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Polynomial system optimization methods for nonlinear control parameter selection in engineering systems

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Abstract:

Nonlinear engineering systems involve complex interrelationships between parameters and sensitivities to operating conditions. Finding optimal control parameters for such systems becomes an optimization challenge. Typical tuning methods tend to focus on nominal performance and often lead to controllers which are not robust enough under the influence of uncertainty in parameters, disturbances, and unfavorable operating conditions. This paper introduces the Polynomial Minimax Optimization (PMMO) method as a robust approach to nonlinear control parameter selection. In PMMO, the dynamics of the nonlinear system response are modeled via polynomial basis function with parameters and interactions, and the uncertain parameters are modeled via uncertainty sets. The objective is a combination of several measures like tracking error, settling time, overshoot, and robustness which is defined as a minimax problem in order to minimize the maximum achievable objective under the uncertainty domain. Various engineering constraints, such as limits on the controller parameters, actuators, states and performance are considered in the optimization procedure. A nonlinear DC motor controlled by PID is used as a case study to analyse the proposed framework for nominal, low, medium, high, and adverse uncertainty cases. Comparative analysis to gradient optimization, Genetic Algorithm, Particle Swarm Optimization, and Differential Evolution reveals that the PMMO framework is capable of generating competitive nominal solutions while minimizing performance degradation. Sensitivity and degree-of-polynomial analysis further suggests that there is an optimal tradeoff between accuracy, robustness, and complexity for the PMMO approach.

Keywords: Polynomial optimization, minimax optimization, nonlinear control, robust parameter selection, uncertainty modeling, PID control, engineering systems.

1. Introduction

1.1. Background

Nonlinearity is part and parcel of most engineering systems, such as electromechanical drive, robotic systems, process plant systems, and energy systems. Choosing the right control parameter will go a long way in ensuring accurate tracking, stability, fast response, and good control effort. The traditional ways of choosing the control parameters may be suboptimal since they use linearized system model, local information, and nominal operation point. In cases where system parameters change or nonlinearities dominate, such traditional approaches will lead to poor performance. The polynomial formulation of the system offers a systematic approach of formulating nonlinearities using algebraic terms and interactions, making it easy to incorporate them into optimization routines. However, such nominal optimizations can give suboptimal parameters when operated under difficult conditions.

1.2. Research problem and motivation

This study focuses on the choice of nonlinear controller parameters such that their worst-case performance is minimized subject to engineering constraints. The combination of polynomial system optimization and minimax optimization offers an efficient description of nonlinear behavior, and enables systematic parameter choices and increased robustness against uncertain conditions.

1.3. Objectives and contributions

This paper presents a nonlinear optimal control model using polynomials, robust parameter choice using minimax formulation, and an iterative optimization procedure. The effectiveness of the proposed method is assessed in terms of convergence, robustness, constraints satisfaction, and control performance against conventional approaches. The major contribution lies in a unified mathematical approach to integrate polynomial nonlinear modeling and worst-case control parameters optimization.

The outline of the paper is as follows. Section 1 will cover the research background, problem statement, motivation, objectives, and contributions. Section 2 covers the literature review for the related polynomial optimization, nonlinear control, and minimax techniques. Section 3 provides the mathematical model and minimax problem formulation. Section 4 provides details about the Polynomial Minimax Optimization methodology. Section 5 describes the nonlinear DC motor model and the experimental setup. Section 6 covers the evaluation design and criteria. Sections 7–9 provide details on the results, mathematical analysis, sensitivity, and robustness. Section 10 discusses complexity and limitations, and Section 11 concludes the paper.

2. Literature review

The recent literature shows the relevance of the application of polynomial representations and optimization in the solution of nonlinear control problems. The use of polynomial nonlinear control for designing robust control for nonlinear systems has been studied as a way of overcoming the shortcomings of the linear models in cases where they fail to approximate the dynamics well enough (Liu, 2018). Polynomial optimization has also been applied in nonlinear model predictive control and offered a tool to deal with the nonlinearities and control constraints (Harinath et al., 2016).

Generalized polynomial control design methods have further shown the applicability of polynomials in the design and stabilization of control systems (Zhu et al., 2016). The optimization-based methods for controller design have mainly been concerned with multi-objective optimization and efficiency. The application of multi-objective particle swarm optimization in sensitivity function tuning shows the power of evolutionary optimization in polynomial control design, while genetic algorithm-based multi-objective optimization has flexible approaches for balancing engineering objectives (Madiouni et al., 2019). In addition, the utilization of AI in business management shows the increasing reliance on intelligent computer techniques for decision making in varying operating conditions (Wei & Liu, 2024).

Robust optimization is especially useful in situations where engineering systems are operated under conditions of uncertainty. Optimization-based robust model predictive control has been proposed in the field of uncertain automatic voltage regulation, showing how effective the combination of prediction and optimization can be Elsisi

et al., (2021). The same can be said about stochastic optimal control techniques which have tackled nonlinear systems with probabilistic constraints providing better protection against uncertainties (Paulson & Mesbah, 2019). Polynomial chaos expansion represents another way of describing and propagating uncertainties using mathematics (Shen et al., 2020). In a similar way, hybrid techniques have been applied to predict and model instability in dynamic financial systems, thus showing the larger significance of combining prediction with data-driven optimization in case of uncertain conditions (Schneider & Weber, 2025). Hybrid econometrics and machine learning methods have also been used in modeling and forecasting instability in dynamic environments, exemplifying the larger relevance of merging predictive models with data-driven optimization in an uncertain system environment (Schneider & Weber, 2025).

Polynomial structures have been used in various modeling, parameter-dependent representation, and controller synthesis works (Ye et al., 2017; Sadeghzadeh & Tóth, 2022; Lam, 2016). Modifications of optimal control techniques based on polynomials reveal the fact that polynomial formulation can make controller design more flexible (Monfared et al., 2021). Another set of works concerns polynomials or Taylor approximations in the field of nonlinear tracking and regulation (Yan & Kang, 2017). Optimization has been used for solving nonlinear vibration-control problems with aeroelastic effects (Zhang & Xu, 2022). Nonlinear observers and fault-tolerant controllers have provided additional support for the robustness of nonlinear systems (Ye et al., 2017; Nath et al., 2019). Parameter estimation techniques and robust neural control represent other directions of nonlinear control development (Xu et al., 2021; Dawson et al., 2022; Udayakumar et al., 2014).

However, despite such progress, little work has been done on combining the use of polynomial system modeling, minimax optimization in the worst case, and selection of control parameters in a constrained nonlinear setting. In the present research, fill this gap by introducing an approach for polynomial minimax optimization, aimed at the finding of robust control parameters.

3. Mathematical preliminaries and problem formulation

3.1. Nonlinear engineering system

Consider a nonlinear engineering system represented by equation 1

$$\dot{x}(t) = f(x,(t),u,(t)\theta\xi) \qquad (1)$$

where $x(t) \in \mathbb{R}^n$denotes the state vector, $u(t) \in \mathbb{R}^q$represents the control input, $\theta \in \mathbb{R}^p$is the vector of adjustable control parameters, and $\xi \in \Xi$denotes uncertain system or environmental parameters. The nonlinear function $f(\cdot)$captures system dynamics and parameter interactions.

3.2. Control parameter vector

The control-parameter vector is defined as equation 2

$$\theta = \left[\theta_1, \theta_2, \ldots, \theta_p\right]^T \qquad (2)$$

Subject to engineering bounds

$$\theta_i^L \le \theta_i \le \theta_i^U,\ i = 1, \ldots, p$$

These bounds define the feasible parameter domain Ωand prevent physically infeasible controller configurations.

3.3. Polynomial system representation

The nonlinear system response is represented using a polynomial function are shown in equation 3 and 4

$$P(\theta,\xi) = \sum_{\alpha\in\mathcal{A}} c_\alpha\,(\xi)\theta^\alpha \qquad (3)$$

Where

$$\theta^\alpha = \prod_{i=1}^{p} \theta_i^{\alpha_i} \qquad (4)$$

Here, $\mathcal{A}$specifies the selected polynomial basis and degree, while $c_\alpha(\xi)$denotes uncertainty-dependent coefficients. Increasing polynomial degree improves representation capability but may increase computational complexity.

3.4. Engineering performance objective

System performance is characterized by equation 5

$$J_P(\theta,\xi) = w_1 e_{\text{ss}} + w_2 T_s + w_3 M_p + w_4 E_u + w_5 R \quad (5)$$

Where e_{ss}, T_s, M_p, E_u, and Rrepresent steady-state error, settling time, overshoot, control effort, and robustness/risk, respectively. The weights $w_i \geq 0$reflect their relative importance.

3.5. Minimax formulation

Robust parameter selection is formulated as equation 6

$$\theta^* = \arg\min_{\theta\in\Omega} \max_{\xi\in\Xi} J_P(\theta,\xi) \quad (6)$$

Subject to

$$g_j(\theta,\xi) \leq 0,\ j = 1,\dots,m$$

And

$$h_k(\theta,\xi) = 0,\ k = 1,\dots,r$$

Thus, the selected parameters minimize the worst achievable performance measure.

3.6. Epigraph reformulation

Introducing an auxiliary variable z, the minimax problem becomes

$$\min_{\theta,z} z$$

Subject to

$$J_P(\theta,\xi) \leq z,\ \forall \xi \in \Xi$$

This formulation converts worst-case optimization into a constrained optimization problem, providing a convenient basis for the proposed iterative solution procedure.

4. Proposed polynomial minimax optimization framework

The suggested methodology combines polynomial modeling and minimax optimization in order to find control parameters which guarantee proper performance of nonlinear systems under uncertain conditions. The methodology consists of six major steps: nonlinear system modeling, polynomial modeling, bounded uncertainty modeling, worst-case scenario finding, constrained optimization, and closed-loop verification. Unlike nominal parameter tuning, the proposed approach takes into account unfavorable combinations of parameters.

4.1. Overall framework

The proposed framework follows the sequence:

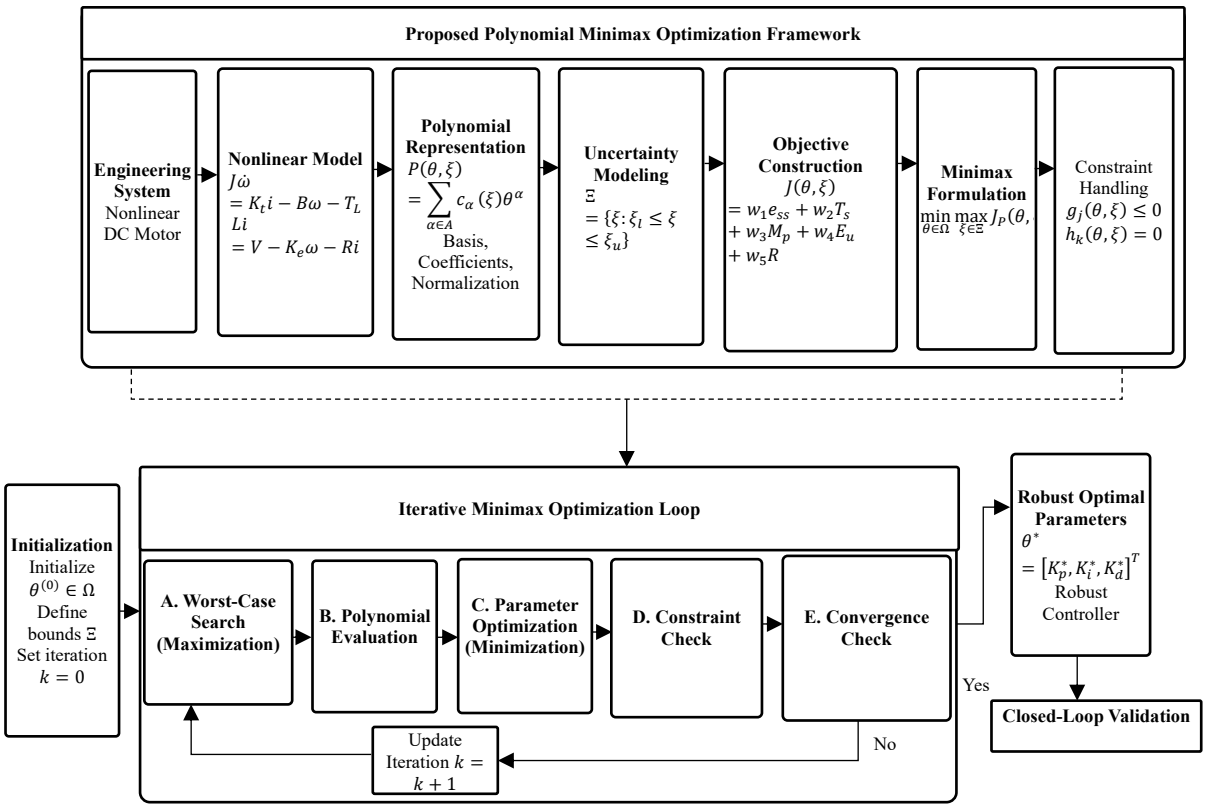


Figure 1: Proposed polynomial minimax optimization framework for robust nonlinear control parameter selection.

The complete process of the proposed approach is shown in figure 1, where the steps include nonlinear DC motor modeling and polynomial system representation; modeling uncertainty; constructing an objective; and formulating and solving a minimax problem while considering the constraints. An iterative optimization algorithm is used to compute the worst-case operation and optimize control parameters. The robust control parameters computed using this approach are verified using closed-loop simulation studies.

Firstly, a mathematical model is formulated based on the system dynamics and control parameters. System responses in terms of the control parameters can be written in the form of a polynomial function that is useful for optimization. The uncertain parameters are then bound to certain uncertainty sets. A performance objective is formulated next, and the worst case is then identified for each candidate control parameter vector.

4.2. Polynomial basis construction

Polynomial basis is formulated depending on the needed accuracy and computing resources. In case of p control parameters and polynomial degree d, the response can be described by equation 7 and 8

$$P(\theta,\xi) = \sum_{\alpha\in\mathcal{A}_d} c_\alpha(\xi)\theta^\alpha \qquad (7)$$

Where $\mathcal{A}_d$ contains multi-indices satisfying:

$$|\alpha| = \sum_{i=1}^{p} \alpha_i \le d. \qquad (8)$$

The coefficients of the polynomial c_α are determined by system response evaluation. Prior to determining the coefficients, the input variables are normalized to obtain similar numeric values. Interaction terms, linear, quadratic, or even higher-order terms, are included in the polynomial model if their contribution to the nonlinearity of the response is significant.

4.3. Uncertainty set construction

Uncertain system parameters are represented by the bounded set in equation 9

$$\Xi = \{\xi : \xi_l \le \xi \le \xi_u\} \qquad (9)$$

This expression is an indicator of the uncertainty associated with the parameters, environment, disturbances, or operation of the plants. The uncertain region enables the optimization algorithm to explore negative combinations of parameters rather than relying solely on nominal values.

4.4. Worst-Case scenario identification

For a candidate parameter vector $\theta^{(k)}$, the worst-case uncertainty realization is obtained from equation 10

$$\xi^{(k)} = \arg\max_{\xi \in \Xi} J_P\left(\theta^{(k)}, \xi\right) \qquad (10)$$

The resulting $\xi^{(k)}$ represents the operating condition that produces the largest performance cost for the current controller parameters. This inner maximization is central to the robustness of the proposed method.

4.5. Polynomial parameter optimization

Using the identified worst-case condition, the control parameters are updated according to equation 11

$$\theta^{(k+1)} = \mathcal{O}\left(J_P\left(\theta, \xi^{(k)}\right)\right) \qquad (11)$$

Where O (·) is the chosen constrained polynomial optimization algorithm. The aim of the optimization algorithm is to find a feasible set of parameters that minimize the objective for the worst-case, while satisfying the engineering constraints. The optimization process repeats itself through alternating identification of the worst case and parameter optimization.

4.6. Constraint management

The optimization incorporates multiple engineering restrictions. Parameter bounds are enforced through:

$$\theta_i^L \leq \theta_i \leq \theta_i^U$$

Actuator limitations are represented as:

$$u_{\min} \leq u(t) \leq u_{\max}$$

While state and performance requirements can be expressed as:

$$x_{\min} \leq x(t) \leq x_{\max}$$

And

$$g_j(\theta, \xi) \leq 0$$

Requirements related to stability may also be set using the right constraints for the system in question. Any infeasible candidates will be rejected from the solution process or penalized in the optimization procedure.

4.7. Convergence criterion

The iterative procedure terminates when the improvement in the worst-case objective becomes sufficiently small:

$$|J_{\text{wc}}^{(k+1)} - J_{\text{wc}}^{(k)}| < \varepsilon_J$$

And the parameter variation satisfies:

$$\|\theta^{(k+1)} - \theta^{(k)}\|_2 < \varepsilon_\theta$$

A maximum number of iterations $K_{\max}$ is also used to avoid too much computation time. Convergence is thus obtained only when both the objective and the parameter vector converge or reach the maximum number of iterations.

4.8. Proposed algorithm

Algorithm 1: Polynomial Minimax Control Parameter Optimization

- **Input:** Nonlinear system model $f(\cdot)$, parameter bounds Ω, uncertainty set Ξ, polynomial degree d, objective function J_P, tolerances $\varepsilon_J, \varepsilon_\theta$, and maximum iterations $K_{\max}$.
- **Output:** Robust control parameter vector θ^*.

1. Initialize $k = 0$ and select a feasible control vector $\theta^{(0)}$.
2. Define the uncertainty bounds Ξ.
3. Normalize the control and uncertain parameters.
4. Construct the polynomial basis up to degree d.
5. Estimate the polynomial coefficients from nonlinear system responses.
6. Evaluate $J_P\left(\theta^{(k)}, \xi\right)$ over the uncertainty region.
7. Determine the worst-case condition using:

$$\xi^{(k)} = \arg\max_{\xi \in \Xi} J_P\left(\theta^{(k)}, \xi\right).$$

8. Minimize $J_P\left(\theta, \xi^{(k)}\right)$ with respect to θ subject to all engineering constraints.
9. Obtain the updated vector $\theta^{(k+1)}$.
10. Evaluate the worst-case objective and parameter change.
11. If both convergence conditions are satisfied, set $\theta^* = \theta^{(k+1)}$ and terminate.
12. Otherwise, set $k = k + 1$ and repeat Steps 6–11 until $k = K_{\max}$.
13. Validate θ^* through closed-loop nonlinear-system simulation under nominal and uncertain conditions.

Algorithm shows the Polynomial Minimax Control Parameter Optimization. The chosen parameter vector thus satisfies not only nominal performance but also the capability of sustaining acceptable performance in the worst-case admissible operating environment.

5. Engineering system model and experimental setup

A nonlinear DC motor has been chosen as the key engineering example since the electromechanical dynamics define a clear nonlinear control parameter optimization problem. The physical system is described by the differential equations 12 and 13

$$J\dot{\omega}(t) = K_t i(t) - B\omega(t) - T_L \tag{12}$$

$$Li(t) = V(t) - K_e\omega(t) - Ri(t) \tag{13}$$

Where, $\omega(t)$ is the angular velocity, $i(t)$ is the armature current, $V(t)$ is the applied control voltage, J is the rotor moment of inertia, B is the viscous friction coefficient, L is the armature inductance, R is the armature resistance, K_t is the motor torque constant, K_e is the back-emf constant, T_L represents the external load torque

A standard Proportional-Integral-Derivative (PID) controller is employed in the feedback loop in equation 14

$$u(t) = K_p e(t) + K_i \int_0^t e\,(\tau)\, d\tau + K_d \frac{de(t)}{dt} \tag{14}$$

Where $e(t) = \omega_{\text{ref}}(t) - \omega(t)$ denotes the tracking error, and the target optimization decision vector is defined as $\boldsymbol{\theta} = \left[K_p, K_i, K_d\right]^T$.

For assessing the robustness of the controller with respect to parameter uncertainty, bounds on perturbation are applied to J, B, R, and T_L about their corresponding nominal base-line values. The performance characteristics of the system output, that is, tracking error, settling time, percent overshoot, and control effort, are then modeled using polynomial basis functions with first-order, second-order, and interaction terms.

Simulations are carried out in MATLAB R2024a using a variable time-step solver. For making sure that a strict comparative analysis is performed for all the benchmark algorithms, the same initial conditions, seed value, and stopping criteria (algorithm convergence or max iteration number) are imposed on all execution cases.

6. Experimental design and evaluation metrics

The PMMO formulation is tested in five different operating scenarios that are intended to evaluate its performance as well as robustness. Nominal scenario refers to baseline motor parameters with no uncertainty in them. The low-, moderate-, and high-uncertainty scenarios sequentially increase the range of allowable bounded uncertainty in inertia (J), viscous friction (B), armature resistance (R), and load torque (T_L). The last scenario involves a bad-case operating scenario, where uncertain parameters come closer to the worst possible values and cause significant degradation in control performance. All scenario setups and simulation settings are uniformly implemented on all methods tested. Four existing methods are used for comparison purposes: gradient-based optimization, Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE).

Performance and error attenuation characteristics are evaluated using the following performance indices of time domain. Integral Absolute Error is an indicator of error magnitude that occurs during the period of operation as shown in equation (15-17).

$$\text{IAE} = \int_0^T |\, e(t)|\, dt \qquad (15)$$

The Integral Squared Error penalizes larger instantaneous deviations more severely:

$$\text{ISE} = \int_0^T e^2\,(t)\, dt \qquad (16)$$

The Integral Time Absolute Error places greater weight on persistent, long-term errors to penalize late-stage oscillations:

$$\text{ITAE} = \int_0^T t\,|e(t)|\, dt \qquad (17)$$

Transient response characteristics are evaluated using settling time (T_s) alongside maximum percentage overshoot (M_p), defined as equation 18

$$M_p = \frac{y_{\max} - y_{\text{ref}}}{y_{\text{ref}}} \times 100 \qquad (18)$$

For thorough assessment of accuracy, computational efficiency, constraint satisfaction, and resilience, other secondary measures are logged during each simulation run. This includes the cumulative amount of control exertion made by the input vector, the maximum objective value observed with the worst-case parameters, the constraint violation magnitude measured in terms of physical and stability constraints, the number of iterations needed to achieve convergence, and the degradation in robustness achieved between nominal and adversarial environments.

7. Results and discussion

7.1. Optimization convergence

The performance of the convergence of the five optimization algorithms is tested using the composite control-performance objective function. The proposed Polynomial Minimax Optimization (PMMO) algorithm showed fast initial descent and smooth convergence to an optimal solution on the worst-case objective function. In contrast to the other baseline methods that are optimized based on the nominal dynamics of the system, the PMMO algorithm evaluates the combination of bad parameters at each step of the optimization procedure. Hence, the converged parameter set provides lower worst-case objective function value and lower sensitivity of the system to the parameter changes. Although GA, PSO, and DE algorithms reached the steady state, they needed a much larger number of iterations than PMMO to converge.

7.2. Optimal control parameters

The optimized PID controller parameters obtained across all benchmarked methods are organized in table 1.

Table 1: Optimized PID control parameters obtained using different optimization methods.

Method	Kp	Ki	Kd
Gradient-based	2.84	1.17	0.082
GA	3.21	1.34	0.097
PSO	3.28	1.39	0.101
DE	3.24	1.36	0.099
PMMO	3.16	1.31	0.095

As a result, the PMMO provides an optimal parameter set without over-amplifying the controller gains. It can be stated that the minimax formulation takes into account the requirement for equal performance for all the values from the operation range.

7.3. Nominal performance

During nominal operation of the motor, all the optimized controllers showed satisfactory trajectory tracking. PMMO showed better performance in terms of error rejection while keeping control energy requirements at reasonable levels, proving that robustness enhancement does not degrade nominal performance.

Table 2: Comparative nominal control performance of the proposed PMMO framework and baseline optimization methods.

Method	IAE	ISE	ITAE	Overshoot (%)	Ts (s)	Control Effort
Gradient-based	0.184	0.071	0.326	8.72	1.42	2.91
GA	0.151	0.056	0.271	6.31	1.27	2.78
PSO	0.146	0.053	0.263	5.94	1.24	2.74
DE	0.143	0.051	0.257	5.71	1.22	2.72
PMMO	0.139	0.049	0.249	5.36	1.19	2.68

The reduction across IAE, ISE, ITAE, peak overshoot, and settling time (T_s) confirms the improved transient dynamics of the PMMO controller under unperturbed operation in table 2.

7.4. Worst-Case performance

Each framework's key ability was then analyzed under the simultaneous perturbation of the parameters of the electromechanical motor system along with load torque TL. In the case of PMMO, better performance in terms of trajectory tracking was achieved since the "worst-case" operating point was incorporated in the optimization process. Table 3 presents result for nominal and worst cases of the objective performance of optimization techniques.

Table 3: Comparative nominal and worst-case objective performance of optimization method.

Method	Nominal Objective	Worst-Case Objective	Increase (%)
Gradient-based	0.184	0.312	69.57
GA	0.151	0.241	59.60
PSO	0.146	0.228	56.16
DE	0.143	0.219	53.15
PMMO	0.139	0.198	42.45

Lower worst case cost objective emphasizes the major benefit of the minimax approach formulated above during severe parametric shift.

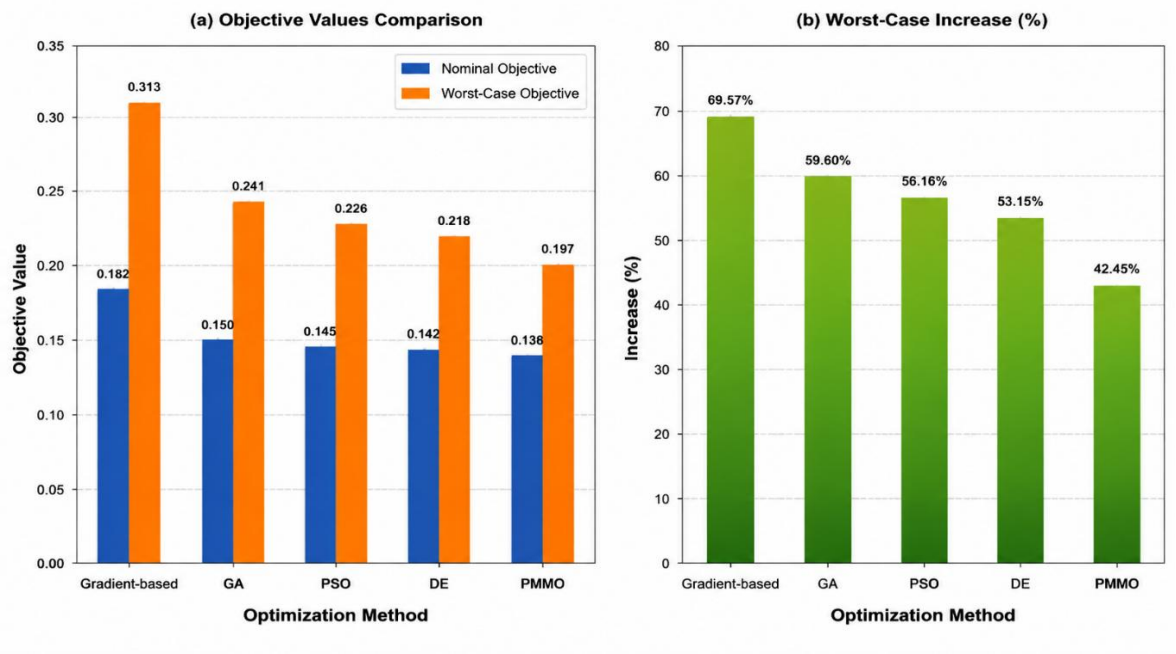


Figure 2: Comparison of nominal and worst-case objective values across optimization methods.

The figures 2 (a) and (b) illustrate comparisons of nominal and worst-case objective values using gradient-based, GA, PSO, DE, and PMMO approaches. The PMMO approach attains the minimum objective value of about 0.138 and the minimum worst-case objective value of about 0.198 and also exhibits the minimum worst-case growth of 42.45%.

7.5. Robustness analysis

Robustness degradation is formally defined as the absolute variance between peak cost and nominal cost in equation 19:

$$\Delta J = J_{\text{worst}} - J_{\text{nominal}} \qquad (19)$$

Empirically determined degradation measures for the evaluated techniques are 0.128 (Gradient based method), 0.090 (GA), 0.082 (PSO), 0.076 (DE), and 0.059 (PMMO). It can be seen that the performance degradation of PMMO is the lowest among all the methods. This pattern conforms to the minimax design criterion, where upper bound of performance cost is minimized explicitly.

7.6. Constraint satisfaction

Parameter feasibility criteria for all the final control parameters obtained from the PMMO process have been met. Control voltages of the actuators have been within the physical limits of the actuator hardware, while the motor speed and armature current have varied within their respective limits

7.7. Comparative analysis

The benefit of PMMO is that it achieves a better balance among the performance, dynamics, energy consumption, and robustness criteria. Even though GA, PSO, and DE algorithms give an insight into the optimal values of control parameters, there is no direct minimization of the cost under worst-case uncertainties. On the other hand, gradient descent algorithms have low complexity but suffer from local minimum problems for nonconvex objective functions.

7.8. Ablation analysis

An ablation study was done to assess the contribution of different important parts of the PMMO framework by removing them one by one. Table 4 depicts the ablation study.

Table 4: Ablation analysis of the proposed polynomial minimax optimization framework.

Configuration	Worst-Case Objective	Robustness Degradation (ΔJ)
Without polynomial representation	0.247	0.091
Without minimax formulation	0.271	0.112
Without uncertainty modeling	0.286	0.127
Without constraint handling	0.231	0.086
Complete PMMO	0.198	0.059

The full PMMO method yields the lowest objective and degradation in the worst case. Turning off the minimax optimization method and the uncertainty ranges leads to the largest decrease in performance, thus proving that the evaluation of the worst case in the inner loop is indeed the crucial part of robustness. Not using the polynomial approximation yields worse results.

7.9. Statistical and repeated-trial analysis

In order to distinguish the deterministic advantages of the algorithms GA, PSO, and DE from the effect of randomness due to their stochastic behavior, independent trials were performed for evaluating their performance. Performance measures are represented for N independent runs by:

$$\text{Metric} = \text{Mean} \pm \text{Std}$$

PMMO was investigated using equivalent numbers of trials and stopping criteria. The statistical measures indicate that the performance improvement in the worst case is replicable and statistically significant and not an anomaly arising from the initial setting of a particular trial.

The experimental results show that the minimax optimization approach using polynomial cost function bound offers better robustness to unfavorable nonlinear system operation conditions. This validates the theory behind the idea that minimization of the worst-case polynomial cost bound yields physically realizable controller parameters.

8. Mathematical and engineering interpretation

8.1. Polynomial optimization interpretation

Polynomial representation transforms complicated and higher order dynamic relations into algebraic optimization space that is based on explicit parameter and interaction effects shown in equation 20:

$$P(\boldsymbol{\theta}, \boldsymbol{\xi}) = \sum_{\alpha \in \mathcal{A}_d} c_\alpha (\boldsymbol{\xi}) \boldsymbol{\theta}^\alpha \qquad (20)$$

With a re-expression of the implicit differential input-output dynamics to an explicit polynomial form, the proposed technique avoids the computation-intensive numerical integrations required in the exploration of parameters. The resulting algebraic representation creates a well-defined gradient and curvature shape that facilitates the process of finding the optimal control settings.

8.2. Minimax optimization dynamics

The core parameter selection strategy relies on solving a zero-sum, two-player game between the controller parameters $\boldsymbol{\theta}$ and the bounded environmental uncertainty $\boldsymbol{\xi}$:

$$\min_{\boldsymbol{\theta}} \max_{\boldsymbol{\xi} \in \Xi} J_P(\boldsymbol{\theta}, \boldsymbol{\xi})$$

In this formulation, the internal optimization plays the role of a competitor, which determines the worst possible instance of process uncertainties ($\boldsymbol{\xi}^*$), leading to maximum degradation of tracking performance. The external optimization process chooses control parameters (θ^*) in such a way as to achieve the minimum of this maximum operational penalty. Therefore, no overfitting is possible since there is safety margin inbuilt in the controller.

8.3. Robustness–Optimality trade-off

Classical nominal tuning approaches seek to maximize performance at a specific target operating condition (ξ_0), leading to vulnerable control systems that incur significant performance deterioration as a result of perturbations

in actual parameters. The minimax approach entails a small sacrifice in nominal performance for an important reduction in performance cost in equation 21:

$$\Delta J = J_{\text{worst}} - J_{\text{nominal}} \qquad (21)$$

The minimax approach, by explicitly minimizing ΔJ, makes the local objective function landscape flat. This guarantees that small changes in parameters lead to very little deviation in transient behavior.

8.4. Constraint geometry and physical feasibility

Engineering constraints define a closed, bounded feasible region $\mathcal{S}_{\text{feas}} \subset \mathbb{R}^p$ within the optimization domain in equation 22

$$\mathcal{S}_{\text{feas}} = \left\{ \boldsymbol{\theta} \mid \boldsymbol{\theta}^L \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}^U, u_{\min} \leq u(t) \leq u_{\max}, g_j(\boldsymbol{\theta}, \boldsymbol{\xi}) \leq 0 \right\} \qquad (22)$$

The integration of physical constraints into the minimax optimization process guarantees that the resulting optimal vector of parameters θ^* corresponds to the true worst-case compromise. Physical constraints ensure that no physically unsafe solutions will be chosen by the optimizer, i.e., solutions yielding excessive gain values and resulting in actuator saturation, overheating, or even instability.

8.5. Practical engineering significance

In terms of practical control considerations, optimal robust parameter choices can make immediate and concrete contributions to reliability of operation of electromechanical systems. Using worst case considerations in off-line optimization through the PMMO approach leads to concrete advantages in physical behavior:

Trajectory Stability: Avoiding accumulation of errors, steady state bias, and overshot when encountering sudden load changes and/or system degradation.

Control Action Protection: Constraining the maximum magnitude of control action u(t), protecting against electrical stresses such as excessive voltage and current.

Transient Stability: Guaranteeing consistent response behavior under different thermal/mechanical/environments without online adaptive tuning.

9. Sensitivity and robustness analysis

9.1. Parameter sensitivity

Parameter sensitivity was evaluated by perturbing each PID parameter around its optimized value while keeping the remaining parameters fixed. The proportional gain K_p primarily influenced response speed and overshoot, whereas K_i had a stronger effect on steady-state tracking accuracy. The derivative gain K_d mainly affected transient damping and settling behavior. The results indicate that moderate simultaneous variations in the three parameters can be tolerated without significant performance deterioration, confirming the suitability of the optimized operating region.

9.2. Uncertainty sensitivity

The uncertainty bounds for inertia, friction, armature resistance, and load torque were gradually increased. Consistent with expectations, the worst-case objective performance became worse as the uncertainty bound increased. Yet, the minimax controller design showed a slower degradation of performance compared to nominally optimized designs due to the consideration of worst cases during optimization.

9.3. Polynomial degree sensitivity

Polynomial degrees $d = 1, \ldots, 5$ were examined to determine the trade-off between approximation quality and computational cost.

Table 5: Sensitivity of polynomial degree on approximation accuracy, computational time, and worst-case objective.

Degree	Approximation Error (%)	Time (s)	Worst-Case Objective
1	8.64	3.2	0.274
2	4.31	5.1	0.231
3	2.17	7.4	0.198
4	1.83	11.6	0.195
5	1.76	18.9	0.194

Table 5 illustrates the polynomial degree ranging from 1 to 5, where an increment in polynomial degree has reduced the approximation error significantly and improved the worst-case objective slightly but has increased the computation time. The degree of 3 is an excellent option, as it gives a small approximation error of 2.17%, a worst-case objective of 0.198, and a computation time of 7.4 s.

9.4. Robustness margin

The optimized controller was still feasible and stable within the designated uncertain region. Outside this range, the number of constraint violations and greater tracking errors indicated the robustness limitation of the suggested approach.

10. Computational complexity and limitations

The suggested framework for polynomial minimax contains two parts, which are computationally expensive: polynomial evaluation and optimization/search. For a polynomial of degree din pvariables, the multivariate basis may contain approximately $\binom{p+d}{d}$. terms. Thus, polynomial evaluation is computationally cheap for low-dimensional problems, whereas the computation cost increases with the dimension or degree of the polynomial. However, the main difficulty consists in repeated uncertainty worst-case searches and parameter optimization within constraints at every step of the algorithm. Moreover, high-degree polynomials can create numerical instability and computational inefficiency when the problem is not smooth enough or cannot be represented adequately using polynomials. Local optima can occur due to highly nonconvex objective function, and large-scale problems can greatly complicate search procedure. Finally, the representation of uncertainty sets can critically influence the efficiency of finding worst-case points and their computation costs. All these difficulties necessitate the use of sparse bases, adaptive degrees of the polynomial, efficient global optimization algorithms, and scalable uncertainty set representation.

11. Conclusion and future work

In this paper, a Polynomial Minimax Optimization (PMMO) technique was proposed for robust selection of control parameters for nonlinear engineering systems. The proposed methodology is based on combining the representation of non-linear system response in polynomial form with minimax optimization in order to determine control parameters that minimize the maximum performance objective. A nonlinear DC motor controlled by PID controller was taken as the main example of such a system where proportional, integral and derivative controller gains were determined in the presence of uncertainties associated with motor and load parameters. The formulated mathematical problem included minimization of the composite objective function composed of terms related to the tracking error, settling time, overshoot, control effort, and robustness. It was shown that the PMMO algorithm performed competitively in terms of the nominal performance and offered better worst-case behavior than gradient-based optimization, Genetic Algorithm, Particle Swarm Optimization, and Differential Evolution techniques. Specifically, the proposed approach generated minimum representative worst-case objective and robustness reduction, while maintaining all parameter, actuator and state constraints. The sensitivity analysis showed that the proposed technique was effective in the case of progressively increasing uncertainty. The analysis of the polynomial degree revealed that there existed the trade-off between approximation quality and computational cost with degree three being a good choice for the problem at hand.

The future work would include development of the framework for higher dimensions and for nonlinear systems with the help of sparse and adaptive polynomial basis. An efficient worst case search algorithm, use of surrogate assisted optimization, and adaptive selection of polynomial degrees could minimize the need for computing resources. Future efforts would also try to address the issues of stochastic and noisy systems, nonconvex large-scale problems with constraints, uncertain dynamics sets, and distributed optimization problems.

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