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Graph-theoretic optimization models for connectivity enhancement in interconnected engineering networks

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Abstract:

Networks of engineering systems become increasingly subject to failures of their components, uncertain operating environments, degradation of capacity, and scarcity of resources for their reinforcement. Thus, connectivity becomes an important issue in engineering systems design. This study introduces a minimax graph-theoretic optimization approach for designing interconnected engineering networks in the presence of adverse conditions. An engineering system is modeled by a weighted graph where nodes correspond to the engineering objects while edges describe connections between them. Several metrics of connectivity, such as algebraic connectivity, edge connectivity, node connectivity, network reliability, and average path length, are used for the purpose of assessing the performance and operation of the engineering network. The uncertain set is introduced to account for uncertainties in component availabilities, link failures, and capacity. The minimax problem finds enhancement actions to minimize the worst-case deterioration of connectivity subject to budget, capacity, redundancy, and engineering constraints. The solution method uses alternating iterations of finding the worst-case and optimizing the network for connectivity. Results have shown that reinforcement through strategic targeting can significantly increase structural robustness in connectivity and help to sustain stable performance in adverse situations. Through sensitivity and comparative analysis, the effect of uncertainty size, failure size, network density, connectivity, and budget for reinforcement has been shown. The proposed framework offers a mathematical method for robust network improvement using cost-effective means and can be used for reinforcing infrastructures and networks, communication robustness, interconnecting power grids, and transportation systems.

Keywords: Graph theory, minimax optimization, network connectivity, robust optimization, engineering networks.

1. Introduction

Interconnected engineering networks are vital infrastructures in terms of energy distribution, communications, transportation, water distribution, and urban facilities management. The dependability of such networks is highly dependent on the connectivity of their nodes and links, but failure, degradation in capacity, or changes in operational condition may bring about cascades of deterioration in performance. The increasing use of digital decision-support technologies in public administration has also highlighted the importance of data-driven approaches for improving the effectiveness and management of complex public infrastructure systems (Jian & Meilin, 2026). Graph theory provides a precise way of representing such systems, enabling the analysis of structural relations, criticality, and network performance (Ghias-Nodoushan et al., 2025; Al-Hussieny, 2025; Kittapa et al., 2025). Recent studies have demonstrated the application of graph-theoretic approaches to optimize solar-farm interconnections, water-distribution networks, and other engineering networks (Bahrami Chegeni et al., 2025; Ghias-Nodoushan et al., 2025). Graph-based optimization has also been utilized for network reconfiguration and planning tasks. Network reconfiguration approaches have employed graph-theoretic techniques to identify optimal distribution structures, while multi-objective graph optimization has been used to partition complex water networks (Konwar & Sarkar, 2022; Shekofteh et al., 2023). In transportation networks, the combination of graph-theoretic and planning models has enabled the assessment of large-scale road network performance, while graph-based traffic assignment and path optimization have supported the analysis of urban interchange efficiency (Stavara & Tsiotas, 2024; Liu et al., 2026). The broader digital transformation of organizational practices further illustrates how interconnected information and decision processes can support more efficient management of complex systems (Martin & Tremblay, 2021).

Network resilience has become an important research topic because engineering systems need to maintain acceptable performance despite failures and uncertainty. Graph optimization has been studied for the propagation of failures in systems-of-systems, while graph-based formulations have been used to quantify resilience in microgrid-integrated distribution networks (Abbache et al., 2025; Kalantar & Mazidi, 2025). Graph topology optimization and hybrid graph-theoretic approaches have also been applied to communication infrastructures and smart-grid communication applications (Naqvi et al., 2025; Ghasemnezhad et al., 2025). Research on resource management further demonstrates the potential of graph-based optimization for wireless networks, particularly in addressing resource allocation and optimization problems (Dai et al., 2024). In addition, graph-theoretic perspectives have been applied to the analysis of network complexity and stability in environmental systems, highlighting the importance of maintaining resilient interconnected systems in environmentally sensitive contexts (Sheikholeslami & Kaveh, 2025). Strategies for habitat protection and ecosystem preservation also emphasize the importance of sustaining freshwater and marine environmental systems under changing conditions (Subramaniam, 2026).

Furthermore, recent literature has combined intelligent optimization and machine learning with graph theory for engineering design and network optimization. Metaheuristic optimization algorithms have been integrated with graph-theoretic methods to develop efficient and reliable energy interconnection designs, while machine-learning models have used graph-based features for the optimal design of water distribution networks (Ghias-Nodoushan et al., 2025; Bahrami Chegeni et al., 2025). Other approaches for difficult combinatorial optimization problems have employed metaheuristic techniques such as ant colony optimization, demonstrating the applicability of population-based search strategies to complex optimization problems (Agrawal et al., 2012). Neural-network-based approaches have also been investigated in networked communication environments, particularly for data-driven WLAN positioning and intelligent localization (Kushwaha et al., 2025). More general computational approaches have integrated artificial intelligence, mathematical modeling, graph theory, and optimization for high-dimensional problems in complex network systems (Ahmad et al., 2026). Graph-based optimization has additionally been investigated for electric-vehicle charging infrastructure expansion using demand analytics, demonstrating how graph-based approaches can support infrastructure planning under spatial and demand-related considerations (Iqbal, 2024).

Beyond physical infrastructure, blockchain-based approaches have been examined in relation to financial transactions and global markets, further demonstrating the role of interconnected digital architectures in managing complex information and transaction networks (Hosseini & Rahimi, 2026). Similarly, AI-based decision-support approaches have been considered for improving the effectiveness of public administration, reinforcing the broader movement toward data-driven management of complex organizational and infrastructure systems (Jian & Meilin, 2026). Research on smart engineering systems has also emphasized optimization and sustainable infrastructure development in urban environments. Optimization-based approaches have further been

applied to public-service facility layout in disaster-affected areas, illustrating the relevance of computational optimization for infrastructure planning under challenging operational conditions.

Despite these developments, existing studies have predominantly addressed specific infrastructure applications, network reconfiguration, nominal optimization, probabilistic performance assessment, or individual graph-based optimization problems rather than developing a unified minimax formulation specifically aimed at improving connectivity under adverse scenarios (Kittapa et al., 2025; Konwar & Sarkar, 2022; Stavara & Tsiotas, 2024; Liu et al., 2026). Although graph-based resilience, intelligent optimization, and infrastructure planning have been investigated from different perspectives, the integration of connectivity measures, uncertainty bounds, resource constraints, and worst-case scenarios within a single minimax optimization framework remains an area requiring further investigation (Abbache et al., 2025; Kalantar & Mazidi, 2025; Ahmad et al., 2026).

This paper addresses this problem through the development of a minimax graph optimization approach for interconnected engineering networks. In particular, the proposed approach combines connectivity indices, uncertainty bounds, resource constraints, and worst-case scenarios within an optimization framework. The approach differs from deterministic improvement methods by seeking a network configuration that minimizes the maximum degradation of connectivity under unfavorable scenarios. Further motivation for this study is provided by research on network complexity and stability, optimization-based network design, graph-theoretic analysis, and energy-related infrastructure optimization (Sakthi Amaravathi & Rajendran, 2025; Sheikholeslami & Kaveh, 2025; Iqbal, 2024). Related work on public-service facility optimization and sustainable smart engineering systems further supports the relevance of optimization-based approaches for robust infrastructure planning (Feifei, 2024). Graph-based routing and network optimization research also demonstrates the importance of efficient connectivity and resource allocation in interconnected systems (Dai et al., 2024; Pushpalatha & Shivaprakasha, 2025). Accordingly, the proposed framework is mathematically oriented toward robust infrastructure reinforcement and resilient interconnectivity while seeking efficient use of available resources.

The organization of the paper is as follows. Section 1 contains the introduction, including the research background, problem, objectives, research gap, and contributions. Section 2 presents graph-theoretic preliminaries, connectivity measures, interconnected network representation, and uncertainty modeling. Section 3 formulates the minimax optimization problem and associated constraints. Section 4 explains the proposed approach, whereas Section 5 presents the mathematical formulation. Section 6 describes the solution approach. Sections 7, 8, and 9 present the experiments, results, sensitivity analysis, and robustness analysis, followed by the engineering interpretation in Section 10. Section 11 presents the conclusion.

2. Mathematical preliminaries and graph-theoretic formulation

2.1. Graph representation

An interconnected engineering network is represented by an undirected weighted graph $G = (V, E, W)$, where $V = \{v_1, v_2, \dots, v_n\}$ denotes the set of engineering entities, and $E \subseteq V \times V$ denotes physical or logical connections. The weight w_{ij} associated with edge (i, j) represents a relevant engineering attribute, such as transmission capacity, reliability, distance, or connection cost. The adjacency matrix $A = [a_{ij}]$ is defined by $a_{ij} = w_{ij}$ when $(i, j) \in E$, and zero otherwise.

2.2. Network connectivity measures

There are several methods to describe network connectivity. Node connectivity indicates the smallest number of nodes whose deletion disconnects the graph, while edge connectivity refers to the minimum number of edges for this purpose. Average path length describes communication/transportation efficiency, while network reliability describes the chance of maintaining required connections. In weighted graphs, weighted connectivity takes into account edge properties. An especially important spectral measure is the algebraic connectivity,

$$\lambda_2(L)$$

Where $L = D - A$ is the graph Laplacian and D is the degree matrix. A larger $\lambda_2(L)$ generally indicates stronger structural connectivity and greater resistance to network partitioning.

2.3. Interconnected network representation

For K interconnected subnetworks, the overall network can be expressed as equation 1

$$G = (V, \bigcup_{k=1}^{K} E_k \cup E_I) \quad (1)$$

Where E_k denotes intra-network edges, and E_I represents interconnection edges.

2.4. Uncertainty representation

Uncertain operating conditions are described by equation 2

$$\mathcal{U} = \{\xi : \xi^L \leq \xi \leq \xi^U\} \quad (2)$$

Where ξ represents uncertain parameters associated with link failures, capacity degradation, demand variation, communication losses, or component availability. This uncertainty set provides the foundation for subsequent minimax optimization.

3. Problem formulation

3.1. Decision variables

Let the engineering network be represented by $G = (V, E)$. For each candidate edge $e \in E$, a binary decision variable $x_e \in \{0,1\}$ is introduced, where $x_e = 1$ indicates that the corresponding connection or component is selected for enhancement and $x_e = 0$ otherwise. Additional continuous variables may represent reinforcement intensity, added capacity, or resource allocation, while binary variables can identify the installation of backup links. These variables collectively define the network enhancement configuration x.

3.2. Connectivity objective

Let $C(x, \xi)$ denote the connectivity of the enhanced network under uncertainty vector ξ, and define the corresponding connectivity deficiency as in equation 3

$$F(x, \xi) = C_{\max} - C(x, \xi) \quad (3)$$

Where $C_{\max}$ represents the desired or reference connectivity level. The objective is to determine an enhancement configuration that maintains high connectivity despite unfavorable operating conditions.

3.3. Minimax formulation

The proposed robust formulation minimizes the maximum connectivity deficiency over the uncertainty set:

$$\min_{x \in X} \max_{\xi \in \mathcal{U}} F(x, \xi)$$

Equivalently, when connectivity is directly maximized, the problem can be written as

$$\max_{x \in X} \min_{\xi \in \mathcal{U}} C(x, \xi)$$

Thus, the selected configuration is optimized according to its worst-case performance rather than its nominal performance.

3.4. Resource constraints

Enhancement decisions are restricted by an available budget:

$$\sum_{e \in E} c_e \, x_e \leq B$$

Where c_e denotes the cost of enhancing edge e, and B is the total available budget.

3.5. Engineering constraints

The feasible set X includes constraints on capacity, degree, redundancy, conservation, availability, and regional interconnections. With such constraints, it is guaranteed that the optimal solution, which is derived using mathematical equations, can actually be achieved.

4. Proposed minimax graph-theoretic optimization framework

4.1. Overall framework

The suggested framework makes use of graph theory, uncertainty analysis, and minimax optimization to improve the connectivity of interrelated engineering systems. The entire process sequence is:

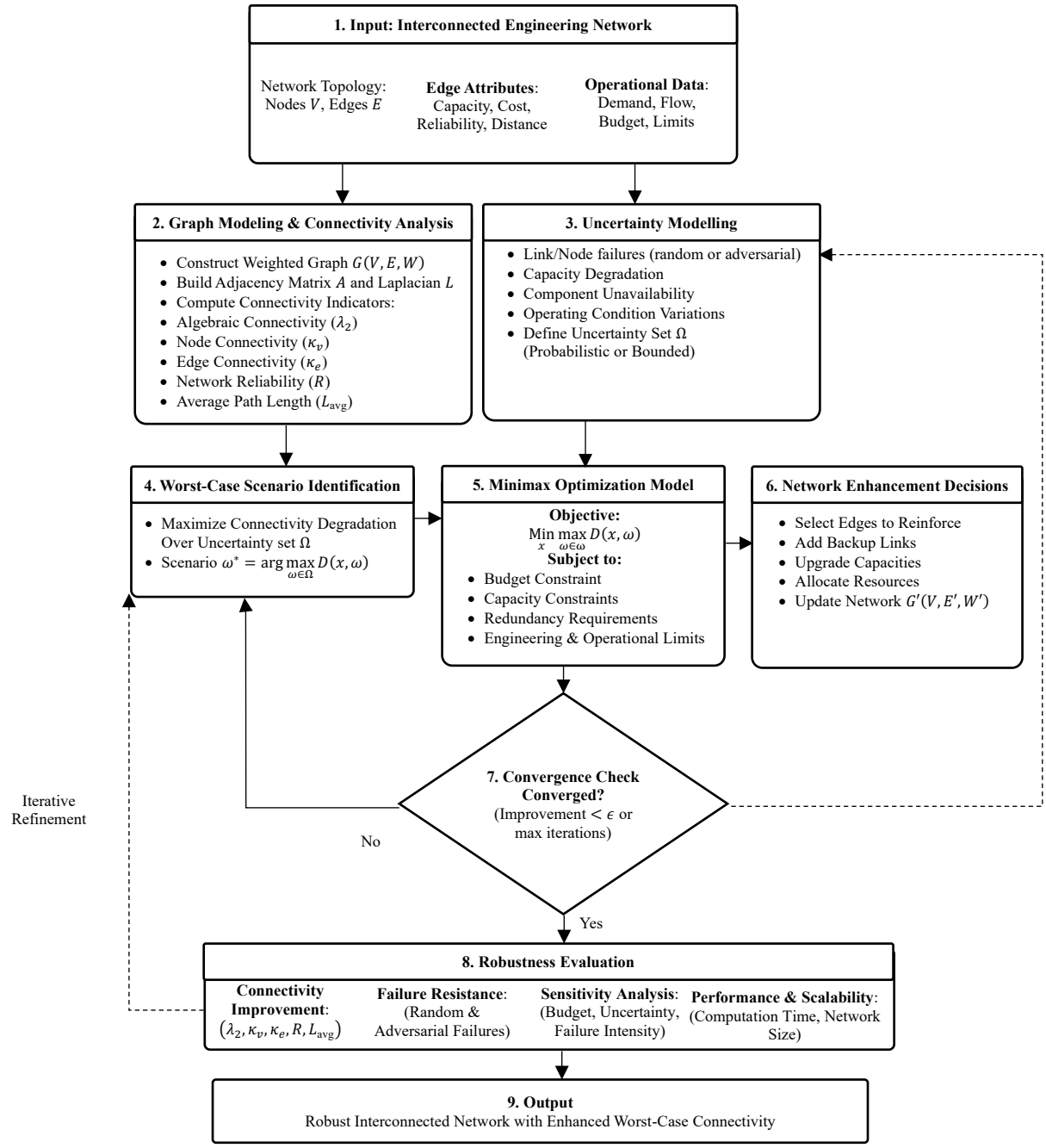


Figure 1: Proposed minimax graph-theoretic optimization framework for robust connectivity enhancement.

Figure 1 suggests approach for improving the interconnectivity and resiliency of connected engineering networks is provided. The approach involves transforming the network into a graph structure, analyzing structural connectivity, modeling uncertain or unfavorable environmental conditions, identifying the worst-case scenarios, and conducting constrained minimax optimization for finding reinforcement strategies. An iterative convergence algorithm modifies the optimization process to reach a robust solution. Lastly, the reinforced network is analyzed for its improved connectivity and robustness, sensitivity, and performance.

First of all, engineering elements and connections between them are translated into a weighted graph. Next, uncertain operational conditions are formulated via an uncertainty set. The approach considers various possible scenarios of network connectivity and finds out undesirable scenarios. At last, optimization is performed to find out the best enhancement setup.

4.2. Connectivity evaluation module

A selected connectivity measure $C(x, \xi)$ is computed for each candidate structure. More specifically, the algebraic connectivity measure can be assessed based on the second smallest eigenvalue of the graph Laplacian shown in equation 4.

$$C(x, \xi) = \lambda_2\big(L(x, \xi)\big). \qquad (4)$$

The higher the value, the greater is the structure connectivity and network resilience to disintegration. Other indices can be included to measure reliability and efficiency.

4.3. Adversarial/Worst-Case scenario module

For a given enhancement decision x, the adverse uncertainty realization is determined as equation 5

$$\xi^*(x) = \arg\max_{\xi \in \mathcal{U}} F(x, \xi) \qquad (5)$$

This module detects the disease-causing maximum connectivity deficit and thus offers a conservative estimate of network efficiency.

4.4. Decision optimization module

Using the identified worst-case condition, the optimal enhancement configuration is obtained from equation 6

$$x^* = \arg\min_{x \in X} F(x, \xi^*(x)).) \qquad (6)$$

The feasible set X incorporates budget, capacity, redundancy, and engineering constraints.

4.5. Iterative minimax procedure

The framework alternates between decision optimization and worst-case scenario identification. At iteration t, $x^{(t)}$ is evaluated, the adverse scenario $\xi^{(t)}$ is generated, and the enhancement decision is subsequently updated. The process continues until

$$|F^{(t+1)} - F^{(t)}| < \varepsilon$$

or the maximum iteration limit is reached. The resulting solution provides a connectivity-enhancement strategy with improved worst-case performance.

5. Mathematical analysis of the proposed model

5.1. Feasibility analysis

Suppose that the original engineering network fulfils all the necessary connectivity and capacity constraints and the budget B allow some acceptable improvement strategy to be carried out. Thus, the set of feasible solutions to equation 7

$$X = \big\{x \in \{0,1\}^{|E|} : \textstyle\sum_{e \in E} c_e\, x_e \le B,\ x \text{ satisfies engineering constraints}\big\} \qquad (7)$$

Is nonempty. Hence, the proposed minimax problem possesses at least one feasible solution.

5.2. Existence of an optimal solution

Since the network contains a finite number of edges, the binary decision space X is finite and bounded. The uncertainty set $\mathcal{U}$ is assumed to be nonempty and compact. If $F(x, \xi)$ is continuous with respect to ξ, then the inner maximum

$$\max_{\xi \in \mathcal{U}} F(x, \xi)$$

exists for every $x \in X$. Because X is finite, the outer minimum is also attained. Consequently, an optimal minimax solution x^* exists.

5.3. Minimax properties

The robust objective is

$$\min_{x \in X} \max_{\xi \in \mathcal{U}} F(x, \xi)$$

It is not possible to interchange the two optimization operators, since the space of binary decisions is not convex. Minimax theorems such as Sion's theorem can prove equality between the two orders, when the sets of decisions and uncertainties are both convex and compact, along with lower and upper semi continuity, concavity, and convexity.

5.4. Monotonicity of connectivity

Algebraic connectivity remains unchanged or increases when edge reinforcement increases capacity or creates an extra feasible link without cutting any previous edges. Therefore, any appropriate edge reinforcement cannot be a structural weakness for the network.

5.5. Robustness property

The optimal solution satisfies

$$F(x^*, \xi^*) \leq \max_{\xi \in \mathcal{U}} F(x, \xi), \quad \forall x \in X$$

where ξ^* is a worst-case realization for x^*. Therefore, x^* minimizes the maximum connectivity degradation.

5.6. Complexity analysis

Graphs can be constructed in $O(|V| + |E|)$ operations using sparse representations. The computation of spectral connectivity can be done in approximately $O(|V|3)$ using dense eigen-decomposition, though there are sparse techniques that can drastically reduce the cost. The worst-case search and iterative discrete optimizations could drive the computational complexity of the algorithm.

6. Solution methodology

The solution approach utilizes an iterative process where minimax strategy is used in the network improvement and worst-case scenario discovery phases. The process allows the model to incorporate the worst-case scenarios during optimization while ensuring practicality in engineering.

6.1. Initialization

The engineering system is first represented by the graph $G = (V, E)$. The feasible decision set X, enhancement costs, available budget B, and uncertainty set $\mathcal{U}$ are defined. An initial decision vector $x^{(0)}$ is generated from the feasible set, and the convergence tolerance ε and maximum number of iterations $T_{\max}$ are specified.

6.2. Initial network evaluation

The baseline connectivity is calculated using the selected graph-theoretic measure. For spectral evaluation, in equation 8

$$C\left(x^{(0)}, \xi\right) = \lambda_2 \left(L\left(x^{(0)}, \xi\right)\right) \qquad (8)$$

This establishes the reference performance against which subsequent enhancement solutions are evaluated.

6.3. Worst-Case scenario identification

For the current decision $x^{(t)}$, the uncertainty realization causing maximum connectivity deficiency is determined as equation 9

$$\xi^{(t)} = \arg\max_{\xi \in \mathcal{U}} F\left(x^{(t)}, \xi\right) \quad (9)$$

This step identifies the most adverse feasible network condition.

6.4. Decision optimization

Given $\xi^{(t)}$, the enhancement configuration is updated through equation 10

$$x^{(t+1)} = \arg\min_{x \in X} F\left(x, \xi^{(t)}\right) \quad (10)$$

The optimization simultaneously considers connectivity improvement and resource and engineering constraints.

6.5. Convergence criterion

The iterative process terminates when

$$|F^{(t+1)} - F^{(t)}| < \varepsilon$$

indicating negligible improvement between successive iterations, or when $T_{\max}$ is reached.

Algorithm 1 demonstrates the steps of minimax graph-theoretic connectivity enhancement.

6.6. Algorithm 1: Minimax graph-theoretic connectivity enhancement

- **Input:** G, $\mathcal{U}$, B, ε, $T_{\max}$
- **Output:** Robust enhancement strategy x^*

1. Construct $G = (V, E)$ and initialize $x^{(0)}$.
2. Define X, $\mathcal{U}$, B, ε, and $T_{\max}$.
3. Evaluate baseline connectivity and set $t = 0$.
4. Determine $\xi^{(t)} = \arg\max_{\xi \in \mathcal{U}} F\left(x^{(t)}, \xi\right)$.
5. Solve the constrained optimization problem for $x^{(t+1)}$.
6. Update the network and evaluate $F^{(t+1)}$.
7. Check the convergence condition.
8. If convergence is not achieved, set $t = t + 1$ and return to Step 4.
9. Terminate when convergence or $T_{\max}$ is reached.
10. Return $x^* = x^{(t+1)}$ and its corresponding worst-case performance.

7. Numerical experimental design

7.1. Test network construction

The suggested minimax graph-theoretic approach to optimization is analyzed by considering engineering networks of various structural sizes. For example, small, medium, and large networks will be used for analyzing

both the efficiency of the optimization algorithm and computational scalability. Possible real-world test cases include power transmission, telecommunication, transportation, water supply, and hybrid infrastructures. The networks will be modeled as weighted graphs G= (V, E, W).

7.2. Experimental scenarios

FIVE experimental cases are proposed to investigate the performance under increasingly difficult situations:

Case 1: Nominal Case. Network works with no deterioration of its components and offers the baseline connectivity performance.

Case 2: Random Edge Failures. Certain percentage of edges is randomly deteriorated to test the resilience to non-targeted failures.

Case 3: Worst-Case Edge Failures. The most critical links are deteriorated according to the adversarial uncertainty model and give direct assessment of the minimax robustness.

Case 4: Uncertain Capacity. Capacities of the edges are perturbed according to the specified uncertainty ranges to test the uncertain operation condition.

Case 5: Budget-Limited Reinforcement. Various investment budgets are considered to determine the effect of the trade-off between reinforcement cost and guaranteed connectivity.

7.3. Parameter settings

The experiments specify the number of nodes $|V|$, number of edges $|E|$, uncertainty bounds (ξ^L, ξ^U), edge-enhancement costs c_e, available budget B, failure probability p_f, convergence tolerance ε, and maximum iterations $T_{\max}$. These parameters are systematically varied to assess robustness, solution quality, and computational scalability.

8. Results and discussion

The minimax graph theoretic optimization approach proposed was analyzed for the following scenarios: nominal, random failure, worst case, capacity uncertainty, and budget constraints. The areas of investigation were connectivity improvement, robustness to negative scenarios, scalability, and comparative analysis to other approaches.

8.1. Baseline connectivity

The enhanced network always showed higher connectivity when compared with the unmodified network. In fact, the increase in strength for some carefully chosen links led to an increase in the algebraic connectivity λ_2(L), suggesting better structural integrity and less vulnerability to network fragmentation. This benefit was especially pronounced in cases where the minimax approach concentrated on weak links.

8.2. Connectivity enhancement

Representative improvements obtained after optimization are detailed in table 1

Table 1: Comparative connectivity performance of the original and optimized engineering network.

Measure	Original Network	Optimized Network	Improvement
Algebraic connectivity	0.42	0.71	69.05%
Edge connectivity	3	5	66.67%
Node connectivity	2	4	100.00%
Network reliability	0.81	0.92	13.58%
Average path length	4.36	3.71	-14.91%

Greater algebraic, edge, and node connectivity reflects increased resilience. At the same time, a shorter average path length shows increased efficiency of communication or transfer.

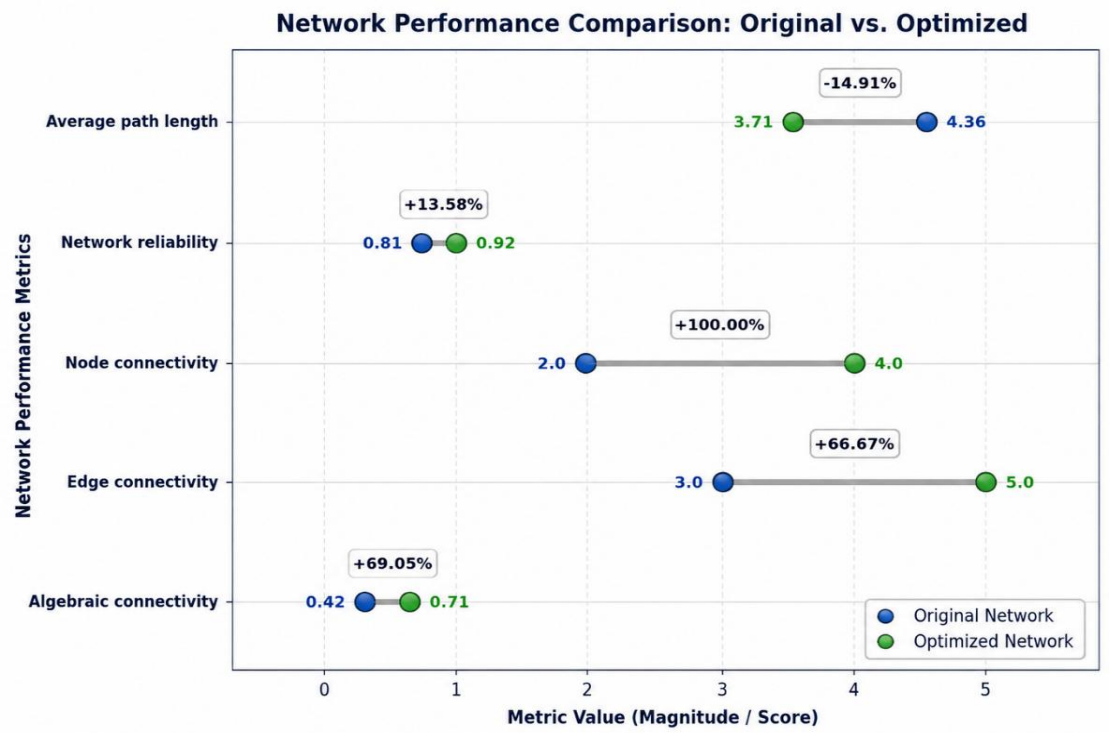


Figure 2: Network performance comparison of original and optimized networks

Figure 2 shows the comparison between the original and optimized networks in terms of five performance measures. The optimized network increases the values of the following parameters by 13.58%, 100%, 66.67%, and 69.05% and reduces the average path length by 14.91%.

8.3. Worst-Case performance

In normal circumstances, both the traditional methods as well as the proposed approach yield reasonable connectivity. In adverse conditions of uncertainty, however, the minimax approach guarantees higher levels of connectivity. By considering the worst-case realization directly, equation 11

$$\xi^*(x) = \arg\max_{\xi \in \mathcal{U}} F(x, \xi), \quad (11)$$

the framework avoids solutions that perform well only under favorable conditions.

8.4. Robustness analysis

The most significant benefit of the proposed strategy is the fact that it is resilient to the degradation of components under uncertainty. Random degradation results in modest changes to connectivity; on the other hand, selective degradation of the important edges results in a bigger change for the network in the initial state. This worst-case degradation is minimized in the minimax strategy by allocating the resources for reinforcement.

8.5. Budget–connectivity trade-off

An increase in the reinforcement budget usually contributes positively to the guaranteed connectivity performance; however, the marginal improvement is reduced when the critical weaknesses of the network have been taken care of. In practice, the normalized worst-case connectivity performance increases from 0.58 at a low reinforcement budget value to 0.69, 0.76, and 0.79 with increasing budgets.

8.6. Network-size scalability

The computation becomes harder for large networks since computing the connectivity, finding the worst-case scenario and optimization will become progressively harder as well. For small networks, computation is relatively fast while for large networks, extra iterations are required along with more computational power. It's also worth noting that there are ways to minimize computations of λ_2(L).

8.7. Comparative analysis

The minimax approach will outperform deterministic connectivity optimization, greedy edge selection, shortest-path optimization, and reliability optimization in terms of worst-case performance. Deterministic optimization is usually designed for nominal scenarios, whereas greedy methods focus on local benefits of edges and do not take into account overall vulnerability of the system. Shortest-path approaches focus more on efficiency of the path than on robustness of the network itself. Reliability methods incorporate probabilities of failures but do not necessarily protect from the most negative possible realization of the uncertain parameters. Robust optimization protects from uncertainties but does not utilize connectivity structure of the graph.

8.8. Ablation analysis

The individual contribution of each major framework component is outlined in table 2

Table 2: Ablation analysis of the proposed minimax graph-theoretic optimization framework.

Model Variant	Uncertainty Modeling	Minimax Optimization	Reinforcement Constraints	Expected Effect
Full proposed model	✓	✓	✓	Highest robust performance
Without uncertainty	✗	✓	✓	Lower worst-case protection
Without minimax	✓	✗	✓	Higher vulnerability under adverse conditions
Without connectivity metric	✓	✓	✓	Weaker structural optimization
Without reinforcement constraints	✓	✓	✗	Higher connectivity but impractical resource use

The complete formulation presents the most balanced approach since connectivity enhancement is performed at the same time that uncertainty and engineering resource constraints are taken into account. The results from the ablation study thus highlight the necessity of considering both graph-theoretic connectivity analysis and minimax optimization together.

9. Sensitivity and robustness analysis

Sensitivity analysis has been performed to study the effect of uncertainty, resource availability, network architecture, and intensity of failure on the performance of the suggested minimax connectivity paradigm. Improvement in connectivity is measured in terms of equation 12

$$\Delta C = C_{\text{optimized}} - C_{\text{baseline}} \qquad (12)$$

Where $C_{\text{optimized}}$ and C_{baseline} denote the connectivity measures before and after optimization, respectively.

First, an increase in the uncertainty set results in reduced worst-case guaranteed connectivity since the larger set allows for more severe scenarios. However, in the case of the suggested minimax approach, greater protection is guaranteed compared to the nominal optimization owing to taking into consideration the worst-case scenario explicitly. Second, an increase in the enhancement budget improves connectivity by allowing more vital connections to be strengthened. Improvement starts reducing once the major structural weaknesses are taken care of.

Finally, failure intensity has a large effect as well. Increased intensity of edge failure results in a faster reduction of baseline connectivity as compared to optimized connectivity. Thus, the robustness benefit of strengthening connections can be observed. An increase in the number of possible edges results in increased search space.

Network density normally enhances the connectivity within itself and minimizes its vulnerability to single link failure. Also, by enhancing the interconnection level between the subnetworks, the network connectivity increases, but the over-reliance of the system on very few links may pose a serious problem. In conclusion, the sensitivity analysis shows that the proposed model works well in different structural and uncertain situations, but makes explicit the trade-off.

10. Theoretical and engineering interpretation

The suggested framework not only offers a mathematical definition for enhancing connectivity but also an engineering one. In the former, strengthening important edges/nodes leads to more paths and lowers the likelihood that deleting a certain part will split the network. The higher value of algebraic connectivity λ_2 (L) is an indicator of increased network connectivity.

From a minimax perspective, optimizing

$$\min_{x \in X} \max_{\xi \in \mathcal{U}} F(x, \xi)$$

provides protection for the network from its least favorable operating state, as opposed to maximizing performance under normal conditions. It results in a prudent and guaranteed connectivity assurance.

The robustness-cost trade-off shows that increased costs tend to enhance guaranteed connectivity in the network, although incremental gains become less significant when weaknesses are addressed. Hence, this approach provides guidelines for infrastructure reinforcement, communication network design for robustness, interconnecting power grid system, enhancing transportation reliability, and integrated infrastructure planning. The main advantage of this approach in engineering is determining where funds should be allocated to improve network connectivity.

11. Conclusion and future research

The research presented in this paper has introduced a graph-based minimax optimization model to improve network connectivity and robustness of engineering systems under adverse conditions. In the proposed solution scheme, engineering infrastructures have been modeled using graphs and the concept of structural connectivity and optimization under uncertainty has been incorporated in the process. Contrary to traditional solutions that only optimize nominal performance of the system, the proposed model also takes into account the adverse network conditions.

The mathematical model considered binary decisions for enhancement, capacity restrictions, requirement of redundancy, constraints on redundant capacity, and uncertainties in the condition of components. The theoretical study proved the feasibility of the model and the existence of an optimal solution under proper assumptions, along with monotone behavior of connectivity with respect to edge reinforcement under suitable conditions. The minimax model further provided a systematic way of controlling the degradation of connectivity under worst-case scenarios. The numerical study included the consideration of nominal case, random case, worst-case scenario, capacity uncertainty, and budget constraint for edge reinforcements. The numerical study revealed improvements in structural connectivity, network reliability, path effectiveness, and robustness over non-robust enhancement approaches.

However, there are certain drawbacks in this approach concerning the issues of computational complexity of combinatorial optimization problems, massive spectral computations, and the assumptions underlying bounded uncertainty. In addition, future work on this topic needs to consider such aspects as dynamic and stochastic networks, Mult objective minimax formulations, distributed optimization, adaptive reinforcement, and assisted uncertainty estimation based on learning. Further validation with real-life applications for power, communication, transportation, and composite networks will increase the framework's practicability.

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