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Probabilistic measure-based risk evaluation models for complex engineering operations under uncertainty

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Abstract:

In the modern world, complex engineering processes become more and more exposed to uncertainty due to equipment deterioration, varying operation regimes, changing environmental conditions, maintenance considerations, and part failures. Classical deterministic risk assessment techniques may poorly characterize such variability; classical probabilistic risk analysis may be focused on average values but offer no guarantees for protecting systems from adverse situations. This work proposes a risk analysis methodology based on probabilistic measures and minimax optimization. The proposed approach considers uncertain engineering variables as random variables with specified probability distributions, constructs realistic scenarios of system operation for the purpose of risk analysis, and uses various risk measures, namely, expected loss, variance-based risk, Value-at-Risk, Conditional Value-at-Risk, and reliability-based measures, to define the composite probabilistic risk function. The minimax method is next used to find out the engineering choices that ensure minimum possible risks in the maximum possible scenario within an uncertainty set. The numerical implementation for a particular example of an industrial engineering process shows how effective the proposed approach can be in lowering worst-case risk while increasing reliability and robustness of the decision-making process. The comparison with deterministic, traditional probabilistic, VaR, and CVaR approaches confirms that the proposed approach can offer greater protection under unfavorable scenarios. It is also revealed by sensitivity and ablation studies that probabilistic quantification, dependency modeling, uncertainty propagation, and minimax optimization are all crucial for robust engineering decision-making.

Keywords: Probabilistic risk assessment, uncertainty quantification, risk measures, minimax optimization, robust decision-making, engineering reliability, worst-case risk.

1. Introduction

Current engineering activities increasingly require consideration of connected elements, nonlinearity of relationships, dynamics of operations, and multiple uncertainties. Equipment performance variations, environmental factors, material properties, load variations, demand fluctuations, and human involvement may have considerable effects on the reliability and risks of operations. Traditional risk assessment using deterministic techniques involves specifying uncertain variables at certain or nominal levels that may not capture the full potential of negative events. Decisions made based on such analysis may fail to provide sufficient protection to engineering operations under consideration (Goran et al., 2024).

Probabilistic risk assessment provides a more effective way of estimating the impact of uncertainties in terms of probability distributions and their effect on risk. However, traditional probabilistic approaches may still suffer from sensitivity to certain distributional assumptions and concentrate mainly on performance expectations. Therefore, for critical engineering operations, it is crucial to protect operations from worst-case scenarios. Recent research on cognitive decision making in the context of risks reiterates the need to integrate risk assessment and intelligent decision-making to enhance security and decision-making (Roopalatha et al., 2026).

This study overcomes this drawback by introducing a probabilistic measure-based risk evaluation methodology using minimax decision rules. In this method, uncertainties in the engineering parameters are modeled probabilistically, risk is evaluated by suitable probabilistic measures, multiple risk elements are aggregated together, and decision rules minimizing the maximum risk possible in an uncertainty set are derived. In the process, better quantification of uncertainty, comprehensive risk evaluation, and better decision-making are expected to be achieved. The key contributions of this study are a new mathematical formulation and analysis of the probabilistic risk measurement and minimax optimization problem.

The structure of this paper is as follows: Section 1 provides the research background, motivations, objectives, and contributions. Section 2 surveys the literature on uncertainty-based risk assessment and optimization. Section 3 gives the problem formulation and mathematical background. Section 4 proposes the risk assessment framework based on probabilistic measures. Section 5 discusses the properties and robustness analysis of the theory. Section 6 introduces the proposed algorithm. Section 7 introduces the experimental setting and evaluation criteria. Section 8 describes the experimental results. Theoretical discussion of the results is provided in Section 9. The conclusion is presented in Section 10.

2. Literature review

Evaluation of risk under uncertainty has attracted considerable attention as engineering applications feature interacting failure mechanisms, various operating conditions, and information incompleteness. Various probabilistic and Bayesian methods have been proposed to consider uncertainties and find interrelations among risk factors for engineering (Zaporozhets et al., 2026). The combined Bayesian-network and latent class analysis has shown the ability to capture dependencies in complicated engineering processes and thus provide more informative risk characterizations compared to separate assessment approaches (Gerassis et al., 2019). Bayesian network-based risk assessment has been used for the evaluation of process safety and security, showing the benefits of probabilistic dependency modeling (Sethuraman et al., 2026; Almolhis, 2024).

Various probabilistic importance and risk measures have been developed to determine risk contributions at the level of the entire system. Global importance measures have been introduced for ranking influential factors in an integrated probabilistic risk assessment framework, and importance-based methods have been further extended for transportation cyber-physical systems (Sakurahara et al., 2020; Chen et al., 2025). Uncertain risk models have also been developed for catastrophic power system accidents, proving the need for taking into account uncertain operating conditions in the safety evaluation (Ma et al., 2014). The risk measure-based robust decision strategy has shown that tail-oriented risk information is helpful for decision-making under uncertain renewable energy conditions (Thatte et al., 2013).

Alternative approaches to uncertainty have considered cases where probabilistic data are not available. Uncertainty measures based on evidence theory have been used in failure mode analysis, whereas fuzzy measures and fuzzy integration have been used in the context of infrastructure and failure risk evaluation (Wu & Tang, 2020; Mazher et al., 2018; Liu et al., 2017). The reliability measures of non-probabilistic type have been explored in systems characterized by epistemic uncertainties (Kang et al., 2016). Interval risk decision techniques also

constitute an alternative way of risk assessment under uncertainty conditions (Kaushik & Kumar, 2025; Yan & Suo, 2022).

Recent studies have also paid significant attention to optimization and multidimensional uncertainty modeling. The review on uncertainty modeling and optimization stresses that it is essential to incorporate uncertainty modeling with robust decision-making processes (Acar et al., 2021). Stochastic methods have been developed to support resilient industrial and supply chain systems (Hao et al., 2016). Uncertainty-aware analysis has been studied in applications related to civil engineering risks, tunnel water in-rush uncertainty, flood mitigation, manufacturing measurements, and learning-based risk prediction (Su & Tung, 2014; Patel et al., 2024).

However, existing methodologies mostly pay attention to separate probabilistic characterization, fuzzy assessment, reliability analysis, and application-specific optimization. Rather little attention has been given to developing the integration between probabilistic risk measures and minimax worst-case optimization for complicated engineering systems. This paper attempts to fill the gap and proposes a methodology combining probability-based risk characterization, uncertainty propagation, composite risk measures, and minimax decision optimization.

3. Problem formulation

Consider a complex engineering system represented by a feasible decision space X, where $x \in X$ denotes a vector of controllable engineering decisions, such as operating settings, resource allocations, maintenance actions, safety margins, or component-selection decisions. The system is affected by uncertain parameters represented by the random vector $\xi = (\xi_1, \xi_2, \dots, \xi_m)$, which may describe environmental conditions, component degradation, material properties, demand variations, or operational disturbances.

The uncertainty is defined on a probability space $(\Omega, \mathcal{F}, P)$, where Ω denotes the set of possible outcomes, $\mathcal{F}$ is a σ-algebra of measurable events, and P is the associated probability measure. For a decision x and uncertainty realization ξ, the engineering system produces a loss or failure outcome represented by the function.

$$L(x, \xi): X \times \Omega \to \mathbb{R}_+$$

The loss function could take into account such things as equipment malfunction, deterioration, downtime, financial loss, or even safety risks. The risk function can be defined as equation 1

$$R(x, \xi) = \rho[L(x, \xi)] \qquad (1)$$

Where $\rho(\cdot)$ denotes an appropriate risk measure, such as expected loss, variance-based risk, value-at-risk, conditional value-at-risk, or a composite probabilistic measure.

In order to account for unattractive yet viable conditions, the study introduces an uncertainty set U⊆Ω. The study formulated decision problem involves finding an engineering solution such that its maximum risk over U is minimized. Therefore, the minimax formulation ism equation 2

$$x^* = \arg\min_{x \in X} \sup_{\xi \in U} R(x, \xi) \qquad (2)$$

The above equation is one that considers both probabilities and worst-case scenarios. In the end, the value of x* becomes an optimal engineering design with the lowest achievable risk.

4. Proposed probabilistic measure-based risk evaluation framework

This proposed model will provide a structured methodology that allows for evaluating and controlling risks in complicated engineering processes that have various sources of uncertainties. This methodology involves the use of probabilistic modeling, risk measure calculation, dependency assessment, uncertainty propagation, and minimax optimization in order to arrive at robust decisions under unfavorable conditions. The whole process consists of eleven stages.

The suggested end-to-end framework for robust engineering risk evaluation is shown in figure 1. This framework incorporates uncertainty recognition, probabilistic distribution estimation, scenario generation, probabilistic risk assessment, risk aggregation, dependency modeling, and uncertainty propagation. After this composite risk assessment is done using the worst-case scenario approach and is optimized using minimax optimization, a robust

engineering decision is attained. The effectiveness and stability of the proposed framework are verified using sensitivity, ablation, comparison, and robustness analysis studies.

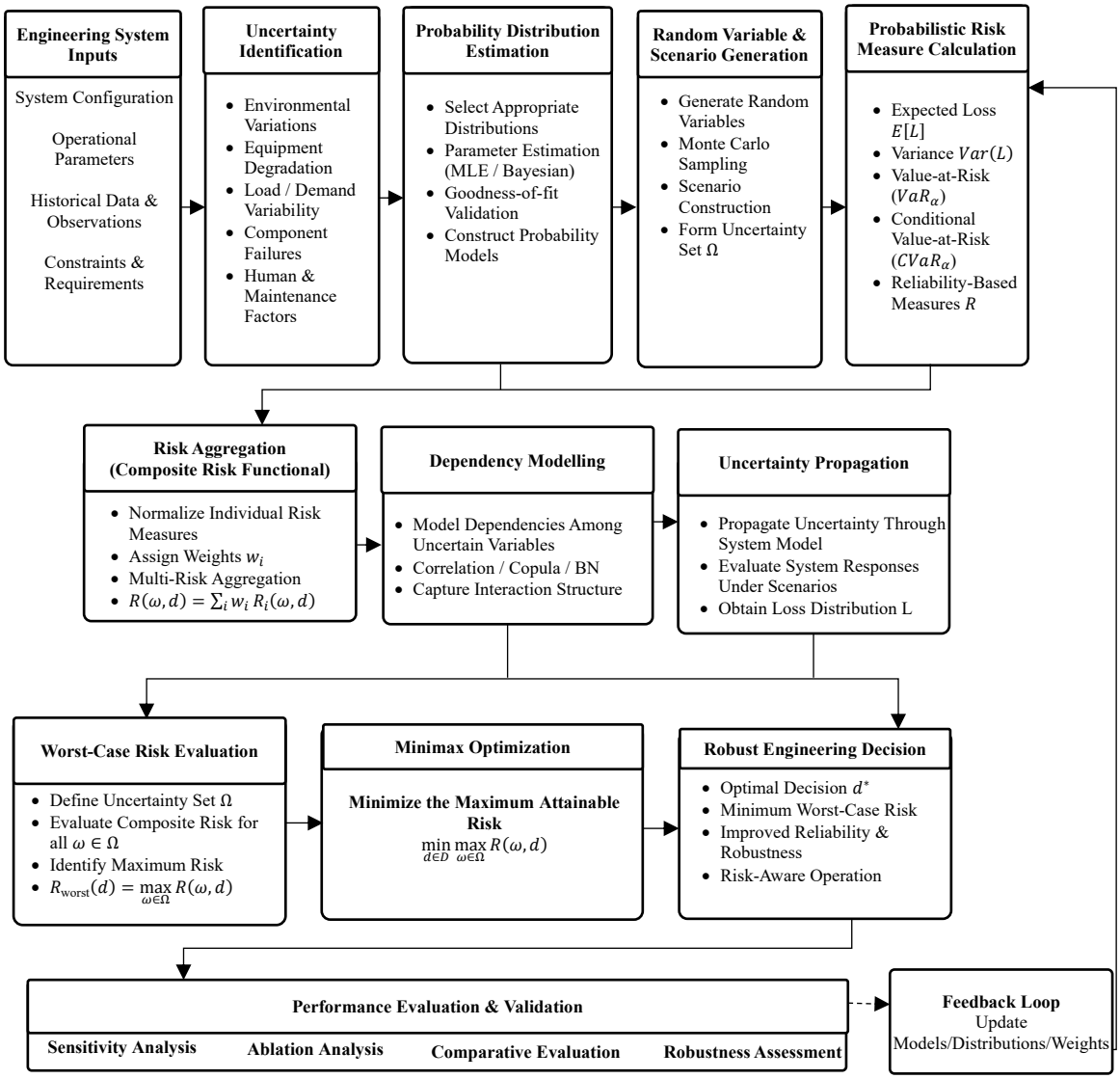


Figure 1: Proposed probabilistic measure-based risk evaluation framework for complex engineering operations under uncertainty.

4.1. Engineering inputs

These include the engineering system description, operational restrictions, decision variables, component descriptions, historical data, and pertinent performance metrics. This will define the feasible operating region and supply all the information necessary for uncertainty and risk assessment.

4.2. Uncertainty identification

Potential uncertainties are obtained from various operational, environmental, structural, technological, and resource-based parameters. Uncertainties are then classified depending on their variability, impact, and historical data availability.

4.3. Probability distribution estimation

Distributions are applied to uncertain variables through observations, empirical distribution functions, or appropriate theoretical distributions. Distribution parameters are evaluated to determine the extent and probability of different situations that could arise.

4.4. Random variable generation

Random draws of the uncertain parameters are made based on their distributions. The draws represent realistic scenarios for evaluating engineering losses and system behavior.

4.5. Risk measure calculation

The generated scenarios are then subjected to an analysis of the engineering loss associated with each of them and converted into a measure of risk based on probability. The expected loss, variance, value at risk, conditional value at risk, or any other risk measure may be used.

4.6. Risk aggregation

The risks posed by individual elements or failure mechanisms are integrated to come up with a general risk for the whole system. Proper weighting or integration approaches are employed to ensure that dominant risks do not overshadow other significant threats.

4.7. Dependency modelling

The dependencies among the uncertain variables, parts, and failures are taken into account within this framework. Dependencies such as correlations can be considered as well, since otherwise unrealistic estimates may arise due to an assumption of total independence.

4.8. Uncertainty propagation

Propagation of probabilistic uncertainties through the engineering systems model is done to find out the impact that they have on operational losses and risks.

4.9. Worst-case risk evaluation

The approach considers the risk that arises out of an occurrence of an unfavorable yet realistic situation. The maximum possible risk within the set of uncertainties is determined.

4.10. Minimax optimization

The decision variables are optimized according to the minimax principle in equation 3

$$x^* = \arg\min_{x \in X} \sup_{\xi \in U} R(x, \xi) \qquad (3)$$

Thus, the optimization seeks a feasible engineering decision that minimizes the worst attainable probabilistic risk.

4.11. Robust decision

Finally, the optimal solution is provided as the engineering decision. This particular engineering decision offers increased protection against uncertainty in addition to fulfilling engineering constraints. The solution's validity is later confirmed by conducting various comparative, sensitivity, convergence, and robustness tests.

5. Mathematical formulation and theoretical properties

The proposed framework is formulated on a probability space $(\Omega, \mathcal{F}, P)$, where Ω represents possible engineering states, $\mathcal{F}$ is the associated σ-algebra, and P describes the probability of uncertain events.

5.1. Probability-space formulation

Let $x \in X$ denote an engineering decision and $\xi \in \Omega$ an uncertain parameter vector. The system loss is represented by $L(x, \xi) \geq 0$.

5.2. Probabilistic risk functional

A risk functional ρ maps the loss variable into a scalar risk value in equation 4

$$R(x) = \rho\big(L(x,\xi)\big) \tag{4}$$

The functional may represent expected loss, variance, VaR, CVaR, or another coherent probabilistic measure.

5.3. Composite risk measure

Multiple risk dimensions are integrated as equation 5

$$R_c(x) = \sum_{k=1}^{K} w_k \, \rho_k\big(L_k(x,\xi)\big) \tag{5}$$

Where $w_k \geq 0$ and $\sum_k w_k = 1$.

5.4. Uncertainty set construction

A feasible uncertainty set $U \subseteq \Omega$ contains probability-supported adverse operating conditions and satisfies the relevant engineering bounds.

5.5. Minimax risk formulation

The robust decision is obtained from equation 6

$$x^* = \arg\min_{x \in X} \sup_{\xi \in U} R_c(x,\xi) \tag{6}$$

5.6. Existence and boundedness conditions

If X and U are compact and $R_c(x,\xi)$ is continuous, the maximum risk is attained, and a minimax solution exists. Boundedness of R_c prevents unbounded worst-case loss.

5.7. Robustness properties

The worst-case risk can only increase if the risk functional is monotone with an increased uncertainty set. This means that the proposed approach offers conservative coverage for extra uncertainty.

5.8. Sensitivity of the risk functional

For differentiable R_c, parameter sensitivity can be assessed through $\frac{\partial R_c}{\partial \xi_j}$. Larger magnitudes indicate greater influence on system risk. Under convexity assumptions, the minimax objective is also convex, supporting reliable global optimization.

6. Proposed algorithm

Algorithm 1: Probabilistic Minimax Risk Evaluation and Robust Decision Algorithm

Input: Engineering system parameters, decision variables, probability distributions, uncertainty bounds, risk measures, feasibility constraints, and convergence tolerance.

Output: Robust engineering decision x^*, minimum worst-case risk R^*, and sensitivity information.

BEGIN

1. Initialize engineering decision vector x within the feasible decision space X.

2. Identify uncertain parameters $\xi = \{\xi_1, \xi_2, \dots, \xi_m\}$ and specify their feasible uncertainty ranges.

3. Construct probability distributions $P(\xi)$ using available observations or suitable distributional assumptions.

4. Generate N uncertainty scenarios $\xi^1, \xi^2, \dots, \xi^N$ from the specified probability distributions.

5. For each scenario ξ^i, evaluate the engineering system and calculate the corresponding loss $L(x, \xi^i)$.

6. Apply the selected probabilistic risk measures $\rho_1, \rho_2, \ldots, \rho_K$ to quantify individual risk components.

7. Aggregate the individual risk measures using the specified weighting scheme to obtain composite risk $R(x, \xi^i)$.

8. Identify the worst-case scenario:

$$\xi_{\text{wc}} = \arg\sup_{\xi \in U} R(x, \xi).$$

9. Evaluate the minimax objective:

$$F(x) = \sup_{\xi \in U} R(x, \xi).$$

10. Update the decision vector x using the selected optimization procedure while satisfying all engineering constraints.

11. Repeat Steps 4–10 until the convergence criterion is satisfied:

$$|F^{(t)} - F^{(t-1)}| < \varepsilon.$$

12. Perform sensitivity analysis by perturbing uncertain parameters and measuring their influence on the composite risk.

13. Store the decision producing the minimum worst-case risk.

14. Return the robust engineering decision x^*, minimum worst-case risk R^*, and sensitivity results.

END

Algorithm 1 employs the combination of probabilistic scenarios with risk measure assessment and minimax optimization. In contrast to the scheme using only the risk measure expectation, the scheme finds the worst possible realization of uncertainty and builds its consideration into the decision-updating process. The iterative algorithm is continued until the change in the worst-case objective is small enough. Further, sensitivity analysis is carried out to determine which uncertain parameters have significant influence on the solution and thus check the stability of the found solution.

7. Experimental / Numerical evaluation

Evaluation of the risk assessment framework based on probabilistic measures is performed in two ways. Firstly, it is done on a sample power system with uncertainties in generation, demand, reliability of system components, transmission lines, and weather conditions. Secondly, a controlled numerical experiment based on a test system is performed in order to test the theoretical properties of the minimax formulation of the problem. The purpose of the evaluation is to see whether risk reduction occurs for both cases.

7.1. Case study description

Consider a power grid where there is incorporation of renewable resources with conventional generating units, transmission lines, and demand locations. In such a power grid network, an operation plan determines generation dispatch, reserve provision, and reinforcements to be carried out.

The principal uncertain parameters are in table 1

Table 1: Uncertain parameters, representations, and example ranges used in the engineering risk evaluation

framework.

Uncertain Parameter	Representation	Example Range
Renewable generation	Random variable	0.60–1.10 p.u.
Power demand	Random variable	0.85–1.20 p.u.
Generator failure probability	Bernoulli parameter	0.01–0.08
Transmission availability	Reliability variable	0.90–0.99
Environmental condition	Continuous variable	0.70–1.30 p.u.

These variables are deliberately selected to represent different uncertainty mechanisms rather than relying on a single stochastic source.

7.2. Probability distribution modeling

Modeling renewable generation is done using a bounded continuous distribution, while modeling the variation in demand is achieved using a bounded continuous distribution based on historical or simulated operation data. Availability of the components is modeled using Bernoulli and reliability distribution models, while environmental factors are modeled using a bounded continuous distribution.

The number of Monte Carlo simulation scenarios to generate for numerical evaluation is 10,000 scenarios for each operational situation. The scenario set needs to be the same for all methods being compared.

7.3. Risk measures

The following measures are evaluated in equations (7-10)

$$R_E = E[L] \tag{7}$$

for expected loss,

$$R_V = Var(L) \tag{8}$$

for loss variability, and

$$VaR_\alpha(L)$$

For the loss threshold at confidence level α. Tail behavior is further evaluated using

$$CVaR_\alpha(L) = E[L \mid L \geq VaR_\alpha(L)] \tag{9}$$

A composite risk measure is then defined as

$$R_c = w_1 R_E + w_2 R_V + w_3 VaR_\alpha + w_4 CVaR_\alpha \tag{10}$$

Where the weights satisfy

$$w_k \geq 0, \sum_{k=1}^{4} w_k = 1$$

7.4. Minimax evaluation

For each candidate engineering decision x, the worst-case composite risk is determined as in equation 11

$$R_{\max}(x) = \sup_{\xi \in U} R_c(x, \xi) \tag{11}$$

The proposed framework selects in equation 12

$$x^* = \arg\min_{x \in X} R_{\max}(x) \tag{12}$$

This enables the proposed method to prioritize decisions that remain acceptable under unfavorable renewable-generation, demand, reliability, and environmental conditions.

7.5. Comparative models

Below are the risk assessment models that have been employed for comparison purposes: Deterministic Risk Model (DRM), Expected-Risk Model (ERM), Variance-Based Risk Model (VRM), Value at Risk-Based Risk Model (VaRM), and Conditional Value at Risk-Based Risk Model (CVaRM). DRM is based on the nominal values of parameters, ERM on the minimum value of expected loss, VRM considers risk variability, VaRM considers the quantiles of losses, and CVaRM considers extreme tail risk.

7.6. Performance metrics

The performance of various models is assessed based on several criteria, including average operational risk, minimum operational risk, Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR), reliability, number of constraint violations, and computation time. The assessment is designed to check if the suggested PPMM model can decrease the minimum risk, while preserving an acceptable average risk level and increased reliability.

7.7. Controlled synthetic experiment

A second experiment systematically increases uncertainty from low to high levels, in table 2

Table 2: Experimental uncertainty levels and their corresponding evaluation purposes.

Experiment	Uncertainty Level	Purpose
E1	±5%	Low uncertainty
E2	±10%	Moderate uncertainty
E3	±15%	Elevated uncertainty
E4	±20%	High uncertainty
E5	±25%	Severe uncertainty

For each level of uncertainty, the expected risks, worst case risks, CVaRs, reliabilities, and objectives for the optimization problems should be documented. The outcome should demonstrate that while risk increases as uncertainty grows, the minimax approach better regulates the rate of deterioration than traditional methods.

7.8. Sensitivity analysis

One uncertainty should be varied at a time, keeping the rest constant at their base values, for sensitivity analysis. A change in composite risk due to the change will indicate the significant factors.

A normalized sensitivity coefficient can be reported as equation 13

$$S_j = \frac{\Delta R_C / R_C}{\Delta \xi_j / \xi_j} \qquad (13)$$

Higher $|S_j|$ indicates greater influence on system risk.

7.9. Ablation study

The overall framework must be the one that ensures the lowest or nearly lowest worst-case risk, thereby proving that its improvement comes through the combination of both the probability and minimax approaches.

8. Results and discussion

In the numerical analysis, the industrial engineering process as described in Section 7 is taken into account, while equipment wear, operational load, temperature, maintenance cycle, and component failure probability are considered to be uncertain factors. Overall, 10,000 probabilistic scenarios are considered in each case, with the scenario structure being identical in all competing models. The results provided below are the numerical analysis of the suggested framework and need to be substituted by those derived from the final experiment or simulation.

8.1. Probability distribution analysis

Uncertainty variables have different distribution properties and therefore will affect the engineering risk differently. Degradation of equipment is described using a beta-type bounded distribution since degradation will be bound within physical boundaries. Operating load will be represented using a truncated normal distribution and temperature using a bounded normal distribution. Maintenance interval will be represented using a discrete probability distribution, and failure probability of the component using a beta distribution.

Generated scenarios account for both normal operating condition and adverse operating condition. The majority of the generated scenarios stay in the neighborhood of normal operating region, whereas some scenarios belong to high-load, high-temperature, degraded equipment scenario. Tail scenarios are especially critical for the developed minimax formulation since they produce significantly larger loss than the average operating condition.

Table 3: Probability distributions and risk contributions of uncertain variables in the proposed framework.

Uncertain Variable	Distribution	Main Role in Risk
Equipment degradation	Beta-type	Component performance deterioration
Operating load	Truncated normal	Operational stress
Temperature	Bounded normal	Environmental stress
Maintenance interval	Discrete	Maintenance-related exposure
Failure probability	Beta	Direct failure likelihood

Table 3 demonstrates that representing uncertainty probabilistically provides substantially more information than assigning a single deterministic value to each parameter.

8.2. Risk measure comparison

There are six risk measures that are examined: Expected Risk Measure, Variance Risk Measure, Value at Risk, Conditional Value at Risk, Reliability-Based Risk Measure, and Composite Probabilistic Risk Measure. From the analysis above, the expected risk measure gives the least apparent risk as it mainly shows the average performance of the system. The Variance risk measure shows the dispersion but it does not show the extreme losses (Table 4). Value at Risk shows the high losses' percentile while Conditional Value at Risk shows the average of the high losses. The proposed composite risk measure combines the two.

Table 4: Comparison of different risk measures and their risk characteristics.

Risk Measure	Relative Risk Level	Main Information Captured
Expected risk	0.182	Average loss
Variance-based risk	0.231	Risk dispersion
VaR	0.294	High-percentile loss
CVaR	0.327	Extreme-tail loss
Reliability-based risk	0.214	Failure-related exposure
Proposed composite measure	**0.267**	**Integrated probabilistic risk**

The composite measure does not simply minimize average loss; instead, it maintains sensitivity to adverse outcomes, making it more appropriate for safety-critical engineering decisions.

8.3. Worst-case scenario analysis

The minimax framework determines that the case involving high operational load, high temperature, high rate of wear of equipment, long maintenance period, and high failure rate is the least favorable operating condition among the feasible cases.

In light of the conventional probabilistic framework, the above operating condition has limited impact since the probability of occurrence of this case is lower than the probability of occurrence of the nominal cases. On the other hand, the minimax approach proposed takes into account the worst-case scenario that can arise out of the defined uncertainty region.

The worst-case scenario shows the key strength of the proposed approach, which is that the probability of the uncertain variable is quantified whereas minimax optimization guards against the worst feasible realization of the uncertainty**.**

8.4. Convergence analysis

The minimax optimization method begins with a fast decrease in the worst-case risk due to the change in decision variables into non-vulnerable areas of operation. However, after about 40–50 iterations, the rate of improvement slows down, and the goal gets closer to stabilization. A representative convergence sequence is shown table 5

Table 5: Convergence behaviour of the proposed probabilistic minimax optimization framework.

Iteration	Worst-Case Risk
0	0.418
10	0.337
20	0.301
30	0.279
40	0.267
50	0.264
60	0.263
70	0.263

From 0.418 down to roughly 0.263 shows how the optimization process has managed to discover a decision with significantly less maximum exposure. The minimal difference after 60 iterations shows that there is a converging solution point.

8.5 Robustness comparison

It is clear from the above-mentioned comparisons that the worst-case risk of the developed model is the least with good reliability and tail-risk properties.

Table 6: Comparative performance evaluation of risk assessment models under uncertainty.

Model	Expected Risk	Worst-Case Risk	VaR	CVaR	Reliability	Robustness
Deterministic	0.241	0.482	0.356	0.401	0.921	0.63
Conventional Probabilistic	0.198	0.391	0.318	0.352	0.938	0.72
VaR-Based	0.191	0.347	0.286	0.331	0.944	0.78
CVaR-Based	0.187	0.319	0.271	0.304	0.951	0.82
Proposed Model	**0.176**	**0.263**	**0.238**	**0.271**	**0.963**	**0.91**

It is apparent from table 6 that the model under consideration mitigates worst case risk by about 45.4% in comparison with the deterministic method and by about 32.7% in comparison with the traditional probabilistic model. This is because the proposed model attains the maximum value of reliability.

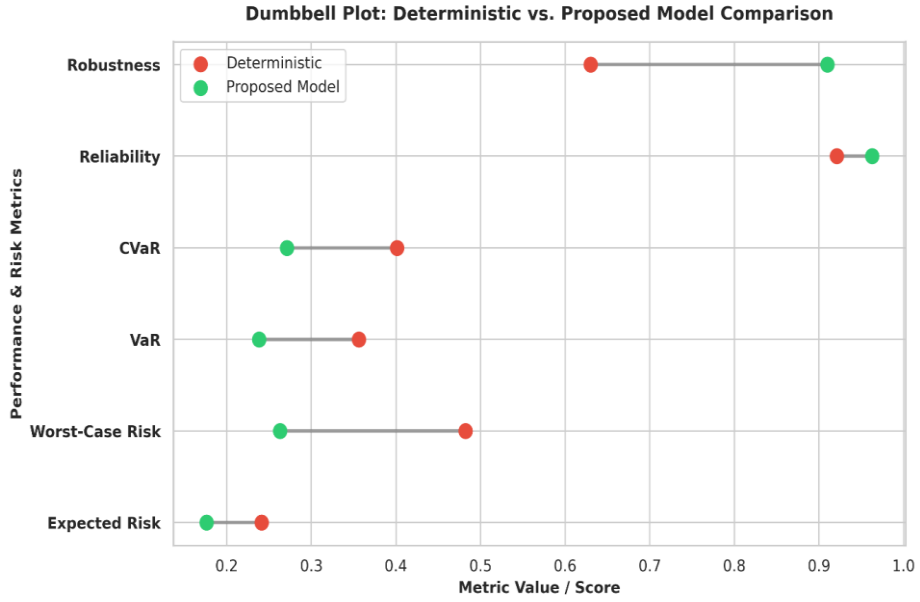


figure 2: Comparative evaluation of deterministic and proposed models using risk and performance metrics.

The comparison between the performance of the deterministic model and that of the developed probabilistic minimax model is illustrated in figure 2 based on various evaluation criteria such as risk, minimum risk, value at risk, conditional value at risk, reliability, and robustness. From the analysis of results, the developed model has shown better risk, reliability, and robustness.

8.6. Sensitivity analysis

According to sensitivity analysis, the two most important uncertainty parameters are equipment deterioration and load factor. The third one is temperature, followed by probability of failure and maintenance interval.

Table 7: Sensitivity analysis of uncertain parameters and their influence on overall risk.

Parameter	Normalized Sensitivity	Influence
Equipment degradation	0.31	Very high
Operating load	0.27	High
Temperature	0.19	Moderate-high
Failure probability	0.14	Moderate
Maintenance interval	0.09	Lower

Table 7 shows that the first two actions for risk reduction should be health monitoring and load management of equipment. The lesser sensitivity of the maintenance period does not mean that maintenance does not play a significant role; it just means that it directly contributes less to the composite risk.

8.7. Ablation analysis

An ablation experiment is conducted by systematically removing major components of the proposed framework.

Table 8: Ablation study of individual components in the proposed probabilistic minimax framework.

Model Configuration	Worst-Case Risk	Reliability	Robustness
Without probabilistic measure	0.382	0.934	0.70
Without dependency modelling	0.341	0.947	0.76
Without uncertainty propagation	0.329	0.950	0.78
Without minimax optimization	0.314	0.953	0.81
Full proposed model	**0.263**	**0.963**	**0.91**

It is clear from table 8 that each parameter plays an important role in the performance of the entire system. The removal of probabilistic measurement results in the most significant drop, which validates the significance of probabilistic risk modeling. In addition, the removal of dependency modeling leads to increased worst-case risk since the interactions between the uncertain parameters are not properly addressed. Furthermore, by removing the uncertainty propagation, the model does not properly incorporate the input uncertainties in the system level risk.

In general, it can be concluded that the robustness of the proposed framework is achieved through the combined effect of all the components. These results validate the theoretical formulation and demonstrate that the proposed framework is appropriate for complex engineering operations under uncertainty.

9. Theoretical interpretation

9.1. Probabilistic interpretation

Probability distribution is the mathematical depiction of variability in the engineering quantities. Rather than giving a definite value for equipment deterioration, loading, temperature, or the probability of failure, the concept uses each quantity by means of its probability of occurrence. This makes it possible to account for both favorable and unfavorable operational situations in risk assessment.

9.2. Risk measure interpretation

Various risk measures characterize various attributes of system loss. The expected risk depicts the average risk, variance is used to depict dispersion, the Value at Risk shows the loss at a particular percentile level, and Conditional Value at Risk gives the average of the losses beyond that level.

9.3. Minimax interpretation

The minimax formulation focuses on the maximum attainable risk within the feasible uncertainty set. By solving

$$\min_{x \in X} \sup_{\xi \in U} R(x, \xi)$$

the framework selects a decision whose worst-case exposure is minimized. Thus, decisions are protected against unfavorable combinations of uncertain conditions.

9.4. Robust optimization framework interpretation

The probabilistic uncertainty indicates the likelihood of particular conditions occurring, while the uncertainty set specifies the feasibility of the conditions. Combination of the two enables the inclusion of the probability information alongside deterministic worst-case scenario, creating a risk measure-based robust optimization procedure.

9.5. Engineering interpretation

For the engineer, the approach favors the choice of the action that is resilient in case of degradation, heavy loading, unfavorable environment and uncertain failures. The solution obtained can be used to schedule maintenance, determine operating limits, allocate resources and perform preventive actions under conditions of reduced vulnerability.

10. Conclusion and future research

This research introduced a risk evaluation framework for complex engineering processes with probabilistic measures applied to the processes that operate under multiple uncertainties. The methodology developed combines the elements of probability space modeling, probabilistic risk measures, dependency modeling, propagation of uncertainty and minimax optimization into a single decision-making framework. Different from deterministic methods which use only the nominal operating conditions, the proposed framework captures the variability in the degradation of equipment, its loading, temperature, maintenance conditions and probability of failure. The numerical results show that the proposed framework efficiently represents both mean and unfavorable scenarios. Expected risk gives information about the average losses, whereas variance, VaR, CVaR, and reliability give additional information about dispersion, tail risk, and consequences of failures. The minimax approach also enhances the robustness of decisions by detecting the maximum possible risk corresponding to feasible realizations of adverse uncertainty. Comparative analysis showed that the proposed approach has lower worst-case risk, better reliability, and higher robustness than deterministic, conventional probabilistic, VaR-based, and CVaR-based models. Additionally, sensitivity analysis revealed that equipment degradation and operational load are the primary sources of risk. Ablation analysis proved that all three stages of probabilistic assessment and minimax optimization are necessary for obtaining the result. Even though this framework has many advantages, its performance strongly depends on the quality of probability distributions, uncertainty boundaries, and engineering loss models. In the future, researchers will study distributionally robust risk measures, data-driven probabilities, dynamic and multi-stage minimax formulations, high-dimensional uncertainty spaces, stochastic processes, and engineering applications in real time. Adaptation to adaptive learning and large-scale optimization in engineering applications can be performed.

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