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# Product Life Cycle Predictions Using A Novel Multi-Stage Fuzzy Product Wise Regression Analysis

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**Abstract**
The project developed in a consecutive order through the stages like planning, initiation, execution, monitoring and completion. Accurate forecasting methods help in making knowledgeable decisions about production, R&D, financeand targeting a product. Proposed a novel approach, called multi-stage fuzzy product-wise regression analysis. g. This research purposes to create a novel forecasting method to predict both the PLT and yearly shipments throughout the entire life cycle of products.

*Keywords:* Antilock Breaking System, Fuzzy linear regression, Product Life Cycle.

## Introduction

This research presents a new linear programming-based regression method using a fuzzy model with symmetric triangular fuzzy parameters. PLC prediction is crucial for effective strategic planning and informed decision-making across businesses, research institutionsand government sectors. Fuzzy linear regression has gained popularity in recent years, and this work proposes a new solution to PLC forecasting using a linearinterval regression model. The necessity problem by using change-points, which are predictable from traditional product-wise regression techniques. The proposed method has some advantages: it can predict non-linear data, regression models simultaneously, offers a global optimal solution and robust than conventional fuzzy regression and determines change-point positions.

## Review Of Literature

Julian Baehr et al. (2024) highlighted that uncertainty assessment methods are not limited to academic Life Cycle Assessment (LCA) experts. They suggested that various methods, like the Q metadata method, could be easily integrated into environmental product declarations (EPD) with minimal effort. Additionally, digital products could provide valuable input features for artificial neural network-based prediction models.

Rehim, Mostafa, and Adel (2021) illustrated the impact of modifying model parameters on the structure of the Product Life Cycle.

Won-Joon Kim, Tai-Yoo Kim and Jeong-Dong Lee (2005), presented a cross-sectional analysis framework, integrating a diffusion model to track demand dynamics over time using time-series analysis.

Solomon, Rajeev &Pecht, Michael (2001)analysed the forecast life cycles methodology of electronic parts is accessible in both years to obsolescence and life cycle stages are predicted.

**Developing the methods for Product Life Cycle predictions**

The possibility theory discussed in this study is fundamentally linked to the fuzzy set theory, where a possibility distribution is interpreted as a fuzzy restriction, serving as a flexible constraint on the potential values of a variable. To overcome certain analytical challenges, practical approaches are proposed: the first utilizes a product-wise model to manage the necessity problem. Next to the fuzzy product-wise regression technique to address automatic change-point detection in nonlinear datasets. Comprehensive details on the general framework and procedures of fuzzy product-wise regression analysis are elaborated in this work.

Analytic methods and framework for multiple Product Life Cycle prediction

The proposed approach, illustrated in Figure 1, involves four main steps:

1st step : estimating the total duration of the PLC
2nd Step : forecasting the yearly number of dispatched specific products
3rdStage : identifying and predicting the distinct phases within the PLC;
Finally, assessing forecasting errors to validate the effectiveness of the multi-stage fuzzy regression approach.

The next section presents the concept of interval arithmetic, which forms the basis of fuzzy product-wise regression. Section C discussed with a fuzzy regression model capable of handling non-linear time series predictions, while Section 5 measures the total efficiency of the forecasting model.

The concept of interval arithmetic

A linear interval model incorporating independent variables is developed using interval parameters to effectively capture data uncertainty.

$Ai(\ I\ = 0,1,2,3, \ldots\ ,\ q)$as

$$Y(x_t) = \sum_{i=1}^{q} \binom{n}{k} x^k a^{n-k} \quad (1)$$

Where, $Y(xt)$is the predicted interval corresponding to the input vector xt and t is the time datum

Where $Y(xt)$denotes the predicted interval associated with the input vector and t represents the corresponding time variable (t=1,2,3,…,n )andxt=( x1t,x2t, x3t…,xqt).

In brief, x=( x1,x2, x3…,xq )represents a q-dimensional input vector, where closed intervals are represented by such as A and B and each interval, as

$$A = [a^L, a^R] \text{means } a^L \leq a \leq a^R \quad (2)$$

whereaLrepresents the left limit and aRrepresents the right limit of (A).

Interval (A) represented by its middle and width as include $a = ac - awa = a_c - a_{wa} = ac - aw$ and a

$= a\,c + a\,w\,a = a_c + a_wa = ac + aaw,$

use:

$A = [a_c - a_w,\ a_c + a_w]$or

$A = \{a : |\ a - a_c\ | \leq a_w\}$

$A = [a_c - a_w,\ a_c + a_w]$

$A = \{a : |a - a_c| a_w\}$

$A = [a_c - a_w, a_c + a_w]$or

$A = \{a: | a - a_c | \leq a_w\}$ (3)

whereacmeansthe center and aw denotes the width (radius, $a_w \geq 0$). (Equation 2 andEquation 3)as

$a_i^c = \frac{a_i^{min} + a_i^{max}}{2}$ (4)

For all i=1,2,3, …, q

i = 1, 2,

$a_i^w = \frac{a_i^R - a_i^L}{2}$ (5)

The subsequent operations involving addition and multiplication are applied from this point forward.

$A + B = (a_c, a_w) + (b_c, b_w)$

$= (a_c + b_c, a_w + b_w)$and(6)

$rA = (r \cdot a_c)a_c + (r \cdot a_w)a_w$ (7)

with r belonging to the set of real numbers.

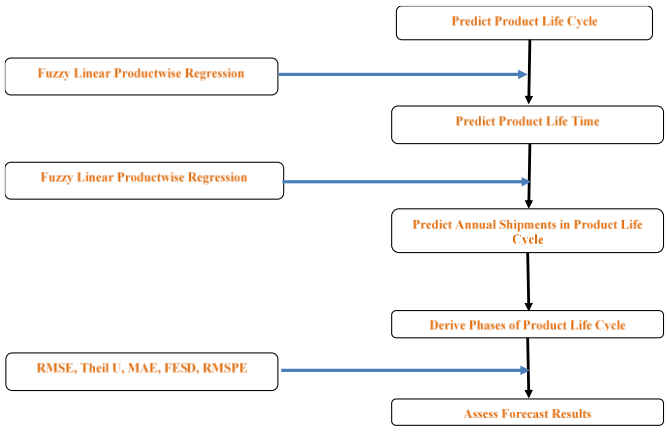



Fig.1. A predictive model for analyzing the Product Life Cycle

**Linear Interval Model**

The specifics of the linear model from Equation 1:

$y(x_t) = A_0 + A_1 x_{1t} + a_{2c} x_{2t} + a_{3c} x_{3t} + \cdots + a_{qc} x_{qt}$ (8)

$y_c(x_t) = a_{0c} + a_{1c} x_{1t} + a_{2c} x_{2t} + a_{3c} x_{3t} + \cdots + a_{qc} x_{qt}$ (9)

$y_w(x_t) = a_{0w} + \sum_{j=1}^{q} a_{jw} . abs(x_{jt})$ (10)

Where:

$A = [A_0, A_1, A_2, A_3, \ldots, A_{0q}]$ T is the vector of coefficients,

$x_t = [x_{1t}, x_{2t}, x_{3t}, \ldots, x_{qt}]$ T is the vector of variables,

$C = [a_{0c}, a_{1c}, a_{2c} \ldots, a_{qc}]T$ is the vector of centers,

$w=[a_{0w}, a_{1wc}, a_{2wc} \ldots, a_{qw}]T$ is the vector of half-widths.

This expresses the equation in a more compact linear form vector of variables.

2) Necessity and Possibility models (Estimation)

The both input data and output data (xt, Y(xt)) can be assessed through multi estimation models. Thepossibilityestimation modelrepresentedas

$$y(x_t) = a_{0w} + \sum_{j=0}^{q} a_j x_{jt}$$

where $a_j = (a_{jc}, a_{jw})$ (11)

thefollowingconditions:

$$\forall_t \in \{1, 2, 3, 4, \ldots, n\},$$

$$y_t = \text{Observed outputs such that}$$

$$y_t \subseteq y*(x_t) \quad (12)$$

TheassessedintervalY∗(xt)bythepossibilitymodel

TheobservedintervalYt(Equation 27). Thenecessityestimatedmodelrepresentedto:

$$y*(x_t) = J = \sum_{j=0}^{q} a_j^* x_{jt}$$

where $a_j^* = (a_{jc}^*, a_{jw}^*)$ (13)

thefollowingconditions:

$$\forall t, f(x_t) \subseteq y_t$$

Where $f(x_t) = Y*(x_t)$ is the prediction function.for each

$T = 1,2,3,\ldots,n$ (14)

intervalYt.

Predicted interval Y∗(xt) is maximized and is included the observed data.

Equations (22), (23) and (24) in Section 2.B.(4). These relations stated asY∗(xt)⊆Yt ⊆Y∗(xt)and illustrated by Figure.2.

3) Forecasting with interval-valued data and addressing the related computational and analytical issues

To predicted interval-valued dataframework of interval regression is formulated and represented by Equations (15), (16), and (17), defining the essential structure of the predictive model

Minimize

$\sum_{t=1}^{n} Y_w(x_t)$ (15)

Subjectto

$\forall y \in Y_t, \quad y \in Y*(x_t),$

$t = 1,\ 2,\ 3 \ldots, n$ (16)

$0 < a^*_{iw}$

$i = 1,2,3,\ldots,q.$ (17)

LPproblemisfollows:

Minimize

$$\sum_{t=1}^{n} a^*_{0w} + a^*_{1w} \mid x_{1t} \mid + a^*_{2w} \mid x_{2t} \mid + \cdots + a^*_{qw} \mid x_{qt} \mid ) \quad (18)$$

Subjectto

$$(a^*_{0c} - a^*_{0w}) = + \sum_{i=1}^{q} (a^*_{ic} - a^*_{iw})\, x_{it} < Y_t^L \quad (19)$$

t=1,2,3,...,n;

$$\left(a^*_{0c} + \sum_{i=1}^{q} a^*_{ic}\, x_{it}\right) + \left(a^*_{0w} + \sum_{i=1}^{q} a^*_{iw}\, x_{it}\right) > Y_t^R \quad (20)$$

$$a^*_{0w}, a^*_{iw} > 0$$

t=1,2,3...,n; (21)

The model used to thesystemshouldhaveandusedtobewarpedandbenttoomuchbyseveralfluctuating data.As more data are gathered, Equations (15)–(21) become progressively fuzzier, leading to the loss of a clear operational definition or interpretative meaning..

4)Necessity regression analysis and the associated problem

The necessity regression presented and formulated problem being familiarized in Equations. (Equation 13 and 14) can be written as:

Maximize

$$\sum_{t=1}^{n} Y^*_w (x_{it}) \quad (22)$$

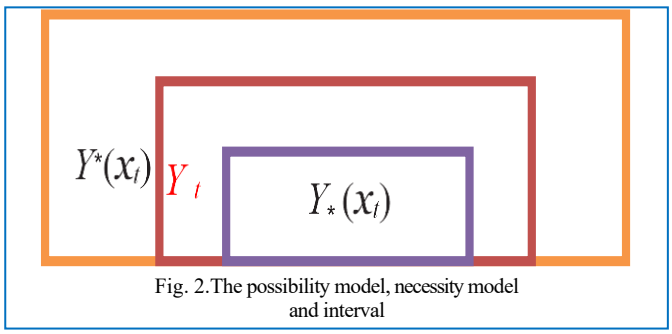


Fig. 2.The possibility model, necessity model and interval

Where, Bi(x) are the basis functions (include hinge functions) and ci are coefficients determined during the model fitting process.

Intercept: The constant vector (−172.08,−323.41)(-172.08, -323.41)(−172.08,−323.41) sets the baseline values for the response.

Linear Term: The vector (184.08, 322.81) (184.08, 322.81) (184.08, 322.81) scales the input xj, introducing linearity into the model.

Hinge Functions: The terms involving $|xj - k| + xj - k|$hinge functions, which are product wise linear functions that change slope at specified " knot " points. These functions allow the model to capture non-linear relationships by introducing kinks at specific values ofxj:

$Y*(x_t) \subseteq Yt,$

$t = 1, 2, 3, \ldots, n$ (23)

$a^*_{iw} > 0 \qquad i = 1, 2, 3, \ldots, q$ (24)

Where,

The corresponding Linear Programming (LP) problem is formulated as follows: Maximize

$$\sum_{t=1}^{n} + \left(a^*_{0w} + \sum_{i=1}^{q} a^*_{jw} |x_{jt}|\right) \quad (25)$$

Subject to

$$\left(a_0^c + \sum_{\{i=1\}}^{q} a_i^c x_{\{it\}}\right) - \left(a_0^w + \sum_{\{i=1\}}^{q} a_i^w x_{\{it\}}\right) > Y_t^L,$$

$t = 1, 2, 3 \ldots, n$ (26)

$$\left(a_0^c + \sum_{\{i=1\}}^{q} a_i^c x_{it}\right) - \left(a_0^w + \sum_{\{i=1\}}^{q} a_i^w x_{\{it\}}\right) > Y_t^L,$$

$$t = 1, 2, 3 \ldots, n$$

$$\left(a^*_{0c} + \sum_{\{i=1\}}^{q} a^*_{ic} x_{it}\right) - \left(a^*_{0w} + \sum_{\{i=1\}}^{q} ax_{it}\right) > Y_t^R,$$

t=1,2,3,...,n;

$$(a0c* + \sum_{ji=1}^{q} a_{ic} x_{it}) - (a0w* + \sum_{ji=1}^{q} a_{iw} x_{it} < YtR,$$

t=1,2,3,...,n; (27)

$$\sum_{t=1}^{n} Yw(x_t) = na_{0w} + \sum_{j=1}^{q} |a_{\{jw\}}| a_{jw} | t = \sum_{t=1}^{n} |xjt|$$

$$\left(a_0^c + \sum_{\{i=1\}}^{q} a_i^c x_{\{it\}}\right) - \left(a_0^w + \sum_{\{i=1\}}^{q} a_i^w x_{\{it\}}\right) > Y_t^L,$$

$$t = 1, 2, 3, \ldots, n$$

$$\left(a0c* + i=1 \sum qaic * xit\right) - \left(a\,0\,w* + i=1 \sum qa_i^w * x_{it}\right) < YtR,$$

$$t = 1, 2, 3, \ldots, n$$

$$a_0^w, a_1^w > 0,$$

$i = 1, 2, 3, \ldots, q$ (28)

Standard necessity regression approach is unsuitable for datasets that display significant variability or frequent fluctuations.

5) The perception of fuzzy product wise regression analysis

TABLE I
AUTOMOBILE PRODUCTION, DOMESTIC SALES AND EXPORTS TRENDS
(2019-20 TO 2024-25)

| Automobile Production Trends | | | | | | |
|---|---|---|---|---|---|---|
| Category | 2019-20 | 2020-21 | 2021-22 | 2022-23 | 2023-24 | 2024-25 |
| Passenger | 3424564 | 3062280 | 3650698 | 4587116 | 4901840 | 5061164 |
| Commercial | 756725 | 624939 | 805527 | 1035626 | 1067504 | 1032645 |
| Three Wheelers | 1132982 | 614613 | 758669 | 855696 | 996159 | 1050020 |
| Two Wheelers | 21032927 | 18349941 | 17821111 | 19459009 | 21468527 | 23883857 |
| Quadricycles | 6095 | 3836 | 4061 | 2897 | 5006 | 6488 |
| Grand Total | 26353293 | 22655609 | 23040066 | 25940344 | 28439036 | 31034174 |
| Automobile Domestic Sales Trends | | | | | | |
| Passenger | 2773519 | 2711457 | 3069523 | 3890114 | 42 18750 | 4301848 |
| Commercial | 717593 | 568559 | 716566 | 962468 | 968770 | 956671 |
| Three Wheelers | 637065 | 219446 | 261385 | 488768 | 694801 | 741420 |
| Two Wheelers | 17416432 | 15120783 | 13570008 | 15862771 | 17974365 | 19607332 |
| Quadricycles | 942 | 12 | 124 | 725 | 725 | 120 |
| Grand Total | 21545551 | 18620233 | 17617606 | 21204846 | 23857411 | 25607391 |
| Automobile Exports Trends | | | | | | |
| Passenger | 662118 | 404397 | 577875 | 662891 | 672105 | 770364 |
| Commercial | 60379 | 50334 | 92297 | 78645 | 65818 | 80986 |
| Three Wheelers | 501651 | 393001 | 499730 | 365549 | 299977 | 306914 |
| Two Wheelers | 3519405 | 3282786 | 4443131 | 3652122 | 3458416 | 4198403 |
| Quadricycles | 5185 | 3529 | 4326 | 2280 | 4178 | 6422 |
| Grand Total | 4748738 | 4134047 | 5617359 | 4761487 | 4500494 | 5363089 |

Source: www.siam.in

It focuses on univariate linear product-wise regression, which is often used in forecasting problems.

$$Y * (x_j) = \left(h(x_j) + \sum_{t=1}^{K-1} B_t^* \cdot \frac{|x_j - P_t| + (x_j - P_t)}{2}\right) (29)$$

This form is commonly used in spline regression or piecewise linear models, where:

$h(x_j)$ is a base function

$B_t^*$ are coefficients,

$P_t$are breakpoints,

The$\frac{|x_j - P_t| + (x_j - P_t)}{2}$

make simpler to:

$\max(0, x_j - P_t)$

To simplify the explanation,

All data points except :P kserve as change points.

Initial necessity regression model :k−1change points.

The main transformation between [Equations (29) and (22)] is introduced in Equation (30), in which represents the product-wise formulation for the data.

$$\sum_{t=1}^{K-1} B_t^* \cdot \frac{|x_j - P_t| + (x_j - P_t)}{2} + \sum_{t=1}^{K-1} B_{tc}^* \cdot \frac{|x_j - P_t| + (x_j - P_t)}{2} + \sum_{t=1}^{K-1} B_{tw}^* \cdot \frac{|x_j - P_t| + (x_j - P_t)}{2}$$
(30)

This says:

Total hinge effect = Component C effect + Component W effect, $t = 1,2,3,\ldots,n$.

The product wise necessity linear programming analysis is defined as follows: Maximize

$$z = \sum_{t=1}^{n} [A0w * + A1w * xj + \sum_{t=1}^{k-1} Bt *$$

$$(|xj - P1| + xj - P1) / 2]$$

Where

$$[\sum_{t=1}^{K-1} B_t^c \frac{|x_j - P_t| + (x_j - P_t)}{2} - [a_{0*}^w + a_{1*}^w xj \sum_{t=1}^{K-1} B_{tc}^w . \frac{|x_j - P_t| + (x_j - P_t)}{2}] > Y\,jL \qquad (31)$$

$$a_{0*}^c + a_{1*}^c x_j +$$

A multivariate product wise linear regression in adding any more independent variables to extended model.

6) Forecast model

Residual error testing method to assess the efficiency and accurateness of the forecasting process.as

$$err\,(x_t) = \frac{|y(x_t) - \hat{y}(x_t)|}{|y(x_t)|} \times 100\,\% \qquad (32)$$

This is known as MAPE when averaged over $t$.

The term err $(x_t)$ represents the percentage error among the real value of $y(x_t)$ and the predicted value of $\hat{y}(x_t)$ for the year wiseshipped product in year t of the PLC. The total forecast error for the completePLC can be measured by calculating these yearly errors.

If err $(j) = |Y_j - \hat{y}_j|$, Then

$$err = \frac{1}{n} + \sum_{j=1}^{n} |y_j - \hat{y}_j| \qquad (33)$$

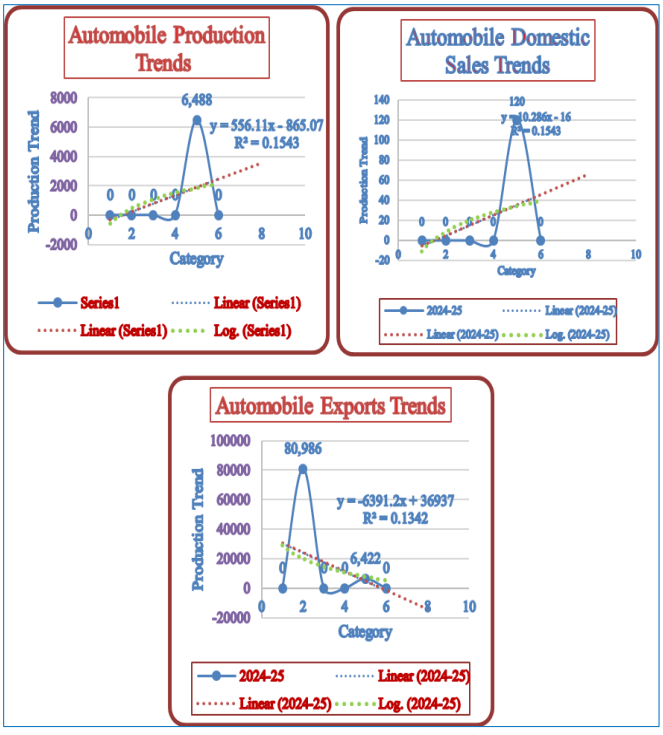



Fig. 3.Automobile Production, Domestic Sales and Exports Trends Between the Years (Forecast) (2019 - 20 to 2024 - 25)

**Automobile industry**

The automobile industry engaged in the design, production, marketing, sales, development, customization, and maintenance of vehicles. India's yearly production of automobiles in past six financial years Automobile Production Trends Between the years (2019-20 to 2024-25) in the following table 1. India has a strong automobile market in terms of exports, domestic salesand production trends.

Automobile Production Trends in India Between the years (2019-20 to 2024-25) in the following Figure 3.

**Production**

The industry produced a total of 2.84 Crore vehicles in Financial Year 23-24, as against 2.59 Crore vehicles in Financial Year 22-23.

**Forecast results**

The Antilock Breaking System was Utilized to test and confirm the effectiveness of the proposed fuzzy regression forecasting methodology, since Antilock Breaking System is the final Product Suspension and Brake Systems which just finished its lifetime in 2024.BaseduponthеProduct Life Cycle data of six Products of main stream electromagnetic,brake pads, rotors, calipers, drums, and hydraulic components provided by the industry,predict the ABS life time in Section 5(B); the YearlyDispatched Products volumes of ABS and change points of the severaldifferent phases of theProduct Life Cycle in Section 5(B) and finally, Section 5(C) measurement of forecast model efficiency.

**Prediction of the Product Life Time**

First,wepredictthePLTofABSforthepredictionofyear wiseShipped Products.PLTforelectromagnetic (Part 1),brake pads (Part 2), rotors (Part 3), calipers (Part 4), drums (Part 5), and hydraulic components (Part 6) havebeen12,10,17,16,18 and 15years,correspondingly.ToppredictthePLTfortheABS products,thelifetime oftheSuspension and Brake Systemsasthefuzzyregression analysis to the product wisechangepoints.

Equation (31) applying the parameters of Equation (29) using LP easily. By applying (Equation 34), PLT of the next ABS Product, by setting xj as seven. The projected average life-time for the product is 12 years; the higher limit is 12.60 years and the lower limit is 11.40 years (Figure. 4). Versus the real life - time for the product was 12 years.

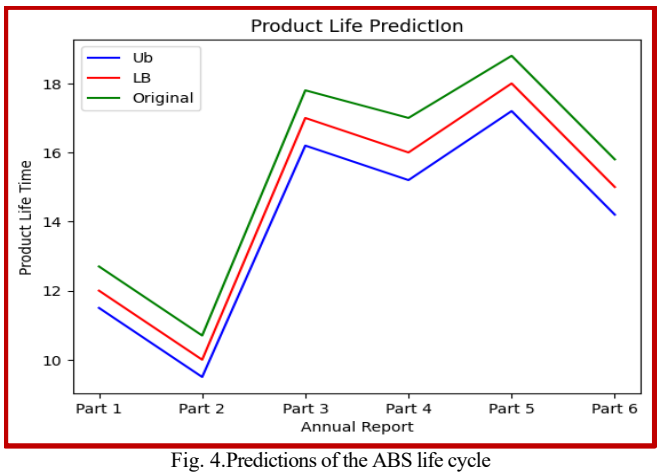



Fig. 4.Predictions of the ABS life cycle

**Prediction of the Yearly Dispatched Products**

After our effective prediction of the Antilock Breaking System Product Life Time, Yearly Antilock Breaking System Dispatched Products were predicted, based upon the YearlyDispatched Products of the earlier Products of electromagnetic, brake pads, rotors, calipers, drums, and hydraulic components. The derived equations (31) for the first to the 12thyear are listed below, as a basis for furthermore calculation of the yearly Dispatched Products and change points.

$Y*(x1)$: = (485204.2, -240264.6)

= add_tuples (result, mul_tuple_scalar((129.7958, 209.5146), xj))

= add_tuples (result, mul_tuple_scalar((-690.7958, -181.4646), abs(xj - 1) + xj - 1))

= add_tuples (result, mul_tuple_scalar((543, -27.15), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((-8, 0.4), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((182, -9.1), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((-303, 15.15), abs(xj - 5) + xj - 5))

Y∗(x2) = (-16462, -21332)

= add_tuples(result, mul_tuple_scalar((21752, 21068), xj))

= add_tuples(result, mul_tuple_scalar((-25034, -20904), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((1715, -85.75), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((2826, -141.3), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((1360, -68), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((-5626, 281.3), abs(xj - 5) + xj - 5))

Y∗(x3) = (-224090, -241930)

= add_tuples(result, mul_tuple_scalar((252100, 240530), xj))

= add_tuples(result, mul_tuple_scalar((-259330, -240170), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((-929, 46.45), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((33503, -1675.1), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-20748, 1037.4), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((-22777, 1138.8), abs(xj - 5) + xj - 5))

Y∗(x4) = (-2523000, -2448000)

= add_tuples(result, mul_tuple_scalar((2580500, 2445100), xj))

= add_tuples(result, mul_tuple_scalar((-2567900, -2445700), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((21344, -1067.2), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((63633, -3181.6), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-86431, 4321.6), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((-6172430, 3086.6), abs(xj - 5) + xj - 5))

Y∗(x5) = (-2137600, -2008300)

= add_tuples(result, mul_tuple_scalar((2214800, 2004500), xj))

= add_tuples(result, mul_tuple_scalar((-2108000, -2009800), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((80490, -4024.5), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((59351, -2967.5), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-457130, 22857), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((252270, -2614), abs(xj - 5) + xj - 5))

Y∗(x6) = (-2288900, -2121100)

= add_tuples(result, mul_tuple_scalar((2358900, 2117600), xj))

= add_tuples(result, mul_tuple_scalar((-2207400, -2125200), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((478660, -23933), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((-715400, 35770), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-16017800, 80085), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((295450, -14773), abs(xj - 5) + xj - 5))

Y∗(x7):

# Original fuzzy regression results

result = (-2288900, -2121100)

= add_tuples(result, mul_tuple_scalar((2358900, 2117600), xj))

= add_tuples(result, mul_tuple_scalar((-2207400, -2125200), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((478660, -23933), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((-715400, 35770), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-16017800, 80085), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((295450, -14773), abs(xj - 5) + xj - 5))

# Extract the low and high values from the result tuple

low, high = result

# Calculate 10% reduction

reduction_factor = 0.10

low_reduced = low - (reduction_factor * low)

high_reduced = high - (reduction_factor * high)

# Return the reduced values

returnlow_reduced, high_reduced

# Example usage:

xj = 3 # Example value of xj

low, high = Y6_star(xj)

print(f”Forxj = {xj}:”)

print(f”Reduced Low Value (10% less): {low}”)

print(f”Reduced High Value (10% less): {high}”)

Y∗(x8) = (-2132200, -1872100)

= add_tuples(result, mul_tuple_scalar((2145200, 1871500), xj))

= add_tuples(result, mul_tuple_scalar((-1861700, -1885700), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((-198120, 9906.2), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((313040, -15652), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-357760, 17888), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((777530, -38876), abs(xj - 5) + xj - 5))

Y∗(x9) = (-2079300, -1835800)

= add_tuples(result, mul_tuple_scalar((2083900, 1835600), xj))

= add_tuples(result, mul_tuple_scalar((-1927300, -1843400), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((-165360, 8268.2), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((476600, -23830), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-486890, 24344), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((720670, -36034), abs(xj - 5) + xj - 5))

Y∗(x10) = (-2499700, -2322500)

= add_tuples(result, mul_tuple_scalar((2502100, 2322400), xj))

= add_tuples(result, mul_tuple_scalar((-2433600, -2325800), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((-12338, 616.9), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((115850, -5792.5), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((-2386, 119.3), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((260640, -13032), abs(xj - 5) + xj - 5))

Y∗(x11) = (-2484300, -2473100)

= add_tuples(result, mul_tuple_scalar((2486600, 2473000), xj))

= add_tuples(result, mul_tuple_scalar((-2488800, -2472900), abs(xj - 1) + xj - 1))

= add_tuples(result, mul_tuple_scalar((67584, -3379.2), abs(xj - 2) + xj - 2))

= add_tuples(result, mul_tuple_scalar((62655, -3132.8), abs(xj - 3) + xj - 3))

= add_tuples(result, mul_tuple_scalar((145810, -7290.7), abs(xj - 4) + xj - 4))

= add_tuples(result, mul_tuple_scalar((-103200, 5159.9), abs(xj - 5) + xj - 5))

Y∗(x12) = (-2480000, -2444300)

= add_tuples(result, mul_tuple_scalar((2482400, 2444100), xj))

= add_tuples(result, mul_tuple_scalar((-2484700, -2444000), abs( xj- 1) + xj- 1))

= add_tuples(result, mul_tuple_scalar((28087, -1404.4), abs( xj- 2) + xj- 2))

= add_tuples(result, mul_tuple_scalar((53410, -2670.5), abs( xj - 3) + xj- 3))

= add_tuples(result, mul_tuple_scalar((75071, -3753.6), abs( xj- 4) + xj- 4))

= add_tuples(result, mul_tuple_scalar((-127020, 6350.8), abs( xj- 5) + xj- 5))

The yearly Dispatched Products of theAntilock Breaking System can be calculated with the results displayed in Table 2 and Figure5.

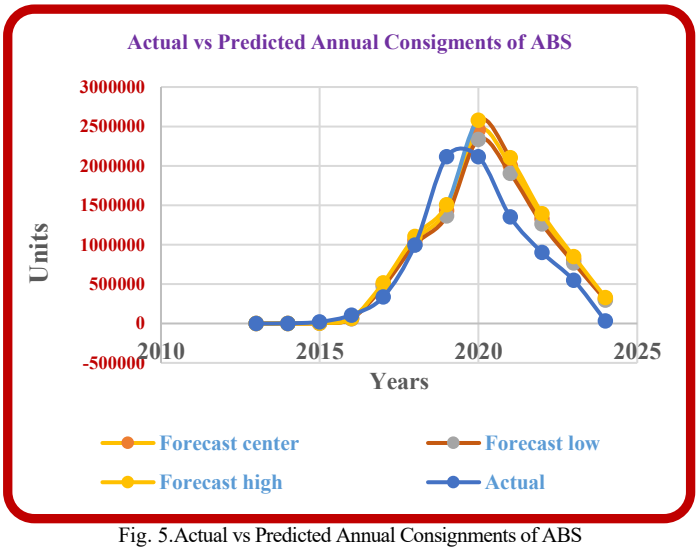



Fig. 5.Actual vs Predicted Annual Consignments of ABS

PLC classifications, the stages of the PLC are change points to development the industrial sales. Thus, the Product Life Cycle of ABS changed from the development to maturity stage. Next to the maturity stage in the 8thyear and finally changed from the maturity to the End of life (decline stage) in the 9th year (Figure. 6).

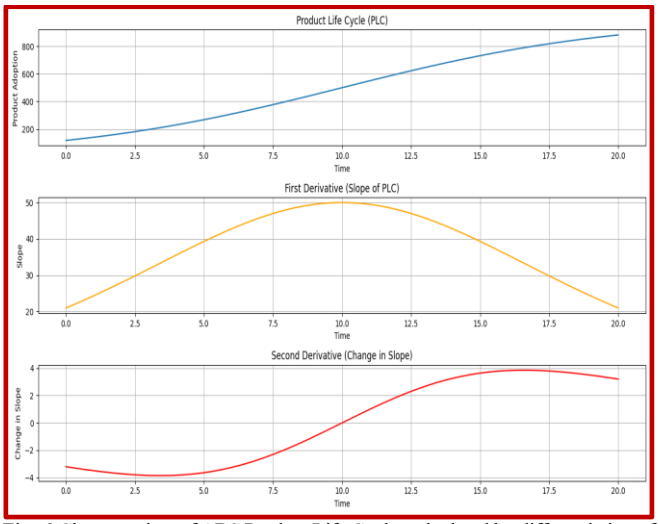



Fig. 6.Change points of ABS Product Life Cycle, calculated by differentiation of slopes

Forecast data, the Product Life Cycle of Antilock Breaking System products changed from the growth to the maturity stage in the ninth year and changed from the maturity to end of life stage in the tenth year (Figure. 7).

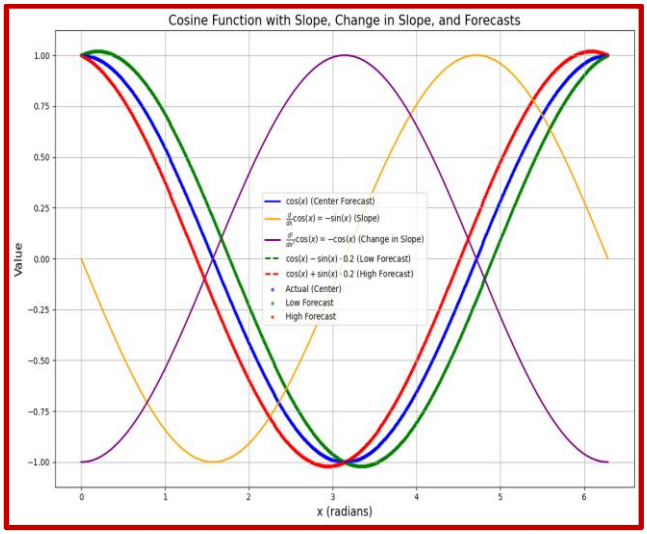



Fig. 7 Product Life Cycles, calculatedbydifferentiationof slopes

**Errors in the forecasts**

Appling Equations (Equation 32andEquation 33), the forecast efficiency for the Antilock Braking System's Product Life Cycle, spanning the past twelve years, has been evaluated based on the forecast results presented in Section 5.A.

**D.Residual Error of Product Life Time**

The residual error over the full product life span is 0%, indicating a perfect prediction of the overall duration of the Product Life Cycle.

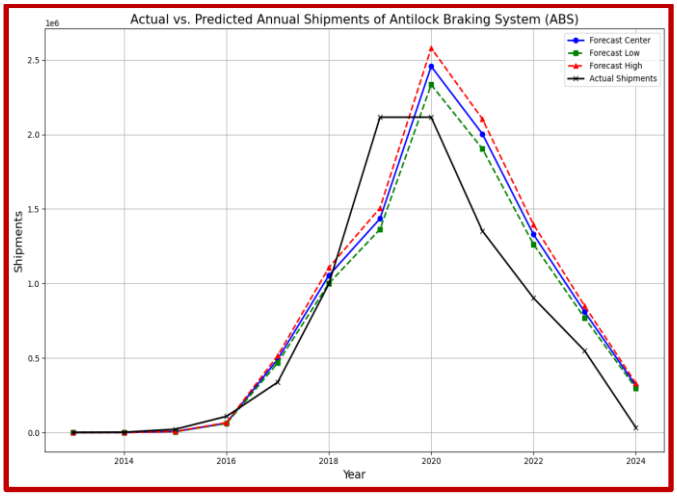



Fig. 8. Actual vs Predicted Annual Shipments of ABS

**E.Trend Accuracy Using Logarithmic Transformation**

To assess the ability of the forecast to capture Dispatched Products trends, the logarithms of the yearly forecasted and actual Dispatched Products values were computed. The residual errors were then derived using Equation (32).

**F.Overall Forecast Error**

Applying Equation (33), the mean forecast error across the entire Product Life Cycle period was calculated to be 15.64%.

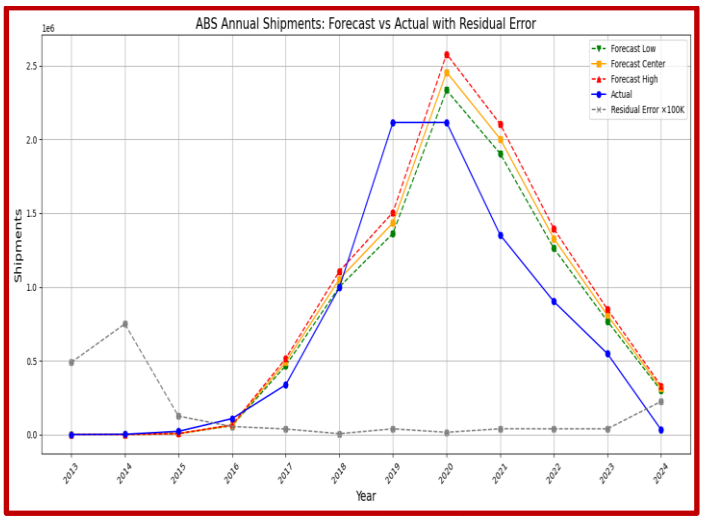



Fig. 9. Actual and Predicted Logarithmic values

## G. Adjusted Forecast Error

The forecast errors Recognizing that the following years (1st, 2nd and 12th) Years are relatively higher due to their Dispatched Products volumes being significantly smaller than the peak values, these years were excluded from the calculation. As a result, the adjusted forecast error drops to 6.98 percentage and reflecting a more accurate estimation of the core trend period of the Product Life Cycle.

**TABLE 2**
**LOGARITHMIC SCALE REPRESENTATION**

| Year | Forecast Center | Forecast Low | Forecast High | Actual | Log (Forecast Center) | log(Forecast Low) | log(Forecast High) | Log (Actual) |
|---|---|---|---|---|---|---|---|---|
| 2013 | 0 | 0 | 0 | 133 | 0.000 | 0.000 | 0.000 | 4.890 |
| 2014 | 0 | 0 | 0 | 1808 | 0.000 | 0.000 | 0.000 | 7.500 |
| 2015 | 6199 | 5881 | 6517 | 21555 | 8.731 | 8.678 | 8.784 | 9.980 |
| 2016 | 62715 | 59735 | 65695 | 108116 | 11.048 | 11.001 | 11.096 | 11.588 |
| 2017 | 491004 | 466755 | 515254 | 337005 | 13.104 | 13.057 | 13.152 | 12.726 |
| 2018 | 1053549 | 1000721 | 1106377 | 996316 | 13.869 | 13.816 | 13.920 | 13.815 |
| 2019 | 1434895 | 1363535 | 1506255 | 2116025 | 14.176 | 14.127 | 14.227 | 14.567 |
| 2020 | 2457340 | 2334375 | 2580305 | 2116140 | 14.715 | 14.665 | 14.764 | 14.567 |
| 2021 | 2004470 | 1904455 | 2104485 | 1353059 | 14.511 | 14.460 | 14.561 | 14.120 |
| 2022 | 1329232 | 1262940 | 1395524 | 902000 | 14.102 | 14.048 | 14.155 | 13.714 |
| 2023 | 808670 | 768191 | 849150 | 548170 | 13.603 | 13.551 | 13.654 | 13.214 |
| 2024 | 313848 | 297985 | 329711 | 33330 | 12.659 | 12.603 | 12.712 | 10.414 |

The logarithmic transformation helps illustrate trend alignment between forecasted and actual values across a wide range of Dispatched Products volumes. The middle years (2016–2020) show the best alignment, confirming that the forecasting model effectively captured the Product Life Cycles peak phase.

Forecast error is higher in the beginning and end years due to unexpected early adoption and a steeper decline than predicted.

This aligns with the earlier summary stating:

Forecast Error (All Years) ≈ 15.64% and Forecast Error (Excluding Years 1, 2, 12) ≈ 6.98%.

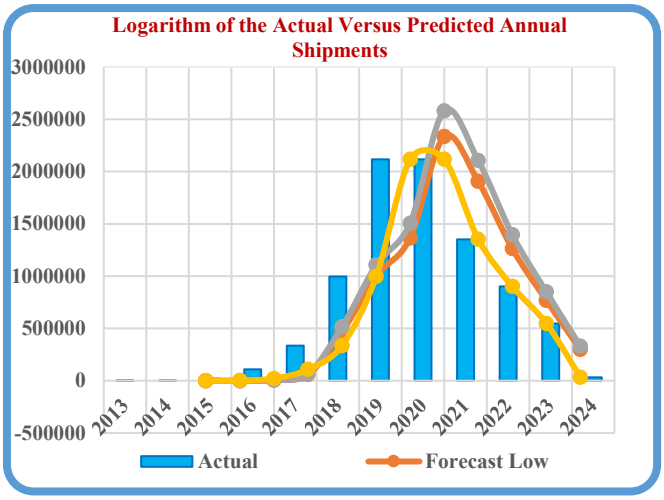



Fig. 10. Actual vs predictedshipments of ABS and forecast errors

**Discussions**

The analytical study introduced a multi-stage fuzzy linear regression approach applied at the product level, capable of forecasting Product Lifetime, annual Dispatch Volumes, and identifying key change points within the Product Life Cycle. This innovative forecasting technique provides a foundational tool that can be leveraged for policy planning, strategic decision-making, inventory optimization, and workforce management.This forecasting approach differs from conventional time-based methods, whose results are time-dependent. Compared with one of the most established and widely used Product Life Cycle prediction techniquesapplied to Antilock Braking Systems and demonstrating strong predictive accuracy—the proposed method offers two key benefits. It is based on the assumptions that (a) a series of evolving products exists, each capable of performing all functions of its predecessor and possibly more, (b) each product follows a time-varying adoption rate, and (c) substitution occurs between actual and potential sales of earlier and later products. Despite these assumptions, the proposed method accurately predicts both Product Life Time and Product Life Cycle using only past data.The actual yearly dispatched products exceeded the forecasted results significantly during the 7th, 8th, and 9th years of the product life cycle. During the growth stage from the introduction of the Antilock Braking System up to its maturity in the 8th year manufacturers should focus on expanding production capacity to maximize revenue, profit, and market share.

**Conclusions**

The proposed model introduces a forecasting methodology for predicting both Product Life Time and nonlinear Product Life Cycle using a multi-stage fuzzy product-wise regression analysis approach. This method proved effective, successfully identifying the change points and accurately estimating the product lifetime as twelve years of the Antilock Braking System life cycle by deriving twelve regression lines from historical data of multiple Antilock Braking System variants.

The results of this forecasting method can be used to guide policy planning and strategy development for each stage of the Product Life Cycle. However, the interval regression analysis has some weaknesses in dealing with data variability, and for the Antilock Braking System, a feasible solution could not be obtained in the necessity regression analysis, showing a limitation of the approach. The interval regression analysis is highly sensitive to outliers (Fig. A1).

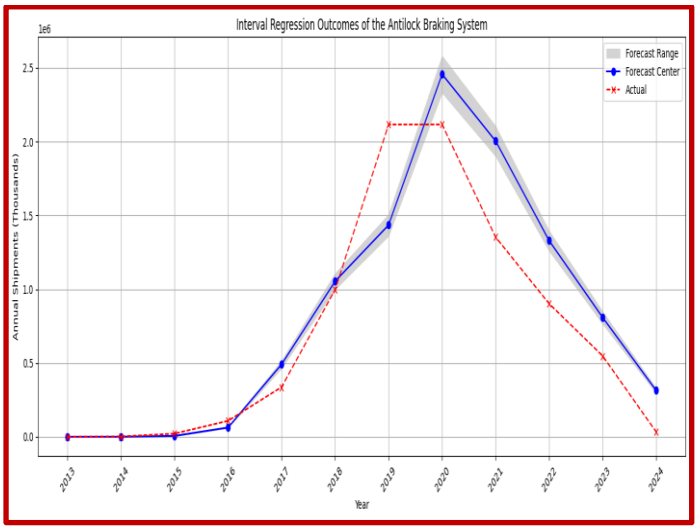



Fig. A1. Recently Consequences of the Collected Data Points

Its results often display greater possibilities than expected and become increasingly fuzzy as more data are added, such as the new data points shown in Fig. A2. Because only the middle and width of the interval data are available andthere is no operational interpretation for the outliers, making it difficult to classify them accurately.

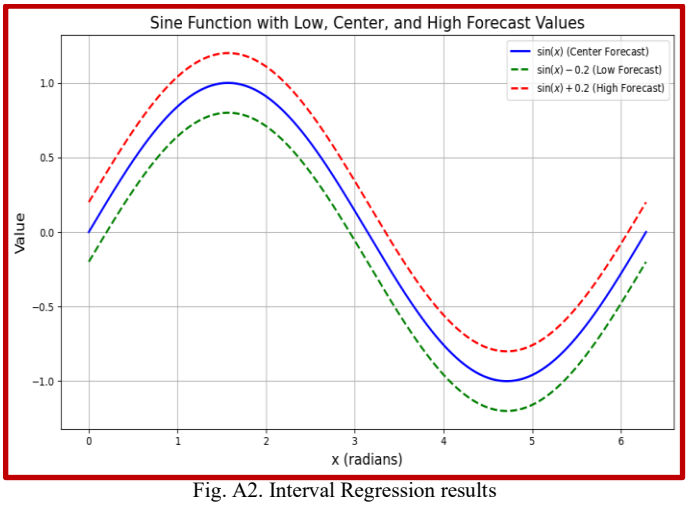



Fig. A2. Interval Regression results

Figure A4 Displays the coefficients of the linear regression model, showing which features have the most influence on the target variable. Feasible solutions can sometimes be found for specific cases (Figure A3); however, they are often lacking when the data shows considerable variation.

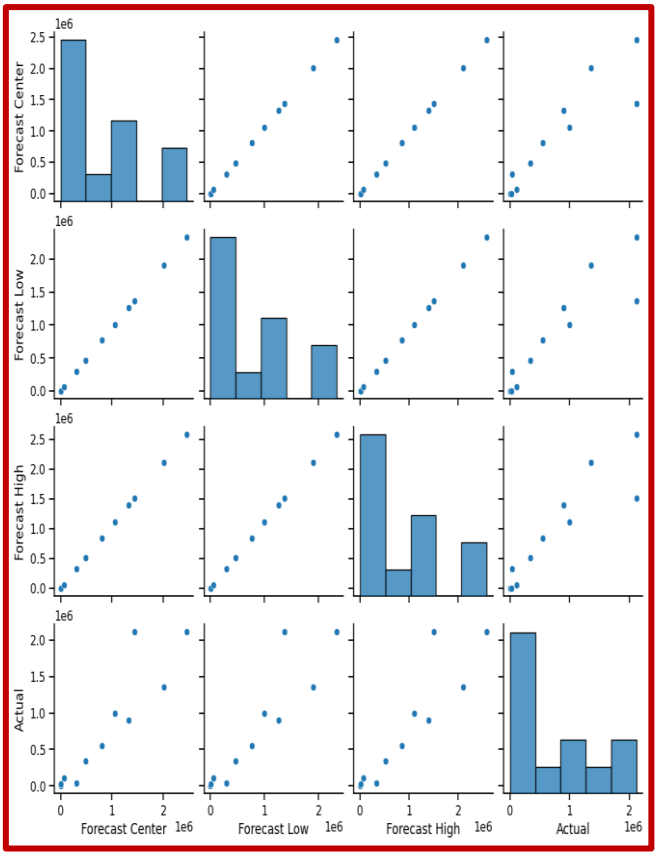



Fig. A3. Feasible Solution

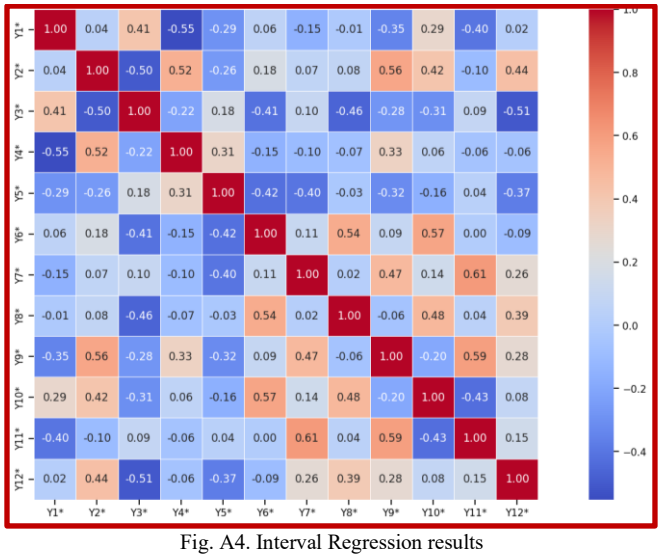



Fig. A4. Interval Regression results

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