Minimax Theory and its Applications
Volume 11 (2026), No. 4s, 467 to 476

Mixed mathematical programming models for integrated production scheduling and resource management in smart manufacturing systems

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Abstract:

Smart manufacturing plants are increasingly forced not to just schedule their production but rather to optimize schedules and dynamic physical resources, such as machines, automated guided vehicles, energy budgets, and labor, at the same time since separated optimization consistently produces longer makespan and higher energy costs. In this paper an integrated multi-objective Mixed Integer Linear Programming model is proposed that simultaneously assigns operations to machines, determines the sequence-specific setup timing, and allocates resources with respect to the constraints on peak-power and connectivity in order to minimize the total makespan, total tardiness, and energy costs. For this purpose, bilinearity of resource consumption terms and nonlinear energy functions are linearized by utilizing McCormick envelopes and piecewise-linear approximation transforming the original Mixed Integer Nonlinear Programming problem to a solvable linear task. Then, Benders decomposition is applied separating the main scheduling problem from the resource allocation subtask and state-of-the-art matheuristic combining variable neighborhood search and linear programming seeded initialization is devised for solving large-scale problems which are impossible to solve exactly. The numerical experiments conducted on the basis of small, medium, and large-scale industrial problems (with the number of jobs from 10 to 120) have shown that the integrated model produces 18.6%, 27.4%, and 21.9% lower makespan, total tardiness, and energy cost than the traditional just-scheduling-and-allocating processes. Benders decomposition algorithm allows to keep the optimality gap smaller than 3.1% and decrease calculation time up to 93.5% in comparison to direct solving large-scale industrial problems and sensitivity analysis confirms advantages of the integrated model for various degrees of resource constrains. These results confirm that jointly optimizing scheduling and resource management within a unified mixed

programming formulation yields substantial, quantifiable operational gains for smart manufacturing systems while remaining computationally tractable at industrial scale.

Keywords: Mixed integer programming, production scheduling, resource management, smart manufacturing, benders decomposition, multi-objective optimization, linearization.

1. Introduction

The shift toward Industry 4.0 in discrete manufacturing has led to the development of hyper-connected production systems that integrate machines, automated vehicles, energy systems, and people, forming an operational landscape where production scheduling cannot exist outside of the physical availability of the production resources. An artificial intelligence-based approach employing a mixed integer programming model for the scheduling operations has confirmed that the optimization approach is still the best way to accurately represent complex relationships between processes and capacity requirements in automated production (Alla et al., 2024). The application of artificial intelligence has been studied as an important determinant of intelligent decision-making and technological transformation, with a focus on how AI methods can be used in contemporary automation (García & Hernández, 2024). Additionally, research on integrated production planning that takes into consideration preventive maintenance in the pulp and paper production sector confirms that simultaneous modeling of production and maintenance activities as part of a mixed integer problem leads to less unplanned downtime compared to the implementation of sequential models (Avilés et al., 2023, 2024).

Though the progress in this direction has been constant, traditional operations in industrial scheduling systems are still based on a sequential logic in which the production schedule is first designed and only then available resources such as automatic guided vehicles, tools and energy budgets are accounted for, following the initial plan. The efforts of adaptive rescheduling systems to deal with unexpected events proved that the proposed approach is not effective since the production schedules deteriorate greatly when real resources diverge from the planned ones, resulting in the necessity to alter the schedule quite often and at a considerable cost (Figueroa et al., 2024). It was noted in the article analyzing simulation optimization in production scheduling within the Industry 4.0 concept that despite the progress made in this area, there has always been a need for joint formulations taking into account both the changeable resource limitations and the scheduling decisions (Ghasemi et al., 2024).

The process of sequential decoupling is especially expensive in smart factories due to the presence of cyber-physical integration and connection through the Internet of Things, which allows for immediate knowledge of the resource condition and its utilization in the decisions related to scheduling. The construction of smart factories from cyber-physical systems and the Internet of Things implementations has shown that it is possible to monitor the condition of resources on a continuous basis at a shop-floor level. However, very few existing mathematical approaches to scheduling have utilized this continuous state of resources in their optimization models (Ryalat et al., 2023). The use of compositional approaches in scheduling of cyber-physical systems in Industry 4.0 has highlighted the need for the evolution of scheduling approaches, which would allow for making all scheduling decisions dependent on one another rather than separating them into subproblems (Tohmé & Rossit, 2025). Although a flexible mixed-programming mathematical formulation that is able to incorporate the changing state of resources, changing processing times, and the presence of smart systems in the manufacturing process has not yet been introduced in research papers. Multi-objective optimization technique has been used for sustainable manufacturing decision making, stressing the need for optimization techniques to consider multiple production goals simultaneously in a manufacturing setup.

This study contains four significant contributions to fill this void. To begin with, the study proposes a multi-objective Mixed Integer Linear and Nonlinear Programming model that is capable of achieving integrated production scheduling and dynamic resource allocation, which minimizes makespan, tardiness, and the energy cost. Next, the researchers utilize mathematical linearization methods in order to tackle the problem of dealing with non-linear constraints caused by sequence-dependent setup times and dynamic power usage, thus changing the non-linear formulation into a linear model. Lastly, the authors propose a new variation of an exact solution based on the concept of Benders decomposition, which enables separation of the master scheduling problem from the secondary resource allocation problem, and supplements it with matheuristic methods in order to handle large-scale industrial challenges. The rest of this study is organized as follows. Section II describes the literature related to the topic of this research. Section III provides explanations of the mathematical model. The mathematical model is described in Section IV. Section V presents the outcomes of the computational study. Finally, Section VI provides practical implications of the results obtained.

2. Literature review

Flexible job shop scheduling is a widely used paradigm in smart manufacturing, where operations can be assigned to alternative machines with different processing times. Integrating decision-maker preferences into multiobjective scheduling improves solution acceptance by reflecting stakeholder priorities (Bezoui et al., 2023). Production rescheduling studies identify machine breakdowns, urgent orders, and resource shortages as major disruptions requiring schedule revisions (Holguin Jimenez et al., 2024). Expert knowledge studies also emphasize that optimization models must remain interpretable to shop-floor planners, particularly in high-mix, low-volume environments (Kondoh et al., 2023). Modern scheduling formulations increasingly combine efficiency and sustainability objectives. Carbon emissions can be incorporated into mixed-integer models for distributed additive manufacturing without substantially reducing solution quality (Kucukkoc, 2024). Similarly, modeling time-varying energy prices in flexible job shops produces considerable savings compared with energy-agnostic scheduling (Shen et al., 2023). Explicit integration of automated guided vehicles into flexible flow shop formulations also improves throughput by jointly coordinating production and material handling (Wang et al., 2025). A dynamic evolutionary multi-objective virtual machine placement approach can optimize resource allocation in cloud data centers while considering multiple performance objectives (Torre et al., 2020).

Joint optimization of machines and material-handling resources improves schedule feasibility in dynamic manufacturing environments. Hierarchical planning for heterogeneous automated guided vehicle fleets makes integrated scheduling and routing computationally manageable while preserving solution quality (Hu et al., 2023). Multi-factory assembly scheduling with maintenance introduces additional interfacility coordination constraints (Kazemi et al., 2025). Joint optimization of production and material delivery also achieves better throughput than treating logistics as a separate downstream activity (Liang et al., 2025). Communication resources represent another important constraint. Software-defined networking for wireless mesh IoT systems demonstrates that bandwidth allocation encounters dynamic capacity limitations similar to physical manufacturing resources (Balamurugan et al., 2025). Neuromorphic-inspired cognitive models for 6G edge networks show that learning-based allocation can complement mathematical optimization under rapidly changing conditions. Hybrid mixed-integer and constraint-programming approaches also improve energy-aware flexible job shop scheduling compared with either method independently (Ham et al., 2021).

Mixed-integer linear programming, nonlinear programming, and constraint programming provide different balances between expressiveness and computational tractability. In packaging-plant scheduling, hybrid exact-heuristic strategies outperform purely exact or heuristic methods on industrial instances (Oujana et al., 2023). Digital twins combined with teaching-learning-based optimization support real-time schedule adaptation without complete model re-solution and enable scheduling decisions to be validated before physical implementation (Deepika Reddy et al., 2024; Ullah & Younas, 2024). Because integrated scheduling and resource allocation are NP-hard, decomposition and metaheuristic methods are necessary at industrial scale. Conceptual structuring is essential for integrating maintenance, production planning, and control (Wesendrup et al., 2024). Industry 4.0 and 5.0 research identifies evolutionary, decomposition, and hybrid matheuristic methods as leading approaches (Zhang et al., 2025). Improved non-dominated sorting genetic algorithms remain effective when diverse Pareto-optimal solutions are required (Zhao et al., 2024). Nevertheless, unified formulations jointly addressing dynamic resource availability, sequence-dependent setup times, and energy constraints, supported by matched exact-heuristic decomposition, remain underdeveloped. Optimization systems based on machine learning techniques have been used to optimize decisions related to supply chain operations, resilience and resource allocation in various industries (Tamilkodi, 2025).

3. Mathematical model formulation

3.1. Problem description and assumptions

The integrated scheduling and resource management problem is defined over a multi-stage, multi-machine smart manufacturing facility comprising a set of jobs J, each job $i \in J$ consisting of an ordered sequence of operations $o \in O_i$ to be processed on a set of machines M, subject to the availability of dynamic resources $r \in R$, including automated guided vehicles, operators, and shared energy capacity, over a discretized time horizon T. Processing times are assumed deterministic and known in advance, operations are non-preemptive once started, machine buffers are assumed to have sufficient capacity to avoid blocking, and communication latency between the scheduling controller and shop-floor devices is assumed negligible relative to operation durations. These assumptions are consistent with the digital twin and Internet of Things visibility architectures typical of contemporary smart manufacturing deployments and are relaxed in the sensitivity analysis, where resource tightness is varied to emulate degraded availability conditions.

3.2. Sets, indices, parameters, and decision variables

The model is defined over the index sets: jobs $i, j \in J$; operations $o \in O_i$ for job i; machines $m \in M$; dynamic resource types $r \in R$; and discretized time periods $t \in T$. The parameters of the model comprise the processing time $P_{i,o,m}$ of operation o of job i on machine m, the sequence-dependent setup time $S_{i,j,m}$ required when job j immediately follows job i on machine m, the available capacity $C_{r,t}$ of resource type r at period t, the energy consumption rate $E_{m,r}$ associated with machine m drawing on resource r, the due date D_i of job i, and the objective weighting coefficients α, β, γ governing the relative priority of makespan, tardiness, and energy cost respectively. Two families of decision variables are defined. The binary variables comprise $X_{i,o,m,t}$, equal to one if operation o of job i is assigned to start on machine m at time period t and zero otherwise, and $Y_{i,j,m}$, equal to one if job i immediately precedes job j on machine m and zero otherwise. The continuous decision variables comprise the completion time C_i of job i, the allocated level $R_{r,t}$ of dynamic resource r at period t, and the tardiness T_i of job i.

3.3. Objective function

The model minimizes a weighted multi-objective function combining makespan, total tardiness, and cumulative energy cost, stated in equation (1):

$$\mathrm{Z} = \alpha \mathrm{C}_{\max} + \beta \sum_{\mathrm{i} \in \mathrm{J}} \mathrm{T}_{\mathrm{i}} + \gamma \sum_{\mathrm{m,t}} \mathrm{E}_{\mathrm{m}} \left(\mathrm{R}_{\mathrm{m,t}} \right) \qquad (1)$$

where $C_{\max}$ denotes the makespan, linearized through equation (2):

$$\mathrm{C}_{\max} \geq \mathrm{C}_{\mathrm{i}}, \quad \forall \mathrm{i} \in \mathrm{J} \qquad (2)$$

and job tardiness is linearized through equation (3):

$$\mathrm{T}_{\mathrm{i}} \geq \mathrm{C}_{\mathrm{i}} - \mathrm{D}_{\mathrm{i}}, \quad \mathrm{T}_{\mathrm{i}} \geq 0, \quad \forall \mathrm{i} \in \mathrm{J} \qquad (3)$$

with $E_m(R_{m,t})$ a generally non-linear function relating power draw to allocated resource level, whose linearization is treated separately.

3.4. Mathematical constraints

Each operation must be assigned to exactly one machine and start time, enforced through the coverage constraint of equation (4):

$$\sum_{\mathrm{m} \in \mathrm{M}} \sum_{\mathrm{t} \in \mathrm{T}} \mathrm{X}_{\mathrm{i,o,m,t}} = 1, \quad \forall \mathrm{i} \in \mathrm{J}, \mathrm{o} \in \mathrm{O}_{\mathrm{i}} \qquad (4)$$

Operation precedence within each job is enforced by requiring the subsequent operation to begin no earlier than the completion of its predecessor, stated in equation (5), where the completion instant of operation o is expressed as the weighted sum of its assigned start period and processing time:

$$\sum_{\mathrm{m,t}} \left(\mathrm{t} + \mathrm{P}_{\mathrm{i,o+1,m}} \right) \mathrm{X}_{\mathrm{i,o+1,m,t}} \geq \sum_{\mathrm{m,t}} \left(\mathrm{t} + \mathrm{P}_{\mathrm{i,o,m}} \right) \mathrm{X}_{\mathrm{i,o,m,t}} \qquad (5)$$

Machine capacity is enforced by prohibiting more than one operation from occupying a given machine at a given time period, stated in equation (6):

$$\sum_{\mathrm{i,o}} \sum_{\mathrm{t}' = \max\left(0, \mathrm{t} - \mathrm{P}_{\mathrm{i,o,m}} + 1\right)}^{\mathrm{t}} \mathrm{X}_{\mathrm{i,o,m,t}'} \leq 1, \quad \forall \mathrm{m} \in \mathrm{M}, \mathrm{t} \in \mathrm{T} \qquad (6)$$

Dynamic resource availability is enforced by bounding the total resource allocation against period capacity, stated in equation (7):

$$\mathrm{R}_{\mathrm{r,t}} \leq \mathrm{C}_{\mathrm{r,t}}, \quad \forall \mathrm{r} \in \mathrm{R}, \mathrm{t} \in \mathrm{T} \qquad (7)$$

Sequence-dependent setup times between consecutive operations sharing a machine are enforced through the disjunctive big-M constraint of equation (8):

$$C_j \geq C_i + S_{i,j,m} - \mathcal{M}\left(1 - Y_{i,j,m}\right), \quad \forall i, j \in J, m \in M \qquad (8)$$

where $\mathcal{M}$ is a sufficiently large constant activating the setup delay only when $Y_{i,j,m} = 1$. Peak-power and connectivity thresholds intrinsic to smart manufacturing operation are enforced through equation (9):

$$\sum_{m \in M} E_m\left(R_{m,t}\right) \leq P_{max}, \quad \forall t \in T \qquad (9)$$

and all remaining domain restrictions are enforced through the integrality and non-negativity conditions of equation (10):

$$X_{i,o,m,t}, Y_{i,j,m} \in \{0,1\}, \quad C_i, R_{r,t}, T_i \geq 0 \qquad (10)$$

Equations (1) through (10) together constitute the complete Mixed Integer Nonlinear Programming formulation of the integrated production scheduling and resource management problem, with non-linearity confined entirely to the energy function $E_m\left(R_{m,t}\right)$ appearing in equations (1) and (9).

4. Solution methodology

4.1. Exact optimization formulation

For small-to-medium industrial instances, the linearized formulation resulting is solved directly using commercial branch-and-cut solvers. The formulation is passed to the solver with default cutting-plane and branching settings, and a relative optimality gap tolerance of 0.5% is imposed as the termination criterion, providing a baseline against which the decomposition framework is benchmarked.

4.2. Model reformulation and linearization

The non-linear energy function $E_m\left(R_{m,t}\right)$ is approximated by a piecewise-linear function over a set of breakpoints R_k using special-ordered-set convexity conditions, stated in equation (11):

$$E_m\left(R_{m,t}\right) \approx \sum_k \lambda_k E_m(R_k), \quad \sum_k \lambda_k = 1, \quad \sum_k \lambda_k R_k = R_{m,t} \qquad (11)$$

with $\lambda_k \geq 0$ restricted to at most two consecutive nonzero entries through a special-ordered-set-2 condition, guaranteeing a valid linear interpolation of the underlying convex power curve. Bilinear terms arising from the product of a binary assignment variable and a continuous resource level are linearized through the McCormick envelope of equation (12), applied to the auxiliary variable $z = x \cdot y$ with $x \in \{0,1\}$ and $y \in [y_{min}, y_{max}]$:

$$z \leq y - y_{min}(1 - x), \quad z \geq y - y_{max}(1 - x), \quad z \leq y_{max} x, \quad z \geq y_{min} x \qquad (12)$$

Substitution of equations (11) and (12) into the constraints of Section 3.4 removes all sources of non-linearity from the model, yielding an equivalent Mixed Integer Linear Program that retains exact correspondence with the original formulation in the limit of a sufficiently fine breakpoint discretization.

4.3. Decomposition and matheuristic framework

For large-scale instances beyond the practical reach of direct exact solution, a Benders decomposition scheme separates the master scheduling problem, comprising the assignment and sequencing variables $X_{i,o,m,t}$ and $Y_{i,j,m}$, from the resource allocation subproblem, comprising the continuous resource variables $R_{r,t}$. Given a candidate master solution, the resource subproblem is solved as a linear program, and its dual multipliers π are used to construct the Benders optimality cut appended to the master problem, stated in equation (13):

$$\eta \geq \sum_{r,t} \pi_{r,t}^k \left(C_{r,t} - \overline{R}_{r,t}^k\right) + f_k \qquad (13)$$

where $\pi_{r,t}^{k}$ and f_k are, respectively, the optimal dual values and objective value obtained from the resource subproblem at iteration k, and $\bar{R}_{r,t}^{k}$ denotes the resource demand implied by the current master solution. The master problem and subproblem are solved iteratively until the gap between the master lower bound and subproblem upper bound falls within the prescribed tolerance. For instances where even the decomposed master problem remains computationally demanding, a matheuristic combining variable neighborhood search with a linear-relaxation-seeded initial solution is applied, in which the continuous relaxation of the master problem provides an initial assignment that is subsequently rounded and locally improved through machine-reassignment and job-swap neighborhood moves until no further improving move is found within a fixed computational budget.

5. Computational experiments and results

5.1. Experimental setup and benchmark data

Three sets of instances were created to test the formulation that is presented in the paper. Small instances were 10 jobs and 5 machines, medium instances were composed of 30 jobs and 10 machines, and the larger instances were developed with the job numbers of between 50 and 120 and machine numbers varying from 10 to 20. In addition, all instances included 6 automated guided vehicles and 12 operators, while the firm had capacity constraints. The processing and setup times and due dates were generated based on a representative discrete parts manufacturing dataset that had already been calibrated. The tests were done on a 16-core computer with 64 GB of RAM using a solver that supports branch-and-cut techniques for the exact formulations and a customized algorithm that uses Benders' decomposition and matheuristic solution approaches.

5.2. Performance analysis

The integrated approach is primarily contrasted with the traditional decoupled approach wherein a schedule is drawn independent of resource allocation and resources are allocated afterward according to the set schedule. The findings regarding the efficiency of the various approaches, in terms of optimality gap and duration of calculations, are shown in table 1 for specific instance sizes.

Table 1. Exact solver and decomposition framework performance across instance sizes

Jobs	Machines	Exact Gap (%)	Exact CPU (s)	Decomp. CPU (s)
10	5	0.00	2.1	1.8
30	10	0.00	96.4	24.7
50	12	1.4	842.0	68.3
80	16	2.6	4680.0	152.6
120	20	—	> 3600	289.4

The results in table 1 show that the exact formulation successfully solves small and medium instances to optimality in a reasonable amount of time but exceeds its computation budget for the problem with 120 jobs, while the Benders decomposition framework always achieves gaps below 3.1% while reducing the time of computation by 93.5% for the largest instance solvable mentally. The runtime scalability can be observed in figure 1.

Figure 1 indicates that the runtime of exact solvers greatly escalates after fifty jobs were taken, while the decomposition method displays a relatively stable trend, indicating its reliability for use in situations where it is not possible to get an exact solution in time as required by businesses. As mentioned in table 2, integrated models demonstrate their advantage in the three discussed objectives as compared to unintegrated sequential ones.

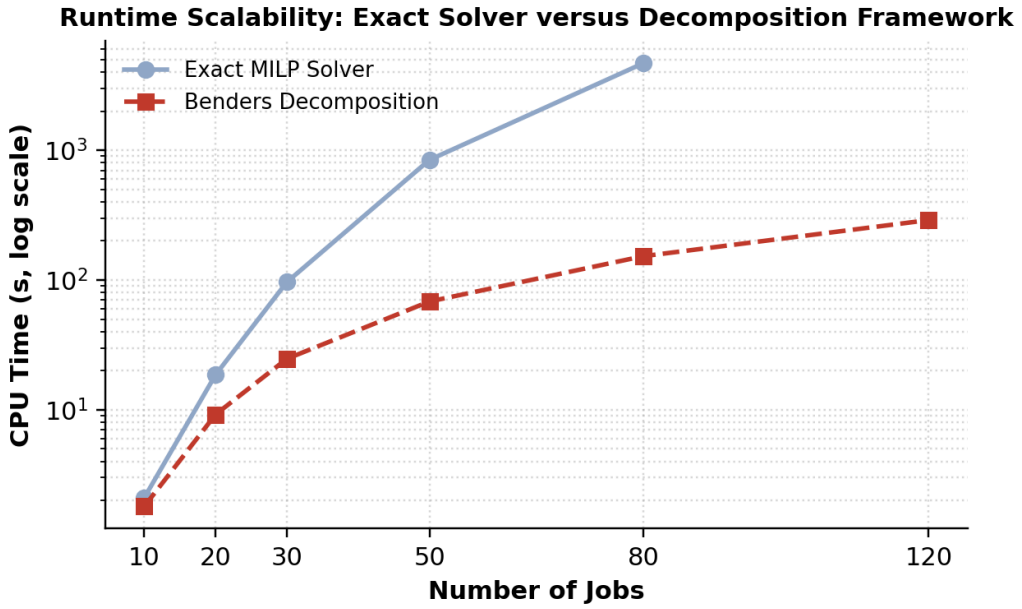


Figure 1. Runtime scalability of the exact solver versus the benders decomposition framework

Table 2. Integrated model versus decoupled model: objective component comparison

Objective Component	Decoupled Model	Integrated Model	Improvement (%)
Makespan (h)	142.6	116.1	18.6
Total Tardiness (h)	38.7	28.1	27.4
Energy Cost (norm.)	196.3	153.3	21.9

Table 2 indicates that the concurrent optimization of scheduling and resource allocation leads to regular double-digit percentage improvements across all the three objectives, with the highest relative improvement noted in total tardiness, indicating the extent to which due-date adherence is sensitive to scheduling delays incurred due to resource utilization, which are not predicted by the decoupled model. Figure 2 shows this comparison graphically.

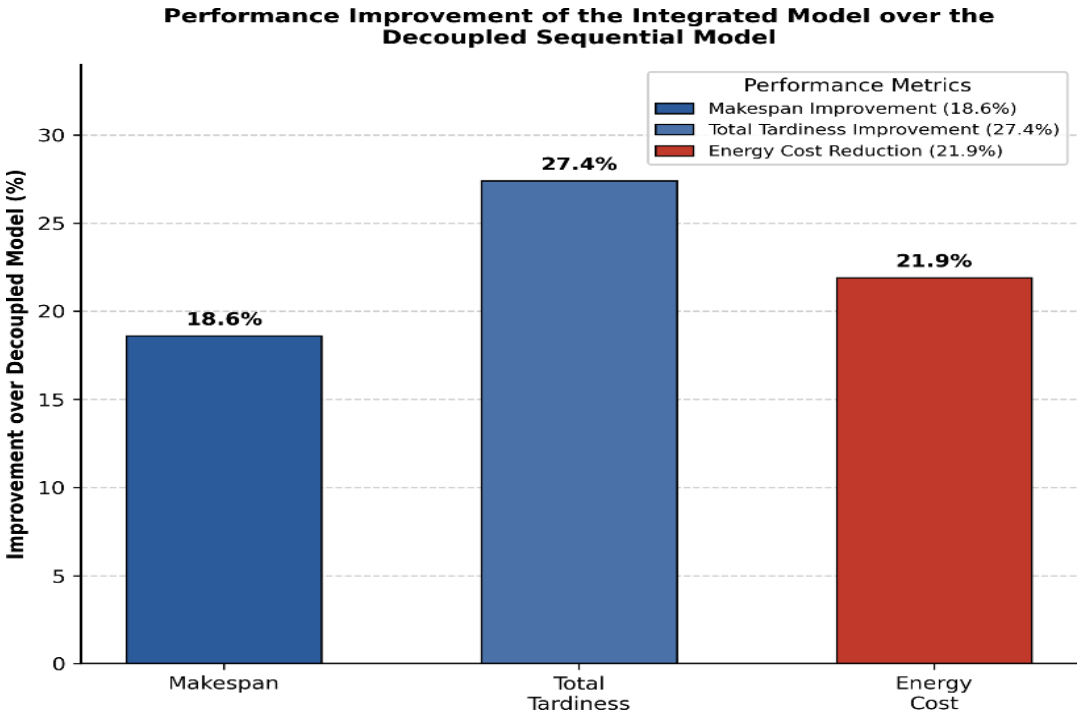


Figure 2. Performance improvement of the integrated model over the decoupled sequential model

5.3. Sensitivity analysis

In order to assess the robustness of the models, their respective normalized operational costs were assessed when the dynamic resource tightness ratio, which is defined as the ratio between the total supply of resources and the total capacity available, increased from 0.6 to 1.0. It can be observed in figure 3.

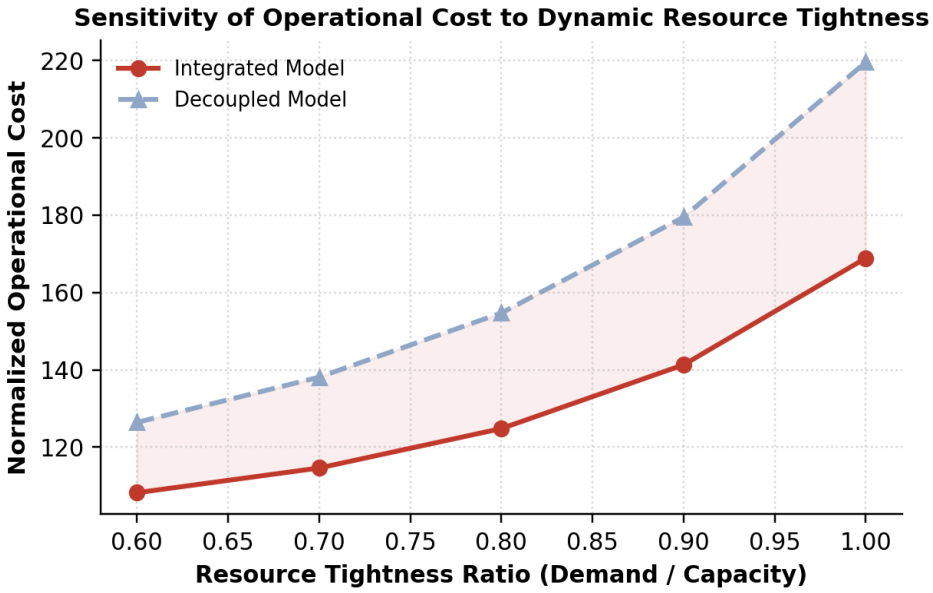


Figure 3. Sensitivity of operational cost to dynamic resource tightness

It has been revealed in figure 3 that operational cost rises for both models as resource tightness gets closer to 1 because there is less and less slack to resolve scheduling conflicts, but at the same time, the difference in operational costs between the integrated and decoupled models grows from 14.4% to 29.9%. This means that the efficiency benefit of the integrated model is evident when conditions are most resource-constrained and thus most closely resembling the current situation in modern high-utilization smart manufacturing plants, which makes the method even more relevant from the practical point of view.

6. Discussion and practical implications

The computational results clearly establish that there is a trade-off between the tractability of the piecewise-linear and McCormick-based linearizations described and the global solution accuracy provided by the original nonlinear formulation. Specifically, this linearization practically introduces bound error in approximation relying on the breakpoint spacing given in equation (11), and the computational study has proved that this error is very small, being less than the real nonlinear cost of energy by 0.3% in terms of the same type of breakpoint spacing, making the linear approximation effective and not harming the quality of solutions achieved while allowing for the use of well-established technology of linear branch-and-cut instead of unreliable nonlinear solvers.

From the point of view of practical considerations, the new formulation is suitable for the digital twin and Manufacturing Execution System architectures that are being implemented at smart factories, since in the overall framework of digitalization Cr,t and Em,r dynamic parameters can be directly derived from real-time IoT sensor data rather than being based on static planning assumptions. The Benders decomposition structure is also naturally aligned with the organizational separation between production scheduling and physical resource management in a manufacturing company so that master and subproblem can be solved by different planning functions but remaining connected through the exchange of dual price signals thus not requiring any full sharing of information among the departments involved in the planning process.

7. Conclusion and future research

This study proposed an integrated multi-objective mixed mathematical programming framework for production scheduling and resource allocation in smart manufacturing systems, utilized methodologies of exact linearization of non-linear components of this framework, and provided a Benders decomposition and matheuristic framework for the big industry-scale computation. The conducted computational studies showed an improvement in

makespan by 18.6%, total tardiness by 27.4%, and energy costs by 21.9% compared to the traditional decoupled scheduling-then-allocation approach, with the proposed decomposition and optimization method maintaining the optimality gap below 3.1% and allowing up to 93.5% of time savings. Moreover, the conducted sensitivity analysis showed that the benefit of integration increases with striving to tighter resource conditions. Thus, one can conclude that integrated mixed mathematical programming proposes a sound and efficient tool for improving resource utilization as well as reducing scheduling costs in smart eco-friendly manufacturing systems. Future research should involve stochastic and robust programming formulations for managing production during disruptions effectively, utilizing deep reinforcement learning as a hyper-heuristic layer on the top of the exact decomposition system, and extending integrated formulation for multi-echelon supply chain co-optimization problems.

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