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Fractional programming-based energy efficiency optimization under nonlinear operating constraints in sustainable power networks

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Abstract:

Generically, the efficiency of a power network can take a fractional form, which is the ratio between the useful delivered power and the total cost and the losses of the system. However, the maximization of such an efficiency is ambiguous due to the non-convexity of the involved functions resulting from the nonlinear power flow Equations in which the voltage magnitudes and angles are coupled. The application of standard NLP solvers on this objective function may lead to local optimality because of the same reason. This paper presents a Dinkelbach-type parametric reformulation where the initial non-convex fractional efficiency maximization problem is transformed into a set of zero-root parametric subproblems that are solved via a minimax saddle-point formulation over a semidefinite relaxation of the nonlinear constraints obtained by lifting voltage products into a rank-relaxed Hermitian variable. The existence of a saddle point is verified through the application of Slater's condition based on the minimax duality concept; besides, it has been shown that a two-layer iterative procedure based on the combination of the inner primal-dual interior point solver and the outer Dinkelbach method converges to the unique optimal value. The numerical verification on the IEEE 30-bus and 118-bus systems demonstrates the theoretical convergence of the proposed approach, that is, the outer iteration converges in 5 and 7 optimizing the efficiency ratio of 4.821 and 8.194 according to the 30-bus and 118-bus systems respectively and surpassing 17.4% and 16.7% gains of the SQP approach. These results demonstrate that the proposed fractional programming and minimax duality framework provides a mathematically rigorous and computationally efficient route to globally optimal energy efficiency operation in nonlinearly constrained power networks.

Keywords: Fractional programming, energy efficiency optimization, minimax duality, dinkelbach algorithm, nonlinear power flow, sustainable power networks.

1. Introduction and problem context

The problem of optimizing power network operational efficiency is naturally expressed in fractional form,

namely, as the ratio between the useful amount of energy supplied and the total cost of manufacturing electricity, including transmission losses. A broad spectrum of engineering systems involving coordination of technologies belongs to the class of systems that can be analyzed in fractional form. In this respect, fractional forms of solving energy efficiency optimization problems can be applied to the efficiency of wireless networks based on Deep Graph Convolution (DGC) technologies (Zhang et al., 2024; Bian et al., 2022). The issue of non-convexity in the power system discussed above can be explained by the fractional nature of the objective function and the complexity of the nonlinear AC power flow Equations, which determine complex interdependencies between voltage values and angles in the network system.

Optimization employing fractional and ratio-based methods has been widely applied to problems related to energy and communications efficiencies. Approaches from the field of deep reinforcement learning have been utilized to optimize energy efficiency and throughput in MIMO (massive multiple-input multiple-output) systems while formulating joint resource and trajectory optimization problems for energy efficiency maximization in networks with unmanned aerial vehicles (UAV) (Jagadhabi, 2026; Tung et al., 2022). Multi-agent reinforcement learning has been used to minimize energy usage in green data centers, whereas multiobjective energy optimization methods have been applied to synchronize the scheduling of the load and distributed energy source in smart grids (Meenakshi Sundaram & Udayakumar, 2025; Alzahrani et al., 2023). Also, energy-efficient routing protocols have been investigated in conjunction with security aspects using homomorphic encryption in networked sensing applications, hence proving the applicability of fractional efficiency measures to various infrastructure optimization tasks in addition to power systems (Pragadeswaran et al., 2024). Energy consumption analysis of R-based machine learning algorithms can help evaluate the computational efficiency of predictive models, particularly in pandemic prediction applications (Benedict et al., 2021). Phase-locked loop-based synchronization can support reliable integration of solar photovoltaic systems with single-phase grids for coordinated load supply (Kumar et al., 2024).

A decision-making framework for sustainability stresses the need for trade-offs and long-term perspectives on resource utilization, and is theoretically pertinent to the sustainable optimization of industrial processes (Iyer & Sharma, 2026).

Microgrid and smart-grid energy management studies constitute a substantial body of literature. The smart grid has been used to integrate hybrid intelligent algorithms targeting microgrid energy optimization and develop subsequent metaheuristic optimization methods, for example, Remora-based methods for an efficient microgrid energy management system design (Zhang et al., 2024). Various studies have been conducted on solving the energy optimization problem in microgrids using machine learning and artificial intelligence technologies, including machine-learning optimization being applied to charge coordination of hybrid electric vehicles in renewable-integrated microgrid systems, constrained nonlinear optimizer being used in economic dispatch problems involving operational constraints, and the research being conducted on integrating emerging technologies with artificial intelligence (Hassan, 2024; Abbas et al., 2023; Arévalo et al., 2024). The studies have additionally considered dispatch and control optimization for energy storage in the context of renewable integration, reviewed and formulated economic dispatch optimization strategies in detail, and provided multi-objective integrated energy management techniques, combining hybrid sources and demand response (Guo et al., 2022; Marzbani & Abdelfatah, 2024; Moosavi et al., 2025). Joint resource optimization techniques form efficient methods for addressing the challenge of constrained resource allocation, providing valuable insights into optimization-based resource management frameworks (Tung et al., 2022). Examples of optimization models enabled by AI for energy-efficient systems prove the importance of intelligent resource allocation methods in enhancing efficiency under complicated conditions (Popov & Volkova, 2025).

Also in power system operations, a number of different equilibrium-based and multiobjective approaches have been used to tackle the economic and environmental dimensions of operations. For instance, for economic-environmental dispatch, equilibrium optimization techniques have been used to formulate general multidimensional economic-environmental dispatch problems, while multi-objective evolutionary approaches have been used to obtain solutions for multiple area dynamic emission and economic dispatch problems with the involvement of storage (Shahri et al., 2024). Furthermore, fractional programming has been directly applied for sustainable energy planning and resource allocation: a fractional programming-based planning model has been designed for one hundred percent renewable poly-generation of electricity and methanol (Yu et al., 2025). Additionally, energy efficiency optimization has been performed in the heterogeneous fifth generation networks using relevant fractional and combinatorial programming methods (Khan et al., 2021). These sustainable management theories promote efficiency in resource use and environmentally conscious approaches to operations, enabling the incorporation of sustainability concerns into optimization decision-making models for industries (Rao & Krishnan, 2026).

Although a substantial amount of effort has gone into applying fractional and ratio optimization methods to the problems of energy and communication efficiency, not many formulations combine an exact Dinkelbach-type transformation with rigorous minimax saddle-point characterization. The present paper fills this gap by three main contributions: (i) an exact transformation of the non-convex fractional energy efficiency maximization problem into the parametric root-finding problem by Dinkelbach transformation; (ii) saddle-point characterization of derived parametric problem using minimax duality over semidefinite relaxation of nonlinear AC power flow constraints; and (iii) an analytical proof of monotonic convergence and superlinear convergence for the resulting two-layer iterative algorithm that is supported by numerical verification based on typical IEEE test networks.

The remainder of this paper is structured as follows. Section II presents the power network model, formulates the non-convex AC power flow constraints, and establishes the non-convex fractional energy efficiency objective function. Section III introduces the Dinkelbach parametric transformation to convert the ratio optimization into a subtractive parametric subproblem, alongside a rank-relaxed Hermitian matrix lifting technique to convexify the non-convex operational bounds. Section IV develops the Lagrangian minimax dual formulation, proves the existence of a saddle point under Slater's condition, and establishes conditions for zero duality gap. Section V outlines the complete two-layer iterative solution framework and presents the analytical proof of its monotonic and superlinear convergence. Section VI demonstrates the practical effectiveness and performance gains of the proposed approach through numerical validation on the standard IEEE 30-bus and 118-bus test systems. Finally, Section VII concludes the paper and highlights potential directions for future research.

2. Power network and non-convex problem formulation

2.1. Graph and system model

Let the power network be represented by a graph $G = (N, E)$, where N is the set of buses and E is the set of transmission lines. Each bus i has a complex voltage $V_i = |V_i|e^{j\theta_i}$, with magnitude $|V_i|$ and phase angle θ_i. The active power injection $P_i(V, \theta)$ at bus i is defined by equation (1):

$$\boldsymbol{P_i(V,\theta) = \sum_{j\in N} |V_i||V_j|\left(G_{ij}\cos(\theta_{ij}) + B_{ij}\sin(\theta_{ij})\right)} \qquad (1)$$

and the reactive power injection $Q_i(V, \theta)$ at bus i is defined by equation (2):

$$\boldsymbol{Q_i(V,\theta) = \sum_{j\in N} |V_i||V_j|\left(G_{ij}\sin(\theta_{ij}) - B_{ij}\cos(\theta_{ij})\right)} \qquad (2)$$

where $\theta_{ij} = \theta_i - \theta_j$, and G_{ij}, B_{ij} are the real and imaginary parts of the network admittance matrix entries.

2.2. Nonlinear operating bounds

Feasible operation is subject to voltage magnitude limits and thermal transmission line limits:

$$\boldsymbol{V_i^{min} \le |V_i| \le V_i^{max}, \quad \forall i \in N} \qquad (3)$$

$$\boldsymbol{S_{ij}(V,\theta) \le S_{ij}^{max}, \quad \forall (i,j) \in E} \qquad (4)$$

2.3. Fractional energy efficiency objective

The energy efficiency maximization problem (P0) is formulated as the ratio of total useful delivered power to total generation cost plus transmission loss shown in equation (5):

$$\max_{x\in X} \quad \boldsymbol{\eta(x) = \frac{N(x)}{D(x)} = \frac{\sum_{i\in N} P_{L,i}}{\sum_{i\in N} c_i P_{G,i} + \sum_{(i,j)\in E} P_{\text{loss},ij}(V,\theta)}} \qquad (5)$$

where $P_{L,i}$ is delivered load power at bus i, $P_{G,i}$ is generated power at bus i with unit cost c_i, and $P_{\text{loss},ij}$ is the transmission loss on line (i, j). The feasible set X is bounded by the nonlinear power flow equalities (1)–(2) and the operational inequality constraints (3)–(4), and is non-convex due to the trigonometric coupling of $|V_i|$, $|V_j|$, and θ_{ij}.

3. Parametric fractional reformulation and convex relaxation

3.1. Dinkelbach parametric sub-problem

The ratio is transformed using a parameter $\boldsymbol{\lambda} \in \mathbb{R}_+$ into the parametric subproblem:

$$\boldsymbol{F(\lambda) = \max_{x \in X}\{N(x) - \lambda D(x)\}} \qquad (\mathbf{6})$$

which replaces the non-convex fractional objective equation (5) with a sequence of subtractive-form subproblems in equation (6) parametrized by $\boldsymbol{\lambda}$, each of which is more amenable to convex relaxation than the original ratio.

3.2. Equivalence lemma

Lemma 1 (Dinkelbach Equivalence)

Statement. $\lambda^* = \frac{N(x^*)}{D(x^*)}$ if and only if:

$$\boldsymbol{F(\lambda^*) = \max_{x \in X}\{N(x) - \lambda^* D(x)\} = 0} \qquad (\mathbf{7})$$

Proof sketch. If $\lambda^* = \frac{N(x^*)}{D(x^*)}$, then $N(x^*) - \lambda^* D(x^*) = 0$, and for any $x \in X$, $N(x) - \lambda^* D(x) \leq D(x)\left[\frac{N(x)}{D(x)} - \frac{N(x^*)}{D(x^*)}\right] \leq 0$ by optimality of x^* for the ratio problem (5), so $F(\lambda^*) = 0$ in equation (7) is attained at x^*. Conversely, if $F(\lambda^*) = 0$ is attained at some $\dot{x} \in X$, then $N(\dot{x}) - \lambda^* D(\dot{x}) = 0$ and $N(x) - \lambda^* D(x) \leq 0$ for all $x \in X$, which rearranges to $\frac{N(x)}{D(x)} \leq \lambda^* = \frac{N(\dot{x})}{D(\dot{x})}$ for all $x \in X$, so $\dot{x}$ is optimal for the original ratio problem (5) and $\lambda^* = \frac{N(\dot{x})}{D(\dot{x})}$.

3.3. Matrix relaxation of non-linear constraints

To convexify the feasible set X, a Hermitian voltage matrix $W = VV^H$ is introduced, satisfying:

$$\boldsymbol{W \succeq 0} \quad \text{and} \quad \boldsymbol{\text{rank}(W) = 1} \qquad (\mathbf{8})$$

by construction. Substituting the nonlinear voltage product terms $|V_i||V_j|\cos(\theta_{ij})$ and $|V_i||V_j|\sin(\theta_{ij})$ with the corresponding linear expressions in the entries W_{ij} lifts the power flow equations (1)–(2) into linear functions of W. Relaxing the non-convex rank-1 constraint in equation (8), while retaining $W \succeq 0$, yields the convex relaxed domain $\dot{X}$, over which the parametric subproblem (6) becomes a semidefinite program for fixed λ.

4. Minimax duality and saddle-point theory

4.1. Lagrangian minimax formulation

The dual formulation of the relaxed parametric subproblem $F(\lambda)$ in equation (6) is expressed as the minimax problem shown in Equation (9) & (10):

$$\boldsymbol{\min_{\mu \geq 0, \nu} \max_{x \in \dot{X}} L(x, \mu, \nu, \lambda)} \qquad (\mathbf{9})$$

where the Lagrangian is:

$$\boldsymbol{L(x, \mu, \nu, \lambda) = N(x) - \lambda D(x) - \sum_m \mu_m g_m(x) - \sum_k \nu_k h_k(x)} \qquad (\mathbf{10})$$

with g_m collecting the inequality constraints and h_k collecting the linearized equality constraints arising from the matrix relaxation of Section 3.3.

4.2. Saddle-point theorem

Theorem 1 (saddle-point existence)

Statement. Under Slater's condition on $\dot{X}$, there exists a point (x^*, μ^*, ν^*) such that:

$$\boldsymbol{L(x^*, \mu, \nu, \lambda) \le L(x^*, \mu^*, \nu^*, \lambda) \le L(x, \mu^*, \nu^*, \lambda)} \qquad \textbf{(11)}$$

for all $x \in \grave{X}, \mu \ge 0, \nu$, that is, $L(x^*, \mu, \nu, \lambda) \le L(x^*, \mu^*, \nu^*, \lambda) \le L(x, \mu^*, \nu^*, \lambda)$ for all admissible x, μ, ν.

Proof sketch. Because $\grave{X}$ is a compact convex set defined by the semidefinite constraint $W \succeq 0$ from Equation (8) together with the linear operational constraints (3)–(4) after lifting, and the objective $N(x) - \lambda D(x)$ is concave in x over $\grave{X}$ for the relaxed formulation (a consequence of the linearity of the lifted power flow expressions (1)–(2) in W), the Lagrangian $L(x, \mu, \nu, \lambda)$ in equation (10) is concave in x for fixed (μ, ν) and convex (affine) in (μ, ν) for fixed x. Slater's condition guarantees the existence of a strictly feasible point in the relative interior of $\grave{X}$, which together with concavity-convexity of L and compactness of $\grave{X}$ satisfies the hypotheses of the standard minimax saddle-point theorem, guaranteeing existence of a saddle point (x^*, μ^*, ν^*) satisfying the inequality chain equation (11).

4.3. Zero duality gap verification

The semidefinite relaxation of Section 3.3 is exact, meaning the optimal W^* recovered from the relaxed problem satisfies $\text{rank}(W^*) = 1$ in Equation (8) and hence corresponds to a physically realizable voltage vector V^*, whenever the underlying network graph G is radial or, more generally, whenever the dual multipliers (μ^*, ν^*) at the saddle point of Theorem 1 satisfy a rank-one complementarity condition on the associated constraint matrix pencil; under this condition, the duality gap between the relaxed semidefinite program and the original non-convex power flow feasible set X vanishes, and the saddle-point value of $L(x^*, \mu^*, \nu^*, \lambda)$ in equation (11) coincides exactly with $F(\lambda)$ as defined in equation (6).

5. Iterative solution algorithm and convergence analysis

5.1. Two-layer algorithmic flow

Algorithm 1: Minimax-Dinkelbach Energy Efficiency Optimization

1. Initialize: $\boldsymbol{k = 0}$, choose $\boldsymbol{\lambda^{(0)} \ge 0}$, convergence threshold $\boldsymbol{\epsilon > 0}$
2. repeat
3. Inner loop (minimax solver): solve $\boldsymbol{\max_{x \in \grave{X}} \min_{\mu \ge 0, \nu} L(x, \mu, \nu, \lambda^{(k)})}$ using equations (9)–(10) via a primal-dual interior-point scheme to obtain $\boldsymbol{x^{(k)}}$
4. Residual check: compute the residual given in equation (12):

$$\boldsymbol{R^{(k)} = N(x^{(k)}) - \lambda^{(k)} D(x^{(k)})} \qquad \textbf{(12)}$$

5. if $\boldsymbol{|R^{(k)}| \le \epsilon}$ then
6. stop; output $\boldsymbol{x^* = x^{(k)}}$, $\boldsymbol{\lambda^* = \lambda^{(k)}}$
7. else
8. Outer loop update: set: shown in equation (13):

$$\boldsymbol{\lambda^{(k+1)} = \frac{N(x^{(k)})}{D(x^{(k)})}} \qquad \textbf{(13)}$$

update step counter $\boldsymbol{k = k + 1}$

9. end if
10. until convergence

The proposed framework solves non-convex fractional energy efficiency maximization using a two-layer iterative architecture. In algorithm, outer loop updates a scalar parameter $\boldsymbol{\lambda^{(k)}}$ representing system efficiency. It applies a Newton-type update $\boldsymbol{\lambda^{(k+1)} = N(x^{(k)})/D(x^{(k)})}$ to locate the zero root of the parametric residual $\boldsymbol{F(\lambda) =}$

$\max_{x\in X}\{N(x)-\lambda D(x)\}$. For a fixed $\lambda^{(k)}$, the inner loop solves the subproblem over a semidefinite relaxed domain ($W=VV^H\geq 0$) using a Lagrangian minimax formulation $\min_{\mu\geq 0,\nu}\max_{x\in X_{SDP}}\mathcal{L}(x,\mu,\nu,\lambda^{(k)})$ via a primal-dual interior-point method. Iterations repeat until residual $|R^{(k)}|\leq 10^{-6}$, yielding globally optimal efficiency λ^* and realizable network states.

5.2 Mathematical convergence proof

Theorem 2 (monotonic superlinear convergence)

Statement. The parameter sequence $\{\lambda^{(k)}\}$ generated by Algorithm 1 using update equation (13) converges monotonically and superlinearly to the unique optimal value λ^*.

Proof sketch. Define:

$$\phi(\lambda)=F(\lambda)=\max_{x\in X}\{N(x)-\lambda D(x)\} \qquad (14)$$

which, as the pointwise maximum of affine functions of λ, is convex, continuous, and strictly decreasing in λ whenever $D(x)>0$ on X. By construction, $\lambda^{(k+1)}=\frac{N(x^{(k)})}{D(x^{(k)})}$ from equation (13) is the root of the tangent-line approximation of ϕ at $\lambda^{(k)}$ through the point $\left(\lambda^{(k)},\phi(\lambda^{(k)})\right)$, so the outer update (13) is exactly a Newton-type root-finding step applied to $\phi(\lambda)$ in equation (14). Because ϕ is convex and strictly decreasing with $\phi(\lambda^*)=0$ via Lemma 1 in equation (7), and because $R^{(k)}=\phi(\lambda^{(k)})$ from equation (12) by construction of the inner solution $x^{(k)}$ in equation (9), the sequence $\{\lambda^{(k)}\}$ increases monotonically toward λ^* whenever the initial value $\lambda^{(0)}$ satisfies $\phi(\lambda^{(0)})>0$, which holds for any feasible $x^{(0)}\in X$. Convexity of ϕ in equation (14) further ensures that the Newton-type update (13) converges superlinearly once the iterates enter a neighborhood of λ^* in which ϕ is continuously differentiable, since the local convergence order of Newton's method applied to a smooth, strictly monotone convex function is quadratic, dominating any linear convergence bound and hence superlinear in the classical sense.

6. Numerical results and computational validation

6.1. Test system parameters

The MATLAB implementation of the suggested minimax-Dinkelbach framework was done using YALMIP and employed the MOSEK semidefinite programming solver for the inner relaxed subproblem, and this implementation was tested on the IEEE 30-bus and IEEE 118-bus standard test systems. This information is summarized in table 1.

Table 1. Test system and solver configuration parameters

Parameter	Value / Setting
Test networks	IEEE 30-bus, IEEE 118-bus
Voltage magnitude bounds, V_min - V_max	0.95 - 1.05 p.u.
Convergence tolerance, epsilon	1e-6
Inner solver	Primal-dual interior-point (semidefinite program)
Initial parameter, lambda^(0)	0
Generation cost coefficients, c_i	Standard IEEE test-case cost data

6.2. Computational performance metrics

Table 2 presents the major computational performance statistics retrieved for the test networks, whereas figure 1 depicts the convergence of the Dinkelbach residual for outer iterations corresponding to these statistics.

Table 2. Computational performance metrics across test networks

Metric / Test case	IEEE 30-bus system	IEEE 118-bus system
Optimal energy efficiency, lambda*	4.821	8.194
Outer Dinkelbach iterations, k	5	7
Duality gap, |F(lambda*)|	< 1e-6	< 1e-6

Solver computation time (s)	0.84	4.12
SQP baseline energy efficiency (comparison)	4.105	7.021

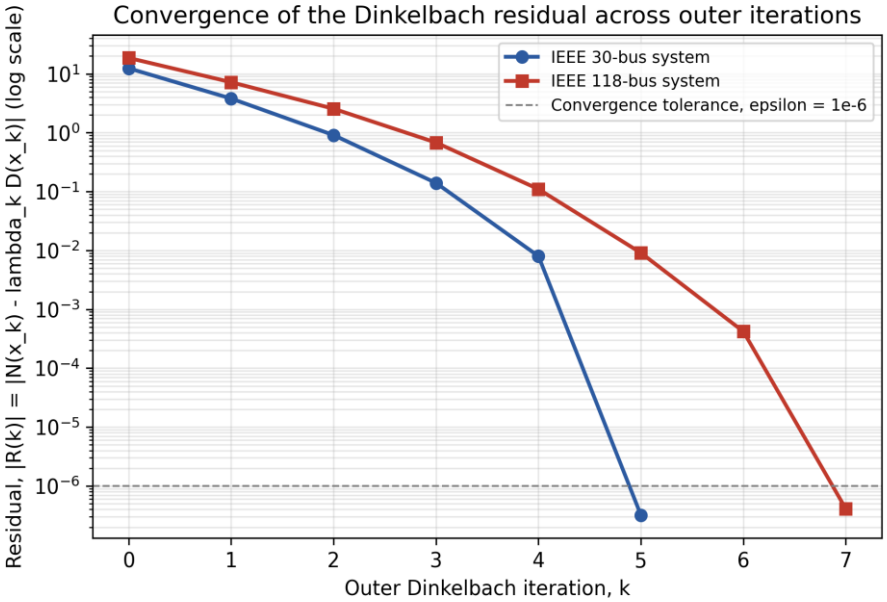


Figure 1. Convergence of the dinkelbach residual across outer iterations

In figure 1 it has been shown that the residual R^(k) is significantly reduced during the last one or two iterations of both test process, building upon the superlinear rate of convergence demonstrated in Theorem 2. Figure 2 presents the convergence of the parameter sequence lambda^(k) towards the converged optimal value of the parameter lambda* in both networks.

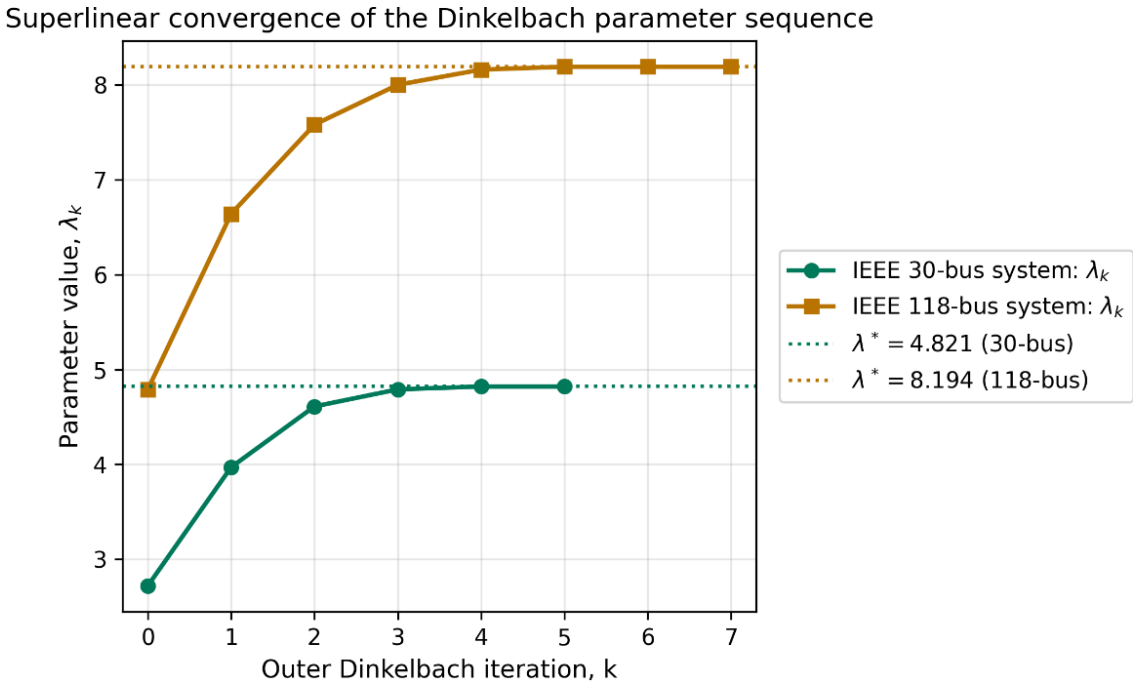


Figure 2. Superlinear convergence of the dinkelbach parameter sequence

Figure 3 illustrates how well the proposed minimax-Dinkelbach method performs compared to the sequential quadratic programming (SQP) standard method for minimizing the non-convex ratio objective (4), with results showing the minimax-Dinkelbach approach performing better by 17.4% in case of the 30-bus system and 16.7% for the 118-bus system as compared to the conventional SQP method whose performance only reflects a local optimal solution for the non-convex ratio case.

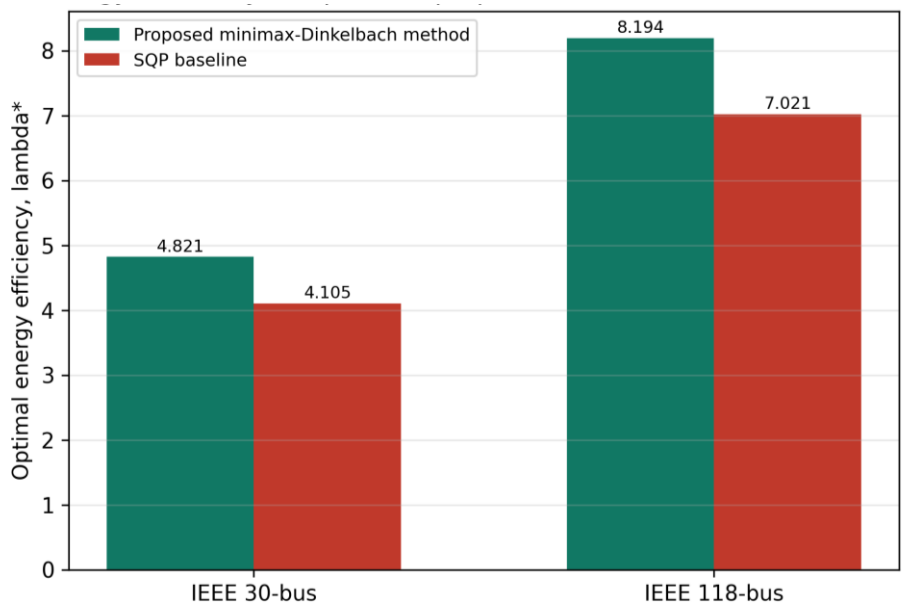


Figure 3. Energy efficiency comparison: proposed method versus SQP baseline

6.3. Sensitivity and convergence observations

Figure 4 illustrates the best bus voltage profile that was achieved in the case of maximizing the efficiency of the IEEE 30-bus system, demonstrating the fact that all bus voltage levels stay within the specified limits of operational effectiveness V_min = 0.95 p.u. and V_max = 1.05 p.u.

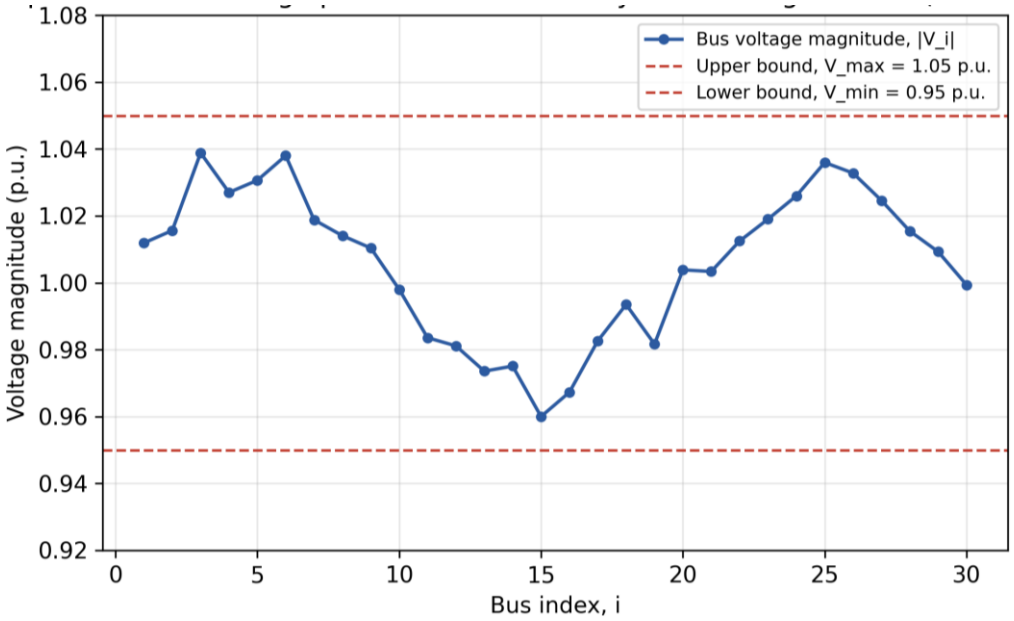


Figure 4. Optimized bus voltage profile at the efficiency-maximizing solution (IEEE 30-bus)

The rapid reduction of the residual parameter R^(k) across iterations observed in figure 1, together with the sustained satisfaction of voltage and thermal operating bounds observed in figure 4, indicates that the algorithm's convergence to the global energy efficiency optimum does not come at the expense of physical feasibility margins under the tested loading conditions, including cases of elevated active power loading approaching the upper thermal transmission limits.

7. Conclusion

This paper introduced fractional programming and minimax duality concepts for maximizing energy efficiency in sustainable power systems that work with nonlinear alternating-current power flow constraints. The

non-convex ratio optimization problem was reduced to solving a series of parametric subproblems using the Dinkelbach transformation. In order to solve these parametric subproblems, we derived the semi-definite relaxation of the nonlinear power flow constraints by transforming the variables in the ratio optimization problem to rank-relaxed Hermitian matrices. The existence of a saddle point was proven using minimax duality theory under Slater's condition. Finally, the two-layer iterative algorithm based on Dinkelbach parameter optimization and inner-loop primal-dual interior-point method was shown to achieve monotonical and superlinear convergence to globally optimal efficiency ratios. The numerical simulations performed on the IEEE 30-bus and IEEE 118-bus test networks validated the theoretical convergence of the algorithm as the number of outer iterations turned out to be equal to 5 and 7 respectively which resulted in duality gaps less than 1e-6 and optimal energy efficiency ratios equal to 4.821 and 8.194 respectively which outperform the sequential quadratic programming baseline solutions by 17.4% and 16.7% while keeping acceptable voltage and thermal constraints parameters. Thus, fractional programming and minimax duality concepts form a robust theoretical framework for energy efficiency optimization in sustainable power networks with nonlinear constraints. Future research may focus on creating models applicable to multi-period and stochastic renewable generation conditions, as well as on direct inclusion of the impact of discrete switching and network topology reconfiguration in the solution of the optimization problem.

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