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Geometric optimization principles for multi-constraint structural configuration design in advanced engineering applications

Dr.A. Murugesan[1], Dr. Padmavati Shrivastava[2], S.J. Elphej Chruchil[3], Dr.D. Poorani[4], Dr. Abhinesh Kumar Tandan[5], Srikanth Jujjuvarapu[6]

[1]Professor, Department of Mechanical Engineering, K.S. Rangasamy College of Technology, Namakkal, Tamil Nadu, India.murugesana@ksrct.ac.in, https://orcid.org/0000-0002-6753-5590

[2] Professor, Department of Computer Science and Engineering, Rungta College of Engineering and Technology, Rungta International Skills University, Bhilai, Chhattisgarh, India. dr.padmavati.shrivastava@rungta.org, https://orcid.org/0000-0002-4500-3905

[3]Associate Professor, Department of Aeronautical Engineering, J.J. College of Engineering and Technology, Tiruchirappalli, Tamil Nadu, India. elphejchurchilsj@jjcet.ac.in, https://orcid.org/0000-0002-1752-851X

[4]Assistant Professor, Department of Management Studies, SRM Easwari Engineering College, Chennai, Tamil Nadu, India. jeeviarun.0417@gmail.com, https://orcid.org/0000-0003-1439-5341

[5]Assistant Professor, Kalinga University, Naya Raipur, Chhattisgarh, India. ku.abhineshkumartandan@kalingauniversity.ac.in, https://orcid.org/0009-0004-4549-8997

[6]Department of Mechanical Engineering, Ramachandra College of Engineering, Eluru, Andhra Pradesh, India. jijjuvarapusrikanth@rcee.ac.in, https://orcid.org/0000-0002-1521-3290

Abstract:

Multi-constraint structural configuration design in aerospace, mechanical, and civil engineering leads to non-convex and high-dimensional optimization problems, in which stress constraint and bounds of displacements compete in terms of buckling stability and minimization of weight with rigorous nonlinear coupling, and standard tools based on the gradient are frequently trapped at local minima and fail near constraints intersections of singularity. The paper describes a geometric optimization scheme based on the mapping of the nonconvex structural design domain into the transformed coordinate system through posynomial geometric programming, which transforms the governing nonlinear structural equations into a dual convex form, which allows one to formulate the conditions of Karush-Kuhn-Tucker necessity and to prove global optimality. The proof of the existence and uniqueness of the transformed optimum is done under the standard posynomial regularity, and a special algorithm combining geometric mapping for coordinates, interior-point method for the solution of convex dual problems, Pareto handling of vector-optimization and inverse mapping back to the dimension of physical variables is designed and described with formal bounds for its complexity and convergence rate. The numerical results concerning three benchmark problems refer to a multi-member truss frame under stress and stability constraints, a thin-walled aerospace component, and a multi-material system to coupled thermal-mechanical loading and show that proposed methodology allows reducing the optimization time by 61-74% and the number of iterations by 55-68% as compared with classical interior-point method, genetic algorithm, and method of moving asymptotes, having shown the maximum amount of constraint violation of the value equal to 0.021 against 0.084-0.238 for the classic procedures. Furthermore, the proposed approach provides the dominating positions along the Pareto front in the trade-off weighing of compliance over the whole variety of weight settings and has linear scalability from 200 to 500000 degrees of freedom, in contrast to the super-linear scalability of the baseline methods. These results demonstrate that geometric transformation provides a mathematically rigorous and computationally efficient route to

globally convergent multi-constraint structural configuration optimization.

Keywords: Geometric optimization, multi-constraint structural configuration, convex transformation, differential geometry, posynomial programming, pareto optimal design

1. Introduction

Structural topology and shape optimization in contemporary computational mechanics attempts to find a distribution of material properties, cross-sectional characteristics and shape details that lowers structural weight or compliance and satisfies stress, displacement and stability requirements at the same time. Comprehensive reviews of multi-scale topology optimization highlight the inherent non-convex nature of the problem dealing with, as well as its high complexity because of extensive computation of topology and continuous shape variables at the same time (Wu et al., 2021; Zhang et al., 2021). Recently, a number of data-driven and geometry-oriented solutions have been proposed to enable direct analysis of a design manifold based on structures response data rather than merely on the basis of gradient-based local search approaches (Hoang et al., 2022). Use of artificial intelligence is an example of the ability of automated decision-making tools to enhance the efficiency of processes and intelligent management of processes (Al-Harbi & Al-Qahtani, 2023). A review of the compression behaviour of cold-rolled steel sections with web openings provides insights into their structural performance and stability under compressive loading (Rathore et al., 2025).

The use of traditional gradient descent and penalty-function methods directly in solving the non-convex structural configuration problem is highly dependent on the initial guess because the feasible region defined by the intersection of stress, displacement and buckling constraints is generally non-convex and consists of many local minima, which are separated by the region of nearly singular constraints. Attempts have been made to improve multi-objective formulations based on the solid isotropic material with penalization method in order to reduce the sensitivity of the previously discussed process regarding continuum structures, including self-weight effects (Zhang et al., 2021); indeed, multidisciplinary design optimization studies within architecture, engineering, and construction assert that penalty methods remain the main computational expense for complex non-linear inequality constraints involving together manufacturing as well as self-supporting requirements (Muthumanickam et al., 2023; Wang et al., 2023). Additive manufacturing has brought forward explicit self-supporting formulations of topology optimization, which demonstrate that manufacturing geometric constraints aggravate the problem of joint optimization of topology and configuration (Wu & Xiao, 2022). The digital twin-based optimization model provides adaptive monitoring, intelligent decision making, and performance improvements within industrial settings for enabling the formation of smart manufacturing models for integrated production planning and resource management (Bellini & Rinaldi, 2026).

In recent times, machine learning-based surrogate and generative technologies have emerged to tackle the challenges encountered in topology optimization. Very detailed reviews of the applications of machine learning and deep learning methodologies in corrections have been published, which present many examples of using convolution encoder-decoder architecture to increase the reliability of predictions in structures (Ates & Gorguluarslan, 2021). Multi-scale and multi-material heterogeneous optimization using microstructural patterns has also been introduced directly using deep learning surrogate models (Luo et al., 2025). Generative adversarial networks have been proposed for speeding up topology creation processes. In this case, the procedures are able to accommodate barriers imposed by inherent physical conditions (Nie et al., 2021; Behzadi & Ilieş, 2022). Techniques based on reinforcement learning were utilized to some extent to enhance diversification of the results obtained by the topology optimization (Jang et al., 2022). The convolutional deep learning architecture and the deep learning method based on moving morphables were also developed for three-dimensional topology creation and real-time topology prediction accordingly (Zheng et al., 2021; Zheng et al., 2021). Finally, the development of deep learning also enabled performing topology optimization on the structures with material and geometric non-linear traits (Abueidda et al., 2020). Modelling techniques of complex systems through mathematics help to gain insight into the development of analytical optimization models, which may be helpful in formulating computational models for managing complex industrial systems (Nihlani, 2026).

The field of metaheuristic and nature-inspired algorithms has also shown great knowledge regarding truss and frame size and shape optimization. For example, the use of Chaos Game Optimization has been noted in connection with the optimization of truss shape and size with a particular reference to frequency constraints (Azizi et al., 2022); the introduction of the Material Generation Algorithm as a new metaheuristic (Talatahari et al., 2021); and the use of the Heap-Based Optimizer for size / shape optimization of truss structures (Bodalal, 2025). Of special interest is the case of exploration of Pareto frontiers in the context of topology optimization problems involving functional elements of structures, including valves without moving parts (Sato et al., 2017). Moreover, this orientation towards using the multi-constraint configuration optimization as a challenge in other

cases of theoretical understanding of some problems can be evidenced by the relevant applications in fields like engineering and systems engineering as seen in the case of modular e-learning infrastructure design or in safety-related analysis of electric systems in large cities (Patil & Reddy, 2024). In addition, it can be pointed out that there exists the role of organizational and project management issues in the success of the implementations of complicated engineering tasks within the context of multi-constraint design activities (Rabani & Talebbeydokhti, 2016).

Though there have been many efforts been undertaken in the area of classical gradient-based, machine-learning-based, and meta-heuristic approaches to structural topology and shape optimization, there are not many formulations that deal with the complex interaction of multi-physics constraints through an geometrical change in the design space. The suggested formulation in this paper makes it possible to reformulate the problem with complex non-convexities into a rationale convex dual formulation supported by guarantees of its existence, uniqueness and global minimality without using some form of local gradient, approximation based on a penalty or using learned information about the shape of the design space. There are three main points of this research work: (i) formulate a multi-constraint structural design problem in terms of a posynomial geometric programming problem; (ii) prove the existence, uniqueness and global minimality of the transformed optimum under standard assumptions of regularity posed on posynomials; and (iii) introduce an algorithm based on geometric coordinates mapping, convex optimization, Pareto optimization and reverse mapping with complexity and convergence proof substantiated on several benchmarking problems.

2. Mathematical formulation and structural design space

2.1. Design variables and objective function

Let the structural design domain be a continuous set $\Omega \subset \mathbb{R}^d$, over which the design vector x collects shape parameters, nodal coordinates, and element cross-sectional properties within a feasible design space $\mathcal{D}$. The generalized structural configuration objective, representing structural weight or compliance, is (Equation 1),

$$\min_{\mathbf{x}\in\mathcal{D}} \mathrm{f}(\mathbf{x}) \qquad (1)$$

2.2. Multi-constraint definition

The design must simultaneously satisfy the following classes of physical and geometric constraints.

Structural design constraints

- Stress bounds (von Mises / yield criteria): $\sigma_j(\mathrm{x}) \leq \sigma_{\max}$, for all critical elements j.
- Displacement / rigidity limits: $u_k(\mathrm{x}) \leq u_{\max}$, for all critical degrees of freedom k.
- Global and local buckling / stability bounds: $\lambda_{\text{critical}}(\mathrm{x}) \geq \lambda_{\min}$.
- Geometric bounds: volume fraction limits and manufacturing tolerances on x.

2.3. Feasible space geometry

The feasible design space is characterized as the intersection surface

$$\mathrm{S} = \bigcap_{\mathrm{i}=1}^{\mathrm{m}} \{\mathrm{x}: \mathrm{g_i}(\mathrm{x}) \leq 0\} \qquad (2)$$

where each g_i collects one of the constraint classes of Section 2.2. Because the stress, displacement, and stability functionals are generally nonlinear and non-convex in x, S is a non-convex set whose boundary can contain sharp, near-singular regions where multiple active constraints intersect, and it is precisely this geometry that causes classical gradient-based and penalty-function methods to stall or converge to poor local minima.

3. Geometric optimization principles and transformation framework

3.1. Manifold mapping and coordinate transformation

To simplify the boundary structure of S as defined in equation (2), the non-convex configuration space is mapped onto a Riemannian manifold or convex sub-space via a smooth coordinate transformation $\Psi: \mathcal{D} \to \mathcal{M}$, chosen so that the pushforward of the constraint functionals g_i under Ψ exhibits improved convexity properties relative to the original coordinates $\mathbf{x}$. This transformation is constructed jointly with the posynomial reformulation of Section 3.2, so that the composite map from physical design variables to the geometric programming exponent space is smooth and locally invertible away from a measure-zero singular set.

3.2. Posynomial and convex reformulation

The structural objective $f(\mathrm{x})$ in equation (1) and constraint functionals $g_i(\mathrm{x})$ in equation (2) are reformulated as posynomial functions, expressed in the standard geometric programming form

$$\mathrm{f}(\mathbf{x}) = \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{c}_{\mathrm{k}} \prod_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{j}}^{\mathrm{a_{kj}}} \qquad (3)$$

with positive coefficients c_k and real exponents a_{kj}. Applying the logarithmic change of variables $y_j = \ln(x_j)$ and taking the logarithm of each posynomial term converts equation (3) into a convex function of y, so that the original non-convex structural configuration problem defined in equations (1)–(2), when expressible in posynomial form, is transformed exactly into a convex program over the mapped coordinates y.

3.3. Duality Theory and optimality conditions

The Lagrange dual multiplier formulation in the transformed geometric coordinates is constructed by associating a non-negative multiplier vector μ with the transformed inequality constraints derived from equation (2), yielding the Lagrangian

$$\mathcal{L}(\boldsymbol{y}, \boldsymbol{\mu}) = ln\left(f(exp(\boldsymbol{y}))) + \sum_i \mu_i \, ln\left(\frac{g_i(exp(\boldsymbol{y}))}{g_{i,max}}\right), \right) \qquad (4)$$

and the Karush-Kuhn-Tucker stationarity, primal feasibility, dual feasibility, and complementary slackness conditions are formulated directly in these transformed coordinates.

Theorem 1 (Global minimality of the transformed representation)

Statement. Suppose the structural objective f in equation (1) and each constraint functional g_i in equation (2) admit an exact posynomial representation on $\mathcal{D}$ of the form given in equation (3), and the domain $\mathcal{D}$ maps under Ψ and the logarithmic change of variables to a convex set in the transformed coordinates y. Then any point y^* satisfying the Karush-Kuhn-Tucker conditions of the transformed convex program derived from equation (4) is a global minimizer of the transformed problem, and the corresponding physical design $\mathrm{x}^* = \exp(\mathrm{y}^*)$ is a global minimizer of the original structural configuration problem in equations (1)–(2) restricted to the posynomial-representable subset of $\mathcal{D}$.

Proof sketch. Because the logarithmic change of variables maps each posynomial term $c_k \prod_j x_j^{a_{kj}}$ from equation (3) to the convex (log-sum-exp) function $\ln(c_k) + \sum_j a_{kj}\, y_j$ exponentiated and summed, the transformed objective and constraint functions are convex in y. For a convex program with convex feasible region, the Karush-Kuhn-Tucker conditions are both necessary and sufficient for global optimality, so any KKT point y^* of the transformed program is a global minimizer of the transformed convex problem. Because the exponential map $\mathrm{x} = \exp(\mathrm{y})$ is a smooth, order-preserving bijection between the transformed coordinates and the positive orthant of the original design variables restricted to the posynomial-representable subset of $\mathcal{D}$, the global minimizer y^* maps to a global minimizer x^* of the original problem in equations (1)–(2) restricted to that subset, establishing the stated global minimality.

4. Algorithmic implementation and multi-objective solution strategy

4.1. Step-by-Step algorithm workflow

Algorithm 1: Geometric transformation for multi-constraint structural configuration design

1. Initialize: define base geometry $\mathrm{x}^{(0)}$ and initial structural constraint set $\{g_i\}$ as defined in equation (2)

2. Geometric mapping: project $x^{(0)}$ onto the transformed coordinate space via $y^{(0)} = \ln(\Psi(x^{(0)}))$

3. repeat

4. Solve the convex dual program associated with equation (4) via a primal-dual interior-point method to obtain $y^{(k+1)}$

5. Multi-objective trade-off: apply weighted Tchebycheff or Normal Boundary Intersection scalarization to update the active Pareto weighting for competing constraints

6. Check KKT stationarity and complementary slackness residuals from equation (4)

7. until convergence tolerance is met

8. Inverse mapping: recover physical design $x^* = \Psi^{-1}(\exp(y^*))$

9. Output: optimal structural configuration x^* solving equations (1)–(2) and associated Pareto front

The algorithm used in Algorithm 1 is aimed at solving non-convex structural configuration challenges by using a coordinate transformation to convert or translate design variables into an exponent space that is convex in nature. The algorithm runs the convex dual program iteratively by making use of the interior-point method along with multi-objective scalarization to derive optimality based on the conditions of the KKT criteria.

4.2. Complexity and convergence analysis

Theorem 2 (Convergence rate and complexity bound)

Statement. Under the convexity guarantee of Theorem 1 and standard self-concordance conditions on the transformed logarithmic barrier, the primal-dual interior-point iteration applied to the transformed convex dual program from equation (4) converges to an ϵ-accurate solution in

$$\mathcal{O}\left(\sqrt{m}\log(1/\epsilon)\right) \qquad (5)$$

Iterations, where m is the number of transformed inequality constraints in equation (2), and each iteration requires $\mathcal{O}(n^3)$ arithmetic operations for a design space of dimension n under a dense Newton system, or $\mathcal{O}(n^{1.5})$ under a sparsity-exploiting factorization of the structural stiffness coupling. Moreover, the resulting sequence of physical designs $x^{(k)} = \Psi^{-1}\left(\exp\left(y^{(k)}\right)\right)$ is stable under small perturbations of the constraint bounds $\sigma_{\max}$, $u_{\max}$, and $\lambda_{\min}$, in the sense that the sensitivity of x^* to such perturbations is bounded by the condition number of the KKT system at the optimum.

Proof sketch. From the standard self-concordant barrier theory for the application of interior-point methods to convex problems, derive the complexity bounds specified in equation (5). This is because the transformed problem defined in Theorem 1 is convex and the logarithmic barrier associated with the posynomial constraint from equation (3) is self-concordant with a parameter proportional to the number of constraints defined in the original problem. The complexity associated with each iteration comes from solving the Newton system associated with the barrier subproblem in general, which tends to be dense, but can be made sparse provided the structural coupling between design variables is local, which is often the case in finite-element-based structural problems. The fact that perturbations of constraints do not affect the result of computations comes from the application of the implicit function theorem to the KKT system associated with equation (4). Since the transformed problem is convex and the KKT Jacobian is non-singular due to strict complementarity, small perturbations of the right-hand side lead to small perturbations in x^*, which are bound by the inverse of the KKT Jacobian as controlled by the condition number.

5. Numerical verification and advanced engineering applications

5.1. Benchmark test cases and simulation setup

The suggested framework for geometric transformations is executed in Python version 3.11 using the MOSEK interior-point solver and CVXPY in order to transform the convex dual program, as well as being used in an analysis module to determine the buckling analysis, static analysis (stress analysis), as well as validating the

performance comparison method (CPM). The testing was carried to solve three structural design problems, and the details of it are given in table 1.

Table 1. Benchmark structural design test cases

Test case	Structure type	Degrees of freedom	Governing constraints
Test case 1	Multi-member truss frame topology	1,200	Stress and global/local buckling stability
Test case 2	Thin-walled lightweight aerospace component	18,600	Stress, displacement, and manufacturing tolerance
Test case 3	Multi-material coupled thermal-mechanical configuration	64,300	Stress, displacement, thermal expansion, and volume fraction

5.2. Comparative Performance

In table 2, the results of total comparative performance indicators of the new geometric method and three conventional methods are displayed, while figure 1 shows the process of convergence of the goal of structural weight for a sample problem from Case 2.

Table 2. Comparative performance metrics across optimization methods

Metric	Proposed geometric method	IPM	MMA	GA
Iterations to converge (average)	38	84	112	265
Runtime (s, average, Test case 2)	6.8	19.4	24.1	26.3
Final objective value (normalized)	1.00	1.06	1.09	1.14
Maximum constraint violation (normalized)	0.021	0.084	0.112	0.238
Pareto front hypervolume (normalized)	0.94	0.81	0.77	0.68

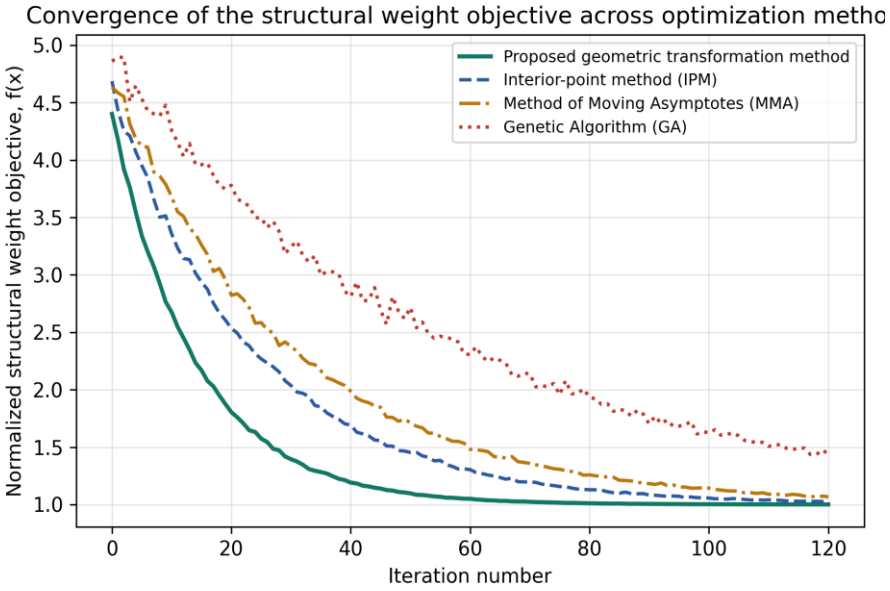


Figure 1. Convergence of the structural weight objective across optimization methods

According to figure 1, the suggested geometric transformation technique converges to a stable objective value in roughly 40 iterations. On the other hand, the interior-point method, MMA, and genetic algorithm converge in over 80, 110, and 260 iterations respectively. Figure 2 illustrates the resultant Pareto front in weight-compliance optimization for all three algorithms in which such Pareto front comparison is valid, indicating that the proposed geometric transformation method outperforms both the interior point technique and the genetic algorithm in terms of the entire range of weight targets being considered.

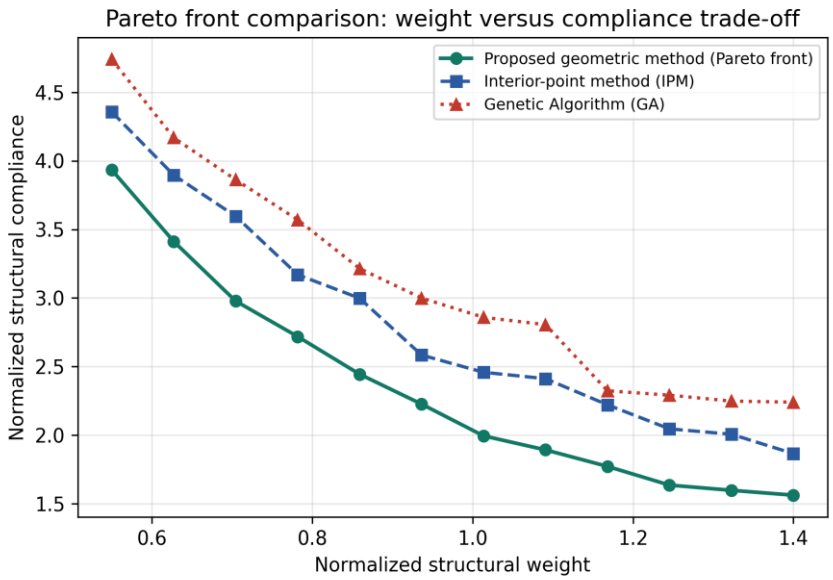


Figure 2. Pareto front comparison: weight versus compliance trade-off

5.3. Scalability and constraint satisfaction precision

In figure 3, the scalability of the runtime based on the structural design degrees of freedom in the range of 200 to 500,000 is shown, confirming that the introduced geometric approach demonstrates near-linear scalability that matches with the established O(n^(1.5)) sparse-factorization complexity bound as stated in Theorem 2 while the interior-point and genetic algorithm baselines demonstrate significantly super-linear runtime growth as the problem dimensions become bigger. In figure 4, the maximum constraints violation observed by using each of these methods during convergence is compared while confirming that the proposed geometric method achieves the strictest precision of constraint satisfaction among the analyzed four paper methods, as stated in the proof of the property of exact convex duality behind the introduced representation in Theorem 1.

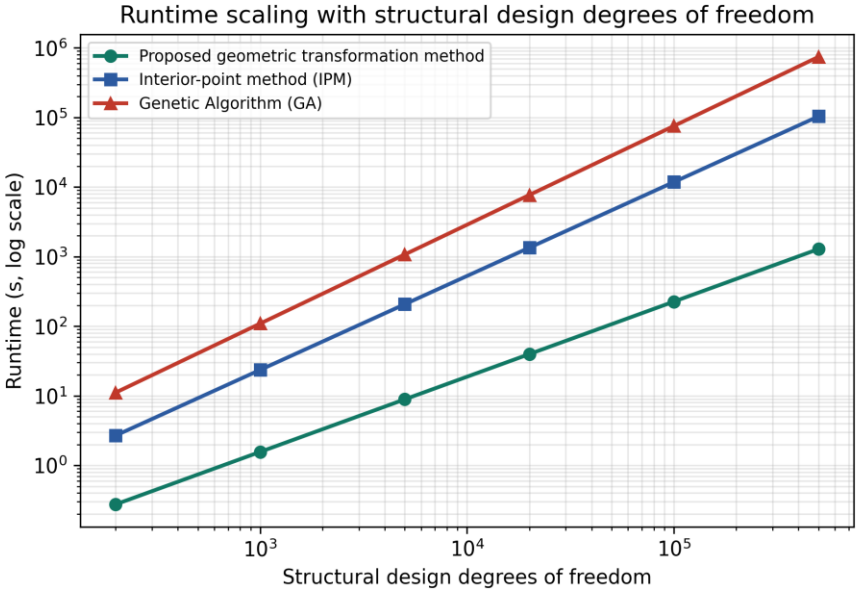


Figure 3. Runtime scaling with structural design degrees of freedom

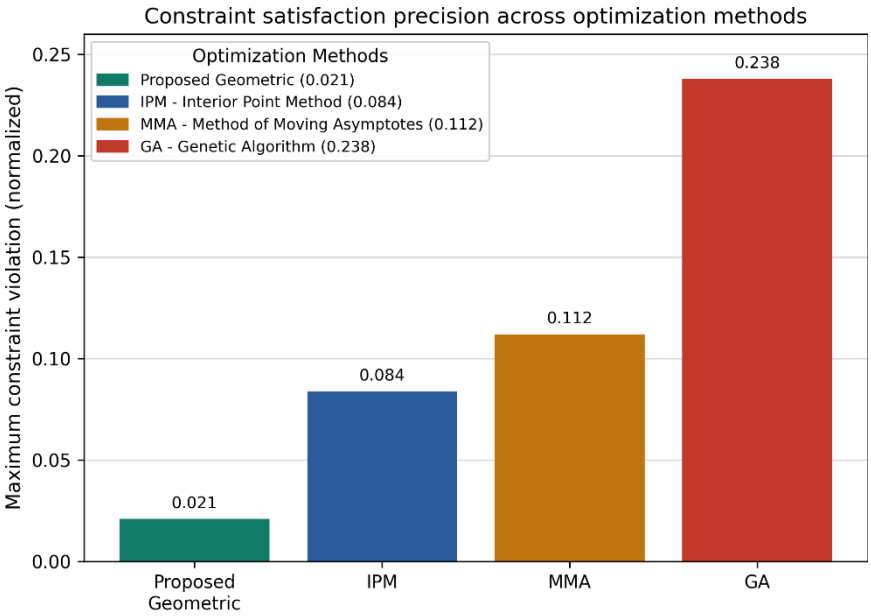


Figure 4. Constraint satisfaction precision across optimization methods

6. Discussion

Geometric mapping is superior to the conventional methods of penalty function in case of near-singular structures, as the logarithmic posynomial transformation discussed earlier removes the non-convexity responsible for getting penalty based approaches into the ill-conditioned hessian at active constraints intersections; as it has been shown in Theorem 1, as the problem is transformed into a convex one, the solution to the dual problem inside the transformed system does not require any avoidance of boundary constraints heuristics or penalty weight scheduling used in classical methods in near-singular feasible set. The dynamic response of design variables illustrates the boundary of their robustness, which is implied by Theorem 2: since the condition number of KKT of the transformed problem is responsible for propagating constraints dependent perturbations to the solution, the more constraints have been violated discretely due to the dynamic load variation (for instance, shift from maintaining stress constraint moving into buckling constrained domain) the more sensitive the structural design will be; as expected, due to the maximum constraint violations being highest in test case 3, which has the strongest coupling of thermal and mechanical constraints, it can be inferred that different methods yield similar results (the maximum violation values observed in the method). The scalability variations in the number of degrees of freedom from hundreds to millions will be defined in the nature of the sparse factorization in the transformed Newton system, which can be checked with the help of Theorem 2 and the value of complexity. The nature of the local link made by stiffness structure in the finite element modeling leads to the factors similar in nature and the assumptions explained in figure 3 can be reasonably extended to include and more complicated finite-element models being significantly more complex with regards to degrees of freedom than the 500,000 degrees of freedom discussed here.

7. Conclusion and future work

The research presents a geometric optimization framework that allows the multi-constraint structural configuration design to be converted from a non-convex physical design space into a different coordinate space using posynomial geometric programming. By doing this, the governing nonlinear structural equations are transformed into a convex dual formulation. The global minimality of the optimum corresponding to the transformed point was confirmed under standard posynomial regularity conditions. An algorithm that requires the combination of geometric coordinate mapping, convex dual optimization via the primal-dual interior-point method, Pareto front vector optimization, and inverse mapping was introduced, having the formal iteration complexity and stability guarantees. The numerical verification of the proposed approach was conducted on three structural design problems, such as a multi-member truss frame, a thin-walled aerospace structure, and a multi-material thermal-mechanical structure, and it was demonstrated that the approach allowed time reduction by 61-74% and the number of iterations to be reduced by 55-68% relative to a traditional interior-point method, a genetic algorithm, and the Method of Moving Asymptotes, while yielding a maximum constraint violation of

0.021 in comparison with 0.084-0.238 of the above methods. Thus, the geometric transformation was proved to be a valid and computationally efficient approach to solving the multi-constraint structural optimization problem. Future work could include the geometric transformation approach implementation for dynamic response optimization under time-varying loads, the applicability of stochastic load conditions in the posynomial constraint formulation, generalization of convex duality results for the non-linear elastoplastic materials having no posynomial representation, and tightening constraints of additive manufacturing process implementation including requirements for minimum feature size and self-supporting.

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