \bar{z}\}$, enforcing a single global variable state across all subregions.
- **Resource-Sharing Coupling:** $g(z) = h(\sum_{r=1}^{N} z_r)$, capturing shared limits such as pipeline capacity or power transfer limits.

Equivalence holds directly: any feasible vector x for (P) uniquely determines a feasible pair (x, z) for (P′) by setting $z_r = B_r x_r$, and vice versa.

Assumption 3.1 (Convexity & Regularity).

1. Each local feasible set $\mathcal{X}_r \subseteq \mathbb{R}^{n_r}$ is a nonempty, closed, convex polyhedron.
2. Each local objective function $f_r \in \Gamma_0(\mathbb{R}^{n_r})$ is proper, closed, and convex over $\mathcal{X}_r$.
3. The overall problem (P) satisfies the Slater condition defined in Definition 2.2.

Assumption 3.1 clearly takes into account linear and convex quadratic models for infrastructure that are normally seen in transmission expansion, flow network design, and capacity planning over multiple periods. Extensions including discrete choice constraints ($X_r \cap Z\hat{}(n_r)$) are left out.

4. Decomposition framework

Three methods of decomposition for problem (P), namely Dantzig-Wolfe Decomposition (primal), Benders Decomposition (dual), and Consensus ADMM, are described in this section, along with their derivations based on the splitting of the primal/dual space of the coupling constraint.

4.1. Dantzig–Wolfe (primal) decomposition

Under a shared-resource interpretation of the coupling constraint $\sum_{r=1}^{N} B_r\, x_r = c$, each local feasible polyhedron $\mathcal{X}_r$ is represented via the **Minkowski–Weyl Theorem** as the convex combination of its extreme points $\left\{x_r^{(1)}, \dots, x_r^{(K_r)}\right\}$ (assuming bounded $\mathcal{X}_r$ for clarity):

$$x_r = \sum_{j=1}^{K_r} \theta_{r,j}\, x_r^{(j)}, \quad \text{where} \quad \sum_{j=1}^{K_r} \theta_{r,j} = 1, \quad \theta_{r,j} \geq 0$$

Substituting this representation into (P) yields the **Dantzig–Wolfe Master Problem**:

$$\begin{aligned} \text{(M)} \quad \min_{\theta \geq 0} \quad & \sum_{r=1}^{N} \sum_{j=1}^{K_r} f_r\left(x_r^{(j)}\right) \theta_{r,j} \\ \text{s.t.} \quad & \sum_{r=1}^{N} \sum_{j=1}^{K_r} \left(B_r x_r^{(j)}\right) \theta_{r,j} = c && \text{(Dual price: } \lambda\text{)} \\ & \sum_{j=1}^{K_r} \theta_{r,j} = 1, \quad \forall r \in \{1, \dots, N\} && \text{(Dual price: } \pi_r\text{)} \end{aligned}$$

Problem (M) is solved via **Column Generation**:

1. **Restricted Master Problem (RMP):** Solved over a subset of generated extreme points to obtain dual multipliers λ (linking constraint prices) and π_r (convexity constraint prices).

2. **Subregion Pricing Subproblem:** Each subregion r solves a local subproblem independently to find a column with negative reduced cost:

$$\min_{x_r \in \mathcal{X}_r} \{f_r(x_r) - \lambda^{\top} B_r x_r\}$$

In the event that the optimal subproblem value is smaller than π_r, the extreme point $x_r\hat{}*$ is added as a new column to the RMP; otherwise, subregion r serves as an optimality proof with respect to the current dual price λ.

4.2. Benders (dual) decomposition

Benders decomposition formulation

When continuous operational subproblems decouple across subregions upon fixing high-level investment decisions $y \in Y \subseteq \mathbb{Z}^p$, the problem takes a two-stage recourse structure:

$$\min_{y \in Y} \left\{ c_y^{\top} y + \sum_{r=1}^{N} Q_r\,(y) \right\}$$

Where the local subregion recourse function $Q_r(y)$ evaluates the optimal operational cost for a given candidate design y:

$$Q_r(y) = \min_{x_r} \{f_r(x_r) \mid x_r \in \mathcal{X}_r(y)\}$$

Benders master problem (M_B)

At iteration k, the master problem uses auxiliary variables $\eta_r \geq Q_r(y)$ to construct lower-bounding cutting planes:

$$\begin{aligned}(\mathrm{M_B}) \quad \min_{y,\eta} \quad & c_y^\top y + \sum_{r=1}^{N} \eta_r \\ \text{s.t.} \quad & y \in Y, \\ & \eta_r \geq Q_r\big(\hat{y}^{(\ell)}\big) + \pi_r\big(\hat{y}^{(\ell)}\big)^\top \big(y - \hat{y}^{(\ell)}\big), \quad \forall r \in \{1, \dots, N\},\ \forall \ell \in \{1, \dots, k-1\}\end{aligned}$$

where $\pi_r\big(\hat{y}^{(\ell)}\big) \in \partial Q_r\big(\hat{y}^{(\ell)}\big)$ is a subgradient of Q_r at the previously evaluated investment plan $\hat{y}^{(\ell)}$, extracted from the dual solution of subregion r's operational subproblem.

Algorithm Workflow per Iteration

1. **Master Step:** Solve the mixed-integer master problem (M_B) to obtain a global candidate investment plan y^k.
2. **Subproblem Step (Parallel):** Fix $y = y^k$ and independently solve each local subproblem r:

$$\min_{x_r \in X_r(y^k)} f_r(x_r)$$

Extract the optimal local operational cost $Q_r(y^k)$ and the corresponding dual subgradient $\pi_r(y^k)$.

1. **Cut Generation:**
   - **Optimality Cut:** If subproblem r is feasible, append the cut $\eta_r \geq Q_r(y^k) + \pi_r(y^k)^\top (y - y^k)$ to (M_B).
   - **Feasibility Cut:** If subproblem r is infeasible, append a hyper-plane certifying infeasibility to eliminate y^k.

4.3. Consensus ADMM

Alternating Direction Method of Multipliers (ADMM) Scheme

In order to eliminate the centralization bottleneck of classical decomposition through the master problem, the formulation of "P" ' gives rise to a completely decentralized ADMM algorithm. For each iteration k=0,1,2, …, the steps involved are the following:

1. Local Primal Update (Parallel across subregions r)

Each subregion $r \in \{1, \dots, N\}$ independently updates its local decision vector by solving:

$$x_r^{k+1} = \arg\min_{x_r \in X_r} \left\{ f_r(x_r) + (\lambda_r^k)^\top B_r x_r + \frac{\rho}{2} \| B_r x_r - z_r^k \|^2 \right\}$$

2. Global Coordination Update

The consensus/coupling variables are updated by solving the centralized or consensus step:

$$z^{k+1} = \arg\min_{z} \left\{ g(z) + \sum_{r=1}^{N} \left(-(\lambda_r^k)^\top z_r + \frac{\rho}{2} \| B_r x_r^{k+1} - z_r \|^2 \right) \right\}$$

When $g(z)$ enforces simple consensus ($z_1 = z_2 = \cdots = z_N = \bar{z}$), this step admits the closed-form average:

$$\bar{z}^{k+1} = \frac{1}{N} \sum_{r=1}^{N} \left(B_r x_r^{k+1} + \frac{1}{\rho} \lambda_r^k \right)$$

3. Dual Price Update (Parallel across subregions r)

Each subregion updates its local multiplier via a simple gradient step:

$$\lambda_r^{k+1} = \lambda_r^k + \rho(B_r x_r^{k+1} - z_r^{k+1})$$

Key Computational Strengths

- **Zero Direct Inter-Subregion Exchange:** The updates to x_r operate independently among the subregions, running in parallel by means of localized optimization algorithms that do not exchange any information.
- **Lightweight Coordination:** z-update involves only basic vector addition/averaging and can be performed either by a lightweight central coordinator or through consensus gossip protocol on the coupling graph $\mathcal{G} = (\mathcal{R}, \mathcal{E})$.

5. Theoretical results

In this present the main theoretical results established in this paper: a theorem on duality gap (Theorem 5.1), a theorem on global convergence of the consensus ADMM scheme (Theorem 5.2), and a theorem on sublinear rate of convergence (Theorem 5.3).

5.1. Zero duality gap

Theorem 5.1 (Strong duality for block-angular infrastructure programs)

Let Assumption 3.1 hold. Then problem (P) has zero duality gap: the optimal value of (P) equals the optimal value of its Lagrangian dual. The dual formulation of the block-angular program (P) is defined as (D) $\max_{\lambda} d(\lambda)$, where the dual function $d(\lambda) = \min_{x_r \in \mathcal{X}_r,\, r=1,\dots,N} \{\sum_{r=1}^N f_r(x_r) + \lambda^\top (\sum_{r=1}^N B_r x_r - c)\}$ fully decomposes into N independent subproblems given by $d(\lambda) = -\lambda^\top c + \sum_{r=1}^N \min_{x_r \in \mathcal{X}_r} \{f_r(x_r) + \lambda^\top B_r x_r\}$; moreover, under the Slater condition (Assumption 3.1), strong duality holds and the dual optimum is guaranteed to be attained at some $\lambda^* \in \mathbb{R}^m$.

Proof of Theorem 5.1 (Strong Duality and Dual Attainment)

Weak Duality

Weak duality, $d(\lambda) \leq \text{val(P)}$ for every λ, holds universally without requiring convexity. For any λ and any feasible primal point x of (P), the linking condition implies $\sum_{r=1}^N B_r x_r - c = 0$, giving:

$$d(\lambda) \leq \sum_{r=1}^{N} f_r(x_r) + \lambda^\top \cdot 0 = \sum_{r=1}^{N} f_r(x_r)$$

Taking the infimum over all feasible x yields $d(\lambda) \leq \text{val(P)}$.

Perturbation Function and Convexity

To prove the reverse inequality, define the perturbation function $p: \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ as:

$$p(u) = \inf\{\textstyle\sum_{r=1}^N f_r(x_r) \mid x_r \in \mathcal{X}_r \quad \forall r, \quad \sum_{r=1}^N B_r x_r - c = u\}$$

so that $p(0) = \text{val(P)}$. Because each set $\mathcal{X}_r$ is convex and each f_r is convex, $p(u)$ is a convex function on its domain (since its epigraph $\text{epi}(p)$ is the linear projection of the convex set $\{(x, u, t) \mid x_r \in \mathcal{X}_r,\, t \geq \sum_r f_r(x_r),\, \sum_r B_r x_r - c = u\}$).

Sub differentiability under Slater's Condition

By Assumption 3.1, the Slater point $\hat{x}$ ensures $p(0) < +\infty$ and places $u = 0 \in \text{relint}(\text{dom}\, p)$. Since a convex function is subdifferentiable throughout the relative interior of its domain, there exists a subgradient vector $-\lambda^* \in \partial p(0)$ satisfying the subgradient inequality:

$$p(u) \geq p(0) - (\lambda^*)^\top u, \quad \forall u \in \mathbb{R}^m$$

Attainment of Dual Optimum

Evaluating the dual function $d(\lambda^*)$ yields:

$$\begin{aligned} d(\lambda^*) &= \inf_{x_r \in \mathcal{X}_r} \left\{ \sum_{r=1}^{N} f_r(x_r) + (\lambda^*)^\top \left(\sum_{r=1}^{N} B_r x_r - c \right) \right\} \\ &= \inf_u \{ p(u) + (\lambda^*)^\top u \} \end{aligned}$$

By rearranging the sub gradient inequality, $p(u) + (\lambda^*)^\top u \geq p(0)$ holds for all u, with equality achieved at $u = 0$. Thus, the infimum over u equals $p(0)$:

$$d(\lambda^*) = p(0) = \text{val(P)}$$

Combining this with weak duality ($d(\lambda^*) \leq \text{val(P)}$) yields $d(\lambda^*) = \text{val(P)}$, establishing strong duality and confirming attainment of the dual optimum at λ^*. ■

As a specific instance of Theorem 5.1, the formulation of MB as the dual of either decomposition is thus validated: the optimal multipliers λ* obtained from either of the two schemes correspond precisely to the shadow prices on the coupling constraints, without any loss of precision compared to the centralised setting, provided that the convexity conditions are met.

5.2. Global convergence of the distributed ADMM iteration

Theorem 5.2 (Global Convergence)

Let $f_r \in \Gamma_0(\mathbb{R}^{n_r})$ for $r = 1, \ldots, N$ (incorporating the indicator functions of $\mathcal{X}_r$) and let $g \in \Gamma_0(\mathbb{R}^m)$. Assume that the unaugmented Lagrangian of (P′) possesses a saddle point (x^*, z^*, λ^*). Then, for any fixed penalty parameter $\rho > 0$, the consensus ADMM iterates (x^k, z^k, λ^k) satisfy:

1. **Primal Residual Convergence:** $r^k = \sum_{r=1}^{N} B_r x_r^k - z^k \to 0$ as $k \to \infty$.
2. **Objective Value Convergence:** $\sum_{r=1}^{N} f_r(x_r^k) + g(z^k) \to \text{val(P′)}$ as $k \to \infty$.
3. **Dual Multiplier Convergence:** $\lambda^k \to \lambda^*$ along a convergent subsequence as $k \to \infty$.

Proof of Theorem 5.2

The proof uses traditional analysis of ADMM convergence in the block-angular convex structure of ("P" ').

• **Step 1: Lyapunov Function Construction**

Define the energy function $V^k = \frac{1}{\rho}\|\lambda^k - \lambda^*\|^2 + \rho\|z^k - z^*\|^2$. Evaluating the optimality conditions for the x-update ($-B_r^\top \lambda_r^{k+1} \in \partial f_r(x_r^{k+1}) + \rho B_r^\top (B_r x_r^{k+1} - z_r^{k+1})$) and z-update, together with the dual step $\lambda^{k+1} = \lambda^k + \rho r^{k+1}$, and applying the monotonicity of the maximal subdifferential operators of each f_r and g, yields the strict non-increasing property:

$$V^{k+1} \leq V^k - \rho\|z^{k+1} - z^k\|^2 - \frac{1}{\rho}\|\lambda^{k+1} - \lambda^k\|^2$$

• **Step 2: Summ ability and Vanishing Differences**

Because $V^k \geq 0$ and V^k is non-increasing, V^k converges to a finite limit $V^\infty \geq 0$. Telescoping the descent inequality from step 1 for $k = 0, \ldots, K$ gives:

$$\sum_{k=0}^{K} \left(\rho \|z^{k+1} - z^k\|^2 + \frac{1}{\rho} \|\lambda^{k+1} - \lambda^k\|^2 \right) \leq V^0 - V^{K+1} \leq V^0 < +\infty$$

Taking $K \to \infty$ confirms series convergence, establishing $\|z^{k+1} - z^k\| \to 0$ and $\|\lambda^{k+1} - \lambda^k\| \to 0$.

• **Step 3: Residual Convergence**

Using the multiplier step relation $\lambda^{k+1} - \lambda^k = \rho r^{k+1}$, the result $\|\lambda^{k+1} - \lambda^k\| \to 0$ directly implies:

$$r^{k+1} = \sum_{r=1}^{N} B_r \, x_r^{k+1} - z^{k+1} \to 0 \quad \text{as } k \to \infty$$

• **Step 4: Objective Optimality & Subsequence Argument**

The boundedness of $\{V^k\}$ ensures $\{z^k\}$ and $\{\lambda^k\}$ are bounded. Together with subregion coercivity and properness, $\{x^k\}$ remains bounded. Extracting a convergent subsequence $\left(x^{k_j}, z^{k_j}, \lambda^{k_j}\right) \to \left(\bar{x}, \bar{z}, \bar{\lambda}\right)$ and passing to the limit in the subproblem optimality conditions certifies that $(\bar{x}, \bar{z})$ is primal feasible and satisfies the KKT conditions for (P′). Hence, $\bar{x}$ is optimal, yielding $\sum_{r=1}^{N} f_r\left(x_r^{k_j}\right) + g\left(z^{k_j}\right) \to \text{val(P}')$. A standard subsequence argument extends objective value and residual convergence to the full sequence.

5.3. Sublinear ergodic convergence rate

Theorem 5.3 ($\mathcal{O}(1/\mathrm{K})$ Ergodic Convergence Rate). Under the hypotheses of Theorem 5.2, define the ergodic running averages $\bar{\mathrm{x}}_\mathrm{r}^\mathrm{K} = \frac{1}{\mathrm{K}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{x}_\mathrm{r}^\mathrm{k}$ for each $\mathrm{r} \in \{1, \ldots, \mathrm{N}\}$ and $\bar{\mathrm{z}}^\mathrm{K} = \frac{1}{\mathrm{K}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{z}^\mathrm{k}$; then there exists a constant $\mathrm{C} > 0$, depending only on ρ and the initial distance to an optimal saddle point $\|(\mathrm{z}^0 - \mathrm{z}^*, \lambda^0 - \lambda^*)\|^2$, such that for every iteration count $\mathrm{K} \geq 1$, the suboptimality and primal infeasibility satisfy $\left(\sum_{\mathrm{r}=1}^{\mathrm{N}} \mathrm{f}_\mathrm{r}\left(\bar{\mathrm{x}}_\mathrm{r}^\mathrm{K}\right) + \mathrm{g}(\bar{\mathrm{z}}^\mathrm{K}) \right) - \text{val(P}') \leq \frac{\mathrm{C}}{\mathrm{K}}$ and $\left\| \sum_{\mathrm{r}=1}^{\mathrm{N}} \mathrm{B}_\mathrm{r} \, \bar{\mathrm{x}}_\mathrm{r}^\mathrm{K} - \bar{\mathrm{z}}^\mathrm{K} \right\| \leq \frac{\mathrm{C}}{\mathrm{K}}$, established by applying Jensen's inequality to the sum of variational inequalities across $\mathrm{k} = 1, \ldots, \mathrm{K}$ and telescoping the Lyapunov energy terms.

Proof of Theorem 5.3. By convexity of each f_r and g, combining the subgradient inequalities characterizing the optimality of the x_r- and z-updates with the multiplier step relation $\lambda^{k+1} - \lambda^k = \rho r^{k+1}$ yields the standard single-step ADMM bound for any $k \geq 0$: $[\sum_{r=1}^{N} f_r\left(x_r^{k+1}\right) + g(z^{k+1})] - [\sum_{r=1}^{N} f_r\left(x_r^*\right) + g(z^*)] + (\lambda^{k+1})^\top r^{k+1} \leq \frac{1}{\rho}(\lambda^k - \lambda^*)^\top(\lambda^{k+1} - \lambda^k) + \rho(z^k - z^*)^\top(z^k - z^{k+1})$. Summing this inequality over $k = 0, \ldots, K-1$ telescopes the right-hand side via the Lyapunov decay argument from Theorem 5.2 to a quantity bounded by $\frac{1}{2}V^0 = \frac{1}{2}\left(\frac{1}{\rho}\|\lambda^0 - \lambda^*\|^2 + \rho\|z^0 - z^*\|^2\right)$, independent of K. Applying Jensen's inequality to the jointly convex objective function $\sum_{r=1}^{N} f_r + g$ and the linear residual map evaluated at the ergodic running averages $\bar{x}^K = \frac{1}{K}\sum_{k=1}^{K} x^k$ and $\bar{z}^K = \frac{1}{K}\sum_{k=1}^{K} z^k$ converts the summed per-iterate inequality into the bound $\left(\sum_{r=1}^{N} f_r\left(\bar{x}_r^K\right) + g(\bar{z}^K)\right) - \text{val(P}') \leq \frac{C}{K}$ and $\|\sum_{r=1}^{N} B_r \, \bar{x}_r^K - \bar{z}^K\| \leq \frac{C}{K}$, where the constant $C = \frac{1}{2}V^0 \cdot \max(1, C_{\text{norm}})$ is explicitly defined by $\|\lambda^0 - \lambda^*\|$, $\|z^0 - z^*\|$, and ρ, confirming the $\mathcal{O}(1/K)$ ergodic rate.

According to Theorem 5.3, the number of ADMM iterations needed to achieve a desired precision of ε in the coupling constraint residual is O(1/ε) and does not depend on the number of subdomains N; but since the major bottleneck of the algorithm is the solution of N parallel local subproblems, its complexity as a function of N is given by the analysis in Section 6, and not by the number of iterations.

6. Convergence and complexity analysis

In order to establish the complexity of the distributed algorithm, consider the theoretical rate of Theorem 5.3 in combination with the per-iteration cost and make the comparison with the classical iteration complexity of the Dantzig-Wolfe and Benders master problem formulations of Sections 4.1-4.2.

6.1. Per-iteration cost model

Under a synchronous, star-coordinated implementation, the wall-clock execution time per ADMM iteration is given by $T_{\text{round}} = \max_{r\in\mathcal{R}} T_{\text{loc}}^{(r)} + T_{\text{coord}} + 2 \cdot T_{\text{comm}} \cdot \text{diam}(\mathcal{G})$, where $T_{\text{loc}}^{(r)}$ denotes the worst-case time required by subregion r to solve its local subproblem (x_r-update), T_{coord} represents the computational latency of the global coordination step (z-update), T_{comm} is the network communication latency incurred when exchanging dual multipliers λ_r and local copies z_r, and $\text{diam}(\mathcal{G})$ accounts for the message propagation delay across the coupling graph diameter in a peer-to-peer consensus implementation (which simplifies to $\text{diam}(\mathcal{G}) = 1$ under a centralized star topology with a direct coordinator).

6.2. Overall complexity

The combination of Theorem 5.3, which proves that $K = O(C/\varepsilon)$ iterations suffice for reaching primal-dual residual accuracy of ε, together with the per-iteration computation, gives a total complexity of,

$$T_{\text{total}}(\varepsilon) = \mathcal{O}\left(\frac{C}{\varepsilon} \cdot \left[\max_{r\in\mathcal{R}} T_{\text{loc}}^{(r)} + T_{\text{coord}} + T_{\text{comm}}\right]\right)$$

Because the Lyapunov energy constant C depends strictly on the initial distance to the saddle point $\|\lambda^0 - \lambda^*\|^2 + \|z^0 - z^*\|^2$ and not explicitly on the number of subregions N, the required iteration count K is, to first order, scale-independent with respect to network partitioning. The total runtime T_{total} scales with N only through $\max_r T_{\text{loc}}^{(r)}$, which assuming balanced subregions grows as a function of local dimension n_r rather than total regional count N. This offers a key complexity advantage over centralized Dantzig-Wolfe and Benders methods, in which the size of the master problem grows dynamically by adding columns or cuts, resulting in centralized master problem (LP/MILP) solving being a bottleneck when N becomes hundreds of subregions.

7. Numerical results

To verify the theoretical results obtained in Sections 5 and 6, constructed an artificial large-scale infrastructure test suite using a transmission-and-storage expansion model for multiple regions, with a total of 500 nodes distributed across $N = 42$ subregions connected through a sparse connectivity graph G (subregion nodes average degree of 3.1) having 187 constraints connecting these tie lines; each subproblem $r \in \{1, \ldots, N\}$ being formulated as a convex quadratic programming problem with nr≈640 decision variables each, solved in a distributed computing setting with realistic distributions for subregion solution times $T"loc"\ ((r)\ )$ and inter-nodal communication delays T_{comm} .

7.1. Convergence behavior

Both the primal residual $\|\sum_{r=1}^{N} B_r\, x_r^k - z^k\|$ and the dual residual $\|\lambda^{k+1} - \lambda^k\|$ decreased monotonically, consistent with Theorem 5.2, reaching a relative residual tolerance of 10^{-4} after 158 ADMM rounds for $\rho = 1.0$, 96 rounds for $\rho = 4.5$ (empirically near-optimal), and 241 rounds for $\rho = 0.2$, illustrating the practical sensitivity of ADMM convergence speed to the penalty parameter ρ despite the ρ-independence of the asymptotic guarantee in Theorem 5.2; moreover, the observed decay of the ergodic objective gap tracked the $\mathcal{O}(1/K)$ bound of Theorem 5.3 closely for $K \geq 40$ rounds, with the empirical gap falling within a constant factor of 1.6--2.3 of the theoretical bound across the tested range of ρ.

7.2. Solution quality and runtime

Comparison of the consensus ADMM algorithm is carried out by comparing it with the centralized interior-point method for solving problem (P) and with the Benders decomposition algorithm of section 4.2 for the investment variable part of the test problem in table 2.

Table 2: Solution quality and runtime.

Method	Wall-clock time (s)	Optimality gap (%)	Peak memory (centralized-equivalent, %)
Centralized interior-point	3,412	0.00 (reference)	100
Bender's decomposition	1,845	0.11	38
Dantzig–Wolfe decomposition	1,660	0.09	41
Consensus ADMM (ρ = 4.5)	985	0.27	9

The consensus ADMM approach led to an approximate 71% reduction in wall clock time required for obtaining the solution when compared with the centralized interior-point approach and a reduction of approximately 41–47% when compared with the Benders and Dantzig-Wolfe approaches, while increasing the optimality gap by just 0.27% as opposed to 0.09–0.11%, in keeping with the asymptotic convergence guarantee of Theorem 5.2 as opposed to the finite convergence guarantees of the master problem based algorithms. The peak memory requirements, primarily due to the factorization of the coupling constraint matrix in the centralized scenario, saw a reduction by about an order of magnitude under the distributed framework, since none of the computational nodes ever possessed the entire decision vector x.

7.3. Sensitivity to problem scale

Experimenting again with N ranging from 10 to 80 regions (with the average local problem size held constant) indicated that the number of iterations of ADMM needed to achieve 10^{-4} residual error increased from 141 to 173, roughly 23% increase for an increase by a factor of 8 in N, confirming the close to N-independence of the iteration complexity suggested in Section 6.2, whereas in the case of Benders' and Dantzig–Wolfe decomposition, the master problem size increased roughly linearly with N.

8. Discussion

To begin with, the decision on whether to use primal decomposition (Dantzig–Wolfe), dual decomposition (Benders), or consensus decomposition (ADMM) must be made considering mainly the structure of the coupling and organizational issues regarding information exchange, and not just based on convergence properties. In the case where there are a small number of subregions N (dozens) and exact convergence is needed – for instance in situations of regulatory requirements for verifiable optimality proofs – the methods discussed in Sections 4.1–4.2, namely the Benders or Dantzig–Wolfe approach, continue to be more appropriate due to Theorem 5.1.

Secondly, in situations where N is large (i.e., hundreds of subregions, as is becoming increasingly common for distribution-level and multi-utility infrastructure coordination), the asymptotically independent of N iteration complexity of the consensus ADMM algorithm proved in Section 6.2 and shown numerically in Section 7.3 favors this approach at the expense of an asymptotic, rather than finite, convergence guarantee and the associated small residual optimality gap. The dependence of the convergence rate on the value of the penalty parameter ρ seen in Section 7.1, even though ρ-independent theoretically according to Theorem 5.2, highlights the practical significance of adaptive penalty parameter strategies, which did not consider in detail in this study but represent an obvious extension thereof.

Finally, the findings hold with a slight modification for mixed-integer local feasible sets Xr, which occur whenever local investment choices (which pipeline section or transmission line to construct) are distributed among subregions. Here, Theorem 5.1 cannot be applied directly, as a nonconvex set Xr would cause a duality gap; the Benders decomposition of Section 4.2, on the other hand, can still be applied without any modification, because it decomposes the integer variables from the master problem and leaves the decomposition of the continuous recourse unaltered, whereas the consensus ADMM algorithm of Section 4.3 needs to solve the xr-subproblem as a (small) mixed-integer problem in each iteration, which is possible for the small-sized local problem of the infrastructure subregions, but does not have the convexity-related convergence guarantees of Theorem 5.2.

Fourth, an inherent constraint in the current study is that subregions and the central coordinator communicate in a synchronous and reliable manner. Infrastructure coordination networks have delays in communication, packet losses, and asynchronous update times owing to the different planning cycles at which jurisdictions function. Future research should explore how to adapt the guarantees of convergence from Theorems 5.2 and 5.3 in an asynchronous and unreliable communication setup, leveraging existing literature on asynchronous-ADMM algorithms, along with methods to ensure privacy of subregions such that they can share only their aggregated prices or flows with the rest of the network.

9. Conclusion

The research intended to design a methodological framework to efficiently solve the optimization problem of large-scale infrastructure systems with distributed decision-making factors, coupling constraints, and high-dimensional solution space. This paper has attempted to overcome the weaknesses of centralized optimization by designing a framework through which decision-making processes can be coordinated among distributed systems without compromising on the quality of the solutions and converging properties. The methodology used in this study included formulating an optimization problem based on the block-angular structure and implementing three decomposition techniques, namely Dantzig-Wolfe decomposition, Bender's decomposition, and Consensus-Based Alternating Direction Method of Multipliers (ADMM). These three methods relied on sub-problem optimization, consensus variable, and coupling constraint to achieve efficient communication among distributed entities through parallel computation. The theoretical investigations of the decomposition techniques in this paper proved strong duality, global convergence properties, and sub-linear convergence properties for the distributed optimization problem. The numerical results of the research have indicated that the consensus ADMM decomposition technique performed better than the centralized and classical decomposition techniques in terms of scalability. It has also been observed that the proposed approach saved about 71% of computational time when compared with the centralized optimization method. Directions for future research include development of the framework to handle asynchronous communication settings, privacy-preserving distributed optimization algorithms, and mixed-integer models of infrastructures. Adaptive parameter settings, decomposition methods taking into account uncertainty, and intelligent decision-support systems are additional improvements that could be included in the framework.

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