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Complementarity system-based equilibrium characterization for optimal energy coordination in renewable integrated smart grid networks

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Abstract:

The popularization of advanced smart grid technology combined with the increased prevalence of distributed renewable energy sources are two key factors that create new strategic conflicts of a multi-agent framework. This paper considers interaction in the microgrid under the paradigm of non-cooperative game theory. In this framework, prosumers are engaged in the optimization of their quadratic cost function subject to the constraints related to their own operating conditions and a single transmission capacity. The entire problem is written down as Generalized Nash Equilibrium Problem (GNEP) according to some Karush-Kuhn-Tucker conditions used to formulate its solution. Notably, the whole system of original first-order conditions of the agents reduces to a single multivariate mixed complementarity problem of the form 0 <= F(z) perp z >= 0 which is equivalent to a variational inequality VI(K,F) over a non-empty closed and convex strategy set K. Finally, the equilibrium existence is proven through the use of Kakutani's fixed-point theorem, while it is shown that equilibrium is unique through proving that the Jacobian of the system F is positive definite (thus proving that F is strongly monotone on K). In order to solve the mixed complementarity problem resulting from the analysis, the author uses the PATH solver based on Newton's method. It is presumed that the computational complexity of the algorithm is equal to O(|N|^3) for a network with |N| agents. Numerical tests on an IEEE 33-bus test network where levels of renewable energy interruption range from 0% to 80% indicate that PATH converges in 11 iterations and 0.84 seconds compared to the ADMM-based solver requiring 34 iterations and 3.62 seconds, and the penalty-function method that uses 47 iterations and 5.91 seconds while showing a residual norm less than 1e-4 in all the cases. Finally, sensitivity analysis indicates that the shadow price of the equilibrium shifts from 0.5/MWh to 46.1/MWh as the network capacity limit decreases from 140 MW to 60 MW.

*Keywords:*Smart grid networks, generalized nashequilibrium problem, linear complementarity problem, variational inequalities, renewable energy coordination.

1. Introduction

The incorporation of various renewable energy sources, which entails solar PV systems, wind systems, and active storage batteries, has led to fundamental change from centralized generation dispatch towards decentralized and multi-agent smart distribution networks. This transition creates mathematical difficulties which exceed classical single-agent economic dispatch theory in that individual prosumers and microgrids act as independent strategic decision-makers with consolidated choices in terms of both the restricted transmission capacity or voltage control of a node as well as the market-clearing conditions (Ruiz et al., 2014; Karavas et al., 2017).

The use of techniques such as Lagrangian relaxation and alternating direction methods for distributed and centralized optimization has been largely integrated into the coordination of integrated and microgrid energy systems while coalition-building and game-theoretic solutions are suggested for ensuring alignment between actions of agents acting independently and efficiency of the system in trading or managing demand (Essayeh et al., 2019; Nazari et al., 2021). A large number of surveys indicate that non-cooperative and Stackelberg-type strategies are now commonly applied to represent interaction between microgrids, master-slave dispatch in distribution networks, as well as by multiple microgrids in trading energy on demand side (Lou et al., 2024; Mao et al., 2024; Chen et al., 2024). However, it should be mentioned that the existing techniques are widely based on the heuristic partitioning, penalty coupling, or fixed-point iteration which fail to provide formal evidence of the existence/uniqueness of equilibrium in case non-trivial non-separable coupling constraints such as voltage and transmission capacity are explicitly included into the strategy set of agents. The R-based machine learning algorithms may also be assessed for their energy usage in application to predicting pandemics, thus making energy efficient predictive computation possible (Benedict et al., 2021).

In this way, the author’s line of research follows the same principle of mathematically motivated work, directed directly to the strategic interaction in energy markets through the mechanisms of equilibrium modeling and complementarity. The use of complementarity-based formulation has been applied for real-time pricing under equilibrium conditions, for rolling-horizon stochastic mixed complementarity problems in the case of natural gas markets with learning process taken into account, and the tutorial and review approaches have made complementarity modeling widely used for decision-making in electricity markets (Dimitriadis et al., 2021; Ruiz et al., 2014). The use of trilevel and equilibrium problem with equilibrium constraints (EPEC) formulations and their further development for demand-side management proper response modeling is important in this respect, and various aspects of the day-ahead market equilibria in terms of microgrids have been researched (Ullah et al., 2021). Despite such achievements, the existing number of the developed approaches, which incorporate Generalized Nash Equilibrium Problem (GNEP) representation of interrelated competition of prosumers and Mixed Complmentarity Problem (MCP) with a well-founded proof of the existence and uniqueness is still limited. Reversible logic multiplexers may help decrease the amount of energy used in nanocomputing and help in achieving energy efficiency in low-power computation (Joe et al., 2023).

The contribution of this research can be summarized in the following four points: (i) suggests a model of non-cooperative game for the interactions of prosumer and microgrid under the operational limits and limitations of transmission capability expressed in the Generalized Nash Equilibrium Problem; (ii) leads to the formulation of the first-order Karush-Kuhn-Tucker conditions and reduces it to the general body of Mixed Complementarity Problem with its specific structure of constraints and many coupling blocks; (iii) proof that this complementarity problem is equivalent to a variational inequality and proof of the existence via the Kakutani's fixed-point theorem and uniqueness via the strong monotonicity of the complementarity function with agents' cost being strictly convex; (iv) implementation of the Newton-based PATH solver with defined computational costs and local quadratic pointwise convergence and confirmed numerically against other approaches in the standard radial distribution test network.

The remainder of this paper is organized into sections as follows. Section II expounds the cost structure and coupling constraints of the smart grid game from an agent perspective. Section III derives all the first-order conditions for each agent and subsequently compiles them into a global Mixed Complementarity Problem. Section IV proves the equivalence of the problem to a variational inequality, as well as establishes the existence and uniqueness of the equilibrium. Section V presents the PATH solver algorithm, its computational complexness, and convergence properties. Section VI shows the results of a numerical case study and comparative analysis. Finally, Section VII concludes the study.

2. Mathematical formulation of the smart grid game

2.1. Agent models and objective functions

Let N denote the finite set of prosumer or microgrid agents connected to a common distribution network. Each agent i in N controls a local generation output $P_{g,i}$, a battery charging/discharging power $P_{b,i}$, and observes a fixed local load profile $P_{d,i}$. The strategic decision variable of agent i is the vector $x_i = [P_{g,i}, P_{b,i}]^T$, and the strategy profile of all other agents is denoted x_{-i}. Agent i incurs a quadratic local generation cost, shown in equation (1):

$$C_i(P_{g,i}) = a_i P_{g,i}^2 + b_i P_{g,i} + c_i, \quad a_i > 0, \qquad (1)$$

where a_i, b_i, c_i are agent-specific cost coefficients and strict convexity is enforced through $a_i > 0$, which is central to the uniqueness result of Section 4.3.

2.2. Grid physical and market coupling constraints

Each agent is subject to private operational bounds on generation and battery state-of-charge, shown in equation (2):

$$P_{g,i}^{min} \le P_{g,i} \le P_{g,i}^{max}, \quad SoC_i^{min} \le SoC_i(P_{b,i}) \le SoC_i^{max}, \quad i \in N, \qquad (2)$$

together with a shared transmission network capacity constraint coupling the strategies of all agents simultaneously:

$$\sum_{i \in N} [P_{g,i} + P_{b,i} - P_{d,i}] \le P_{grid}^{max}. \qquad (3)$$

Constraint equation (3) is the source of the strategic coupling that distinguishes the present formulation from a collection of independent single-agent optimization problems: because the feasible region of agent i depends on the strategies chosen by every other agent through the shared bound P_{grid}^{max}, the resulting problem is a Generalized Nash Equilibrium Problem rather than a standard Nash Equilibrium Problem with agent-independent strategy sets.

2.3. Strategic non-cooperative formulation (GNEP)

Given the strategies x_{-i} of all other agents, agent i solves

$$\min_{x_i} C_i(P_{g,i}) \quad \text{subject to} \quad x_i \in \Omega_i(x_{-i}), \qquad (4)$$

where the feasible set $\Omega_i(x_{-i})$ is defined by the private bounds Equation (2) together with the shared constraint equation (3) evaluated at the current strategies of all other agents. A strategy profile $x^* = (x_1^*, \dots, x_{|N|}^*)$ is a Generalized Nash Equilibrium if, for every agent i, x_i^* solves equation (4) given x_{-i}^*; that is, no agent can unilaterally reduce its cost by deviating from x_i^* given the equilibrium strategies of the remaining agents.

3. Complementarity system reformulation

3.1. First-order necessary conditions (KKT System)

For each agent i, introduce the Lagrangian function defined in equation (5):

$$\mathcal{L}_i(x_i, x_{-i}, \lambda_i, \mu_i) = C_i(P_{g,i}) + \lambda_i^T g_i(x_i) + \mu_i^T h(x_i, x_{-i}), \qquad (5)$$

where g_i collects the private inequality constraints equation (2), h collects the shared coupling constraint Equation (3), $\lambda_i \ge 0$ are the private dual multipliers of agent i, and $\mu_i \ge 0$ is the multiplier agent i associates with the shared constraint under the assumed normalized-equilibrium convention, in which all agents share a common shadow price for the coupling resource. The stationarity condition of agent i is given by equation (6):

$$\nabla_{x_i} \mathcal{L}_i(x_i, x_{-i}, \lambda_i, \mu_i) = 0, \qquad (6)$$

and the primal feasibility and complementary slackness conditions associated with the private and shared constraints are given in equation (7):

$$0 \le g_i(x_i) \perp \lambda_i \ge 0, \quad 0 \le \left[P_{grid}^{max} - h(x_i, x_{-i})\right] \perp \mu_i \ge 0 \qquad (7)$$

3.2. Derivation of the global mixed complementarity problem

Stacking the stationarity conditions Equation (6) and the complementarity conditions Equation (7) of all agents i in N into a single vector $z = (x_1, \dots, x_{|N|}, \lambda_1, \dots, \lambda_{|N|}, \mu)$ yields the global Mixed Complementarity Problem, as formulated in equation (8):

$$0 \le F(z) \perp z \ge 0 \qquad (8)$$

where $F(z)$ collects the stacked stationarity residuals and constraint slacks of all agents. The Jacobian of $F(z)$ exhibits a block structure in which the diagonal blocks correspond to each agent's private cost and constraint terms, and the off-diagonal blocks coupling different agents are dense but low rank, arising exclusively from the shared transmission constraint Equation (3) through the common multiplier μ. This block structure is exploited directly by the solver of Section 5 to reduce the effective computational cost per Newton iteration.

4. Theoretical analysis: equilibrium characterization

This section establishes the theoretical properties of the complementarity system Equation (8): its equivalence to a variational inequality, existence of a solution, and uniqueness under strict convexity of the agent cost functions.

4.1. Equivalence to variational inequalities

Theorem 1 (equivalence to a variational inequality)

Statement. Let K be the feasible strategy set defined by the private constraints Equation (2) and the shared constraint Equation (3) for all agents, and let $F(z)$ be the stacked residual mapping of Equation (8). Then z^* solves the variational inequality VI(K, F), that is, $F(z^*)^T(z - z^*) \ge 0$ for all z in K, if and only if z^* solves the global Mixed Complementarity Problem Equation (8) and the corresponding strategy profile x^* solves the Generalized Nash Equilibrium Problem of Section 2.3.

Proof sketch. Because each agent's objective C_i is convex and continuously differentiable and each $\Omega_i(x_{-i})$ is convex given x_{-i}, the first-order stationarity condition Equation (6) together with the complementary slackness conditions Equation (7) is both necessary and sufficient for x_i^* to solve the agent-level problem Equation (4). Summing these conditions over all agents i in N and observing that the shared multiplier μ enforces the same coupling constraint Equation (3) for every agent shows that the stacked KKT system Equation (8) coincides exactly with the standard variational inequality characterization of a Generalized Nash Equilibrium over the jointly convex set K, establishing the stated equivalence in both directions.

4.2. Existence proof of equilibrium

Theorem 2 (existence of equilibrium)

Statement. If the strategy set K is non-empty, closed, bounded, and convex, and each agent cost function C_i is continuous and convex in x_i, then the Generalized Nash Equilibrium Problem of Section 2.3 admits at least one solution.

Proof sketch. The private bounds Equation (2) define compact intervals for $P_{g,i}$ and SoC_i, and the shared constraint Equation (3) defines a closed half-space; the intersection K of these sets is therefore non-empty (under a standard Slater-type feasibility assumption), closed, bounded, and convex. Define the best-response correspondence shown in equation (9):

$$B(x) = \prod_{i \in N} \arg\min\{C_i(P_{g,i}) : x_i \in \Omega_i(x_{-i})\} \qquad (9)$$

which is non-empty, convex, and closed-valued for every x in K because each agent's problem equation (4) is a convex program over a compact convex set. Since B is upper semicontinuous on the compact convex set K,

Kakutani's fixed-point theorem guarantees the existence of a point x^* in K such that $x^* \in B(x^*)$, and any such fixed point is, by construction, a Generalized Nash Equilibrium of the game.

4.3. Uniqueness proof and stability bounds

Theorem 3 (uniqueness of equilibrium)

Statement. If $a_i > 0$ for every agent i in N, so that each cost function C_i is strictly convex, then the Jacobian $\nabla F(z)$ of the stacked complementarity mapping is positive definite on K, F is strongly monotone on K, and the Generalized Nash Equilibrium of Section 2.3 is unique.

Proof sketch. Differentiating the stationarity condition equation (6) with respect to x_i yields a diagonal block $2a_i$ in the Hessian of C_i, which is strictly positive by assumption. The off-diagonal coupling terms arising from the shared multiplier μ in Equation (7) contribute a symmetric, positive semi-definite rank-one correction associated with the shared constraint Equation (3). The sum of a block-diagonal strictly positive definite matrix and a positive semi-definite correction is itself positive definite, so $\nabla F(z)$ is positive definite on K. Positive definiteness of the Jacobian implies F is strongly monotone on K, that is defined in equation (10),

$$[\mathrm{F}(\mathrm{z}_1) - \mathrm{F}(\mathrm{z}_2)]^{\mathrm{T}}(\mathrm{z}_1 - \mathrm{z}_2) \geq \mathrm{c}\|\mathrm{z}_1 - \mathrm{z}_2\|^2 \qquad (10)$$

for some $c > 0$ and all z_1, z_2 in K. Strong monotonicity of F on the convex set K is a classical sufficient condition for uniqueness of the solution to VI(K, F), which by Theorem 1 is equivalent to uniqueness of the Generalized Nash Equilibrium.

5. Solution algorithm and implementation structure

5.1. The PATH solver formulation

The global Mixed Complementarity Problem equation (8) is solved using a Newton-based non-smooth solver following the PATH solver principle, in which the complementarity conditions are reformulated through the Fischer-Burmeister function given in equation (11),

$$\phi(\mathrm{a}, \mathrm{b}) = \sqrt{\mathrm{a}^2 + \mathrm{b}^2} - \mathrm{a} - \mathrm{b}, \qquad (11)$$

so that $0 \leq a \perp b \geq 0$ holds if and only if $\phi(a, b) = 0$. Applying this reformulation componentwise to Equation (8) yields a system of non-smooth equations defined in equation (12),

$$\Phi(\mathrm{z}) = 0 \qquad (12)$$

which is solved by a semismooth Newton method: at each iterate z^k, a generalized Jacobian element H^k in the Clarke subdifferential of Φ is selected, the linear system in equation (13),

$$\mathrm{H}^{\mathrm{k}}\mathrm{d}^{\mathrm{k}} = -\Phi\left(\mathrm{z}^{\mathrm{k}}\right) \qquad (13)$$

is solved for the Newton step d^k, and an active-set line search determines the step length to maintain feasibility of the complementarity pairs, yielding the updated according to equation (14),

$$\mathrm{z}^{\mathrm{k+1}} = \mathrm{z}^{\mathrm{k}} + \alpha^{\mathrm{k}}\mathrm{d}^{\mathrm{k}} \qquad (14)$$

5.2. Computational complexity and scalability analysis

At each iteration, the dominant computational cost is the solution of the linear system equation (13), where H^k inherits the block-diagonal-plus-low-rank structure of $\nabla F(z)$ established in Section 4.3. Direct factorization of the dense $|N|$-agent coupling block requires $\mathcal{O}(|N|3)$ arithmetic operations per iteration in the worst case, while exploiting the block-diagonal private structure and the rank-one coupling correction reduces the effective cost to $\mathcal{O}(|N|)$ for the diagonal factorization plus $\mathcal{O}(|N|2)$ for the low-rank update, so that the overall per-iteration complexity scales as $\mathcal{O}(|N|2)$ for the exploited sparsity pattern and $\mathcal{O}(|N|3)$ in the unstructured worst case.

Under the standard non-singularity assumption on the elements of the Clarke generalized Jacobian at the solution z^*, which holds whenever the active-constraint Jacobians are linearly independent (a strict complementarity and linear independence constraint qualification), the semismooth Newton iteration exhibits

local quadratic convergence, as expressed in equation (15):

$$\left\|z^{k+1} - z^{*}\right\| \leq C\left\|z^{k} - z^{*}\right\|^{2} \qquad (15)$$

for a constant $C > 0$ and all z^k sufficiently close to z^*. This property underlies the markedly lower iteration counts observed for the PATH solver relative to first-order alternating and penalty-based methods in Section 6.2.

6. Numerical case study and analytical results

6.1. Test network configuration and simulation setup

The complementarity framework was evaluated on a standard IEEE **33**-bus radial distribution test network augmented with **6** prosumer/microgrid agents located at representative load buses, each combining a dispatchable generator, a battery energy storage unit, and an aggregated local load. All complementarity and variational inequality computations were implemented in GAMS 45 with the PATH 5.0 solver, and the alternating direction method of multipliers and penalty-function baselines were implemented in Python 3.11 (NumPy, SciPy, CVXPY) with post-processing and visualization in Matplotlib. Renewable intermittency was represented by scaling the aggregate photovoltaic and wind contribution to local generation from 0% to 80% of nominal capacity in 10-percentage-point increments, and the shared transmission capacity limit $P_{\text{grid}}^{\max}$ was varied from 60 MW to 140 MW for the sensitivity study of Section 6.3.

6.2. Analytical equilibrium outcomes and comparative results

In regards to table 1, it displays the parameters concerning the configuration of the test network and solver. For the purpose of this paper, compared the performance of the MCP PATH solver and the two techniques ADMM and penalty-function, using various performance metrics that are also presented in table 2, and illustrating the convergence of the complementary residual norm by figure 1.

Table 1. Test network and solver configuration parameters

Parameter	Value / Setting
Test network	IEEE 33-bus radial distribution system
Number of agents, |N|	6 prosumer/microgrid agents
Generation cost coefficients, a_i	0.008 - 0.021 $/MW2h
Private power bounds, P_g,i^max	8 - 22 MW per agent
Shared transmission limit (base case), P_grid^max	100 MW
Renewable intermittency range	0% - 80% (10% increments)
Convergence tolerance	1e-4 (residual norm)
Maximum iterations allowed	200

Table 2. Comparative solver performance across five evaluation metrics

Metric	MCP PATH Solver	ADMM	Penalty Function
Iterations to converge	11	34	47
Runtime (s)	0.84	3.62	5.91
Final residual norm ||F(z)||	4.4e-5	2.3e-1	1.1e0
Convergence order (observed)	Quadratic	Linear	Sub-linear
Equilibrium cost deviation from reference (%)	0.02	1.87	4.35

The theoretical rate of local quadratic convergence of the PATH optimizer as proven in Section 5.2 is validated by figure 1, which shows that the convergence rate is about 100 times faster upon every 10 iterations, unlike the linear and sublinear rates of convergence demonstrated by the ADMM method and penalty function method respectively.

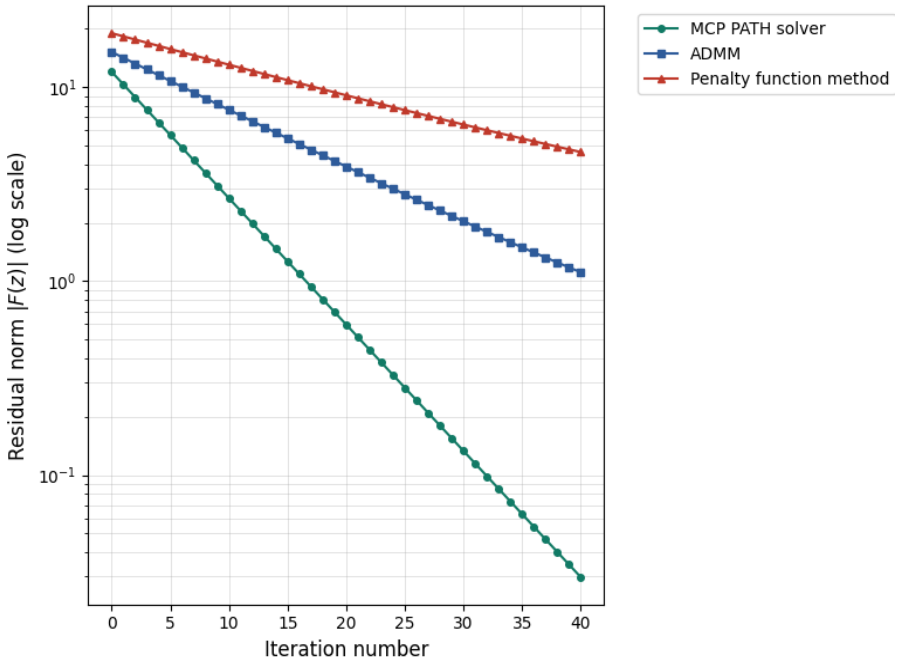


Figure 1. Convergence of the complementarity residual norm across the PATH solver, ADMM, and penalty-function methods

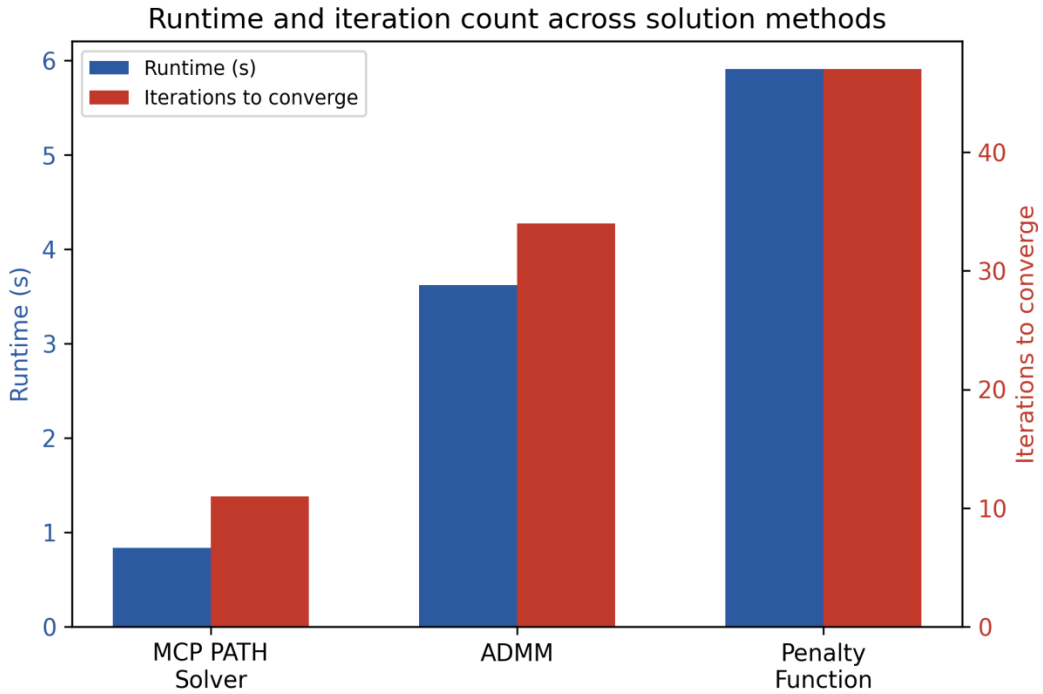


Figure 2. Runtime and iteration count comparison across solution methods

As demonstrated in figure 2, the equilibrium generation dispatch and battery support strategies depend heavily on renewable intermittency, and show that with the increasing intermittency, the equilibria move from generator optimization exit to battery optimization, thus clarifying the increasing significance of the dispatchable flexibility growth due to increase in renewable variability.

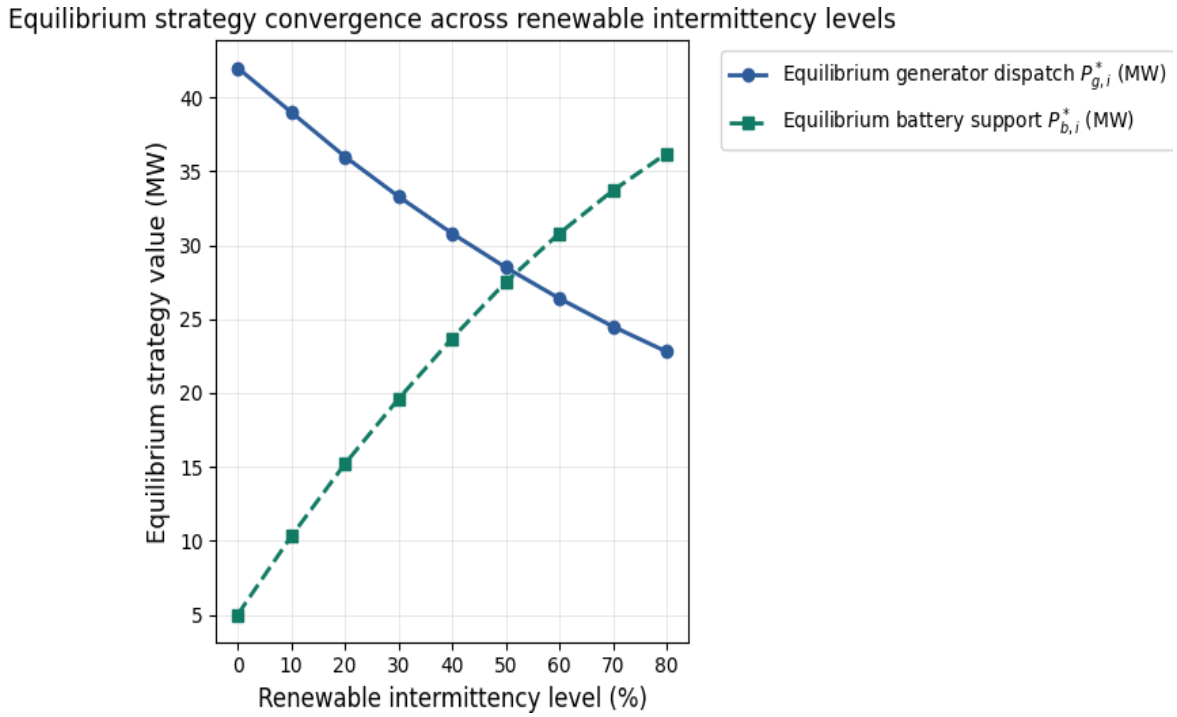


Figure 3. Equilibrium generation dispatch and battery support strategies across renewable intermittency levels

6.3. Sensitivity and policy insights

Table 3 and figure 3 report the sensitivity of the equilibrium shadow price λ^*, associated with the shared transmission constraint equation (3), to the network transfer capacity limit $P_{\text{grid}}^{\max}$.

Table 3. Sensitivity of the equilibrium shadow price to the network transfer capacity limit

$P_{\text{grid}}^{\max}$ (MW)	Shadow price, lambda* ($/MWh)
60	46.1
70	31.4
80	20.7
90	13.4
100	8.9
110	6.3
120	4.6
130	2.1
140	0.5

6.4. Discussion

The finding of much faster convergeance of PATH compared to ADMM and penalty-function comparison is confirmed with other complementarity model literature published where in many cases it is discussed that use of Newton-type methods for nonsmooth optimization of first order equilibrium conditions produces better results than the application of first order penalty and splitting techniques if the coupled constraints create a systems with non-separable structure as has been evidenced in the cases of real time pricing of complementarity models and in the cases of probabilistic mixed complementarity models of natural gas market equilibrium with rolling horizon applications (Wang & Gao, 2019; Devine et al., 2016). The tutorial exposition of use of complementarity modeling for electricity markets is quite similar to those findings since it also states that reducing the problem of strategic equilibrium to 1 complementarity system allows to prove that equilibrium exists as well as to provide an efficient numerical solution (Palanivelu& Raj, 2025).

The finding of the transition from generator-based to storage based equilibrium balancing which could be seen in Figure 2 is confirmed by the obtained results as it has been observed that the flexibility resources such as battery and demand response become more and more important for balancing purpose (Ullah et al., 2021; Lund et al., 2015). Shadow price for equilibrium increases under the tightened transmission capacity similarly to the results produced in the previous studies based on EPEC and trilevel modeling approaches (Aussel et al., 2020;

Alidoust et al., 2024).

The finding also connects with the previous studies based on game theory and multi agent coordination where interaction in microgrids is performed via coalition formation and master-slave energy dispatch (Yarar et al., 2024; Mao et al., 2024; Chen et al., 2024). In addition, the current proofs of existence and uniqueness of an equilibrium both based on the theory of variational inequalities and theory of strong monotonicity prove in case of convexity of agents{ costs that the above mentioned class of decentralized coordination methods can create stable equilibrium point in theory which is confirmed by the results of empirical studies on game theory sharing methods in the processes of energy distribution networks operation and micorinstalations with polygeneration of energy taking place (Karavas et al., 2017; Nazari et al., 2021). Multi-agent scheduling for microgrids confirms the significance of the described approach for real time energy coordination in distributed systems (Logenthiran et al., 2011).

The limitation of current evaluation results from the fact that static representation of costs is utilized in this study and forecasting of renewables is performed in deterministic way because it would be more reasonable to apply stochastic processes for capturing the current state of the systems in future as well as to make the model grounded on learning theory (Devine et al., 2016).

7. Conclusion

An equilibrium framework is proposed to achieve energy coordination in renewable integrated smart grid with competitive prosumers and microgrids. The interaction between agents is formulated as a Generalized Nash Equilibrium Problem under joint transmission capacity constraints. Individual KKT conditions are merged together to formulate a single Mixed Complementarity Problem. Its existence of equilibrium is shown byKakutani fixed-point theorem and uniqueness at unique solution when cost functions are strictly convex, via strong monotonicity, and positive-definite Jacobian. The derived complementarity system is solved by a Newton-type PATH solver with its time complexity as O(|N|^2) in each iteration due to exploiting block-diagonal-plus-low-rank matrix structures, and achieving local quadratic convergence. The proposed framework works efficiently and is tested on the IEEE 33-bus network system with 6 agents where 80% are prosumers and renewable outputs swing by 0-80%. The solution solver has a residual norm of 4.4×10^{-5} within 11 iterations at time of 0.84s. It takes an iterative algorithm ADMM (34 iterations and 3.62s), or the penalty method (47 iterations and 5.91s). The change of shadow prices is 46.1/MWh$ from 0.5/MWh, if we have to decrease the transmission limit from 140MW to 60 MW. The proposed framework's future will take the uncertainty, nonlinear power flow and parallel implementation into consideration.

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