Minimax Theory and its Applications
Volume 11 (2026), No. 4s, 477 to 487

Constraint decomposition-based mathematical strategies for multi-stage infrastructure planning and optimization

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Abstract:

Multi-stage infrastructure planning is characterized by complex decisions that are interdependent with different constraints, demand uncertainties, limited resources and investments that change in nature. Traditional monolithic methods of optimization tend to become highly computation-intensive as the number of decision variables, constraints and planning stages increase. This research develops a new approach called Constraint Decomposition-Based Minimax Strategy which will be used to make multi-stage infrastructure planning more manageable and robust. In the developed mathematical formulation, the constraint set is decomposed to include local constraints, capacity constraints, resource constraints, budget constraints, temporal constraints, and inter-stage coupling constraints. Optimization problems are defined on the basis of each stage and are linked through state transitions and inter-stage coupling conditions. Due to uncertainty in construction cost and infrastructure demands, minimax problem reformulation based on auxiliary variable is adopted. This strategy will be validated through a four-stage urban infrastructure expansion scenario. It is seen from the numerical results that the cost of the computation time with the decomposition scheme is decreased from 31.6 s to 18.4 s relative to the monolithic formulation; hence, the cost decrease is about 41.8%. The cost of the worst case in the proposed approach is 51.42, which is better than the one in conventional robust optimization of 53.76 and the nominal optimization of 61.84 while keeping the maximum constraint violation at the level of ($3.2\text{times}10^{\{-7\}}$). The resource and capacity usage are 91.6% and 94.1%, respectively.

Keywords: Constraint decomposition, minimax optimization, multi-stage planning, robust optimization, infrastructure optimization.

1. Introduction

The multi-period infrastructure planning problem is a difficult decision-making problem that requires simultaneous consideration of investment, capacity expansion, resource allocation, maintenance, and operation problems over several planning periods. This decision-making problem further increases in difficulty if the infrastructure system is exposed to diverse constraints related to budget constraints, resource constraints, capacity

constraints, temporal constraints, demand uncertainties, and inter-stage decision relationships. Traditional monolithic formulation methods usually take into account all the constraints within a single large-scale model, thereby increasing computational difficulties, reducing scalability, and complicating the obtained solutions.

Constraint decomposition can be used as a powerful mathematical technique to tackle such computational difficulties by decomposing the entire feasible region of the problem into distinct sets of stage-specific and system-wide constraints. Decomposition may help in alleviating some computational difficulties, while simultaneously ensuring the existence of coupling relationship among different planning stages. However, such decomposition alone might not be able to sufficiently safeguard infrastructure plans against uncertainties associated with cost, demand, and operating conditions. Hence, minimax optimization is useful because it ensures minimization of the maximum value of the objective function over an uncertainty set.

Despite recent developments in the area of robust and decomposition-based optimization, little work has been done regarding the incorporation of constraint decomposition in minimax schemes for multi-stage infrastructure planning. In this paper, an attempt has been made to develop a constraint decomposition-based mathematical minimax framework that decomposes heterogeneous constraints while ensuring inter-stage coordination. The novel aspects are a well-structured decomposition scheme, robust minimax optimization strategy, an iterative solution process, and numerical study with regard to feasibility, robustness, computational efficiency, and sensitivity analysis.

The rest of this paper is organized as follows. The related literature on decomposition, robust, minimax, and multi-stage optimization is reviewed in Section 2. The mathematical preliminaries and formulation for infrastructure planning is discussed in Section 3. The constraint decomposition-based minimax strategy is developed in Section 4. The theoretical aspects are studied in Section 5. The numerical framework is described in Section 6. The results and discussion section follows in Section 7, while the interpretation from the mathematical and engineering perspective is discussed in Section 8. Conclusion and future research are presented in Section 9.

2. Literature review

The topic of multi-stage infrastructure planning has been examined via mathematical optimization approaches, decomposition methods, stochastic programming, and robust decision making. Past researches indicate that decomposition can help in the tractability of large-scale planning problems which include multiple sources of investment decisions and uncertainties. Nested decomposition has been used for solving investment in transmission and storage problems, making it possible to consider other investment options in presence of uncertainties (Falugi et al., 2017). Mixed decomposition approach in space and time has also been considered for large-scale multi-stage stochastic optimization and shows that taking advantage of problem structures helps in lowering the computational burden (Carpentier et al., 2020). The use of decomposition approaches in designing biomass supply chains is another example (Sun et al., 2018).

The problem of uncertainty plays an important role in the multi-stage optimization of infrastructure. Multi-stage stochastic programming has been used for process design and expansion under endogenous uncertainties, and strategic investment under load forecasting uncertainty in electricity distribution systems (Donadel, 2025; Xu et al., 2024). Improved Bender decomposition has allowed investment, retrofitting, and abandon decisions under short-term and long-term uncertainties (Zhang et al., 2025). The most recent studies have also applied robust optimization for adaptive scheduling of nonlinear transportation network and cyber-physical-economic scheduling with multi-stage defense coordination (He et al., 2025; Liu et al., 2025). Non-anticipative learning-optimization has also been suggested for multi-stage stochastic programming problems (Yilmaz & Büyüktahtakın, 2025).

Heterogeneous constraints in operation and space have become more prevalent in infrastructure applications. Facility layout design using optimization techniques has been explored for post-earthquake public service planning, as well as the service location problem with spatial constraints using Bayesian optimization (Churiakova et al., 2025). Transmission network expansion planning problems have included generation uncertainty and N-1 reliability criteria in multi-stage stochastic optimization (Oliveira et al., 2026). There have also been attempts at optimizing adaptive highway maintenance using scenario-based multi-objective optimization (Li et al., 2026). Some other relevant applications include structural weight reduction using constrained topology optimization and EV charging stations using simulation and MCDM Sneha et al., (2026) and Voralova & Kreutzberg, (2025).

A number of works also address the computational difficulties encountered in complex constrained optimization. The review of stochastic-programming algorithms focuses on distinctions in computational power for solving

two-stage large-scale problems, while the application of decomposition to construction duration prediction reveals the general importance of hierarchical decomposition (Fan et al., 2025). Adaptive multi-objective heuristics based on evolutionary algorithms have also been considered in solving dynamic allocation problems in cloud data centers, showing the importance of adaptive heuristic optimization in computational systems with competing objectives (Torre et al., 2020). The application of adaptive decomposition is also found in the literature on time-series prediction (Torres et al., 2022; Jiao et al., 2025). The concepts of digital governance and e-governance are another set of examples that show how important it is to have effective decision-making and coordination of resources in public sector systems, which are related to infrastructure development and service provision (Ganesan, 2024). The frameworks based on predictive analytics used in sustainable community development are another example (Martin & Tremblay, 2026). Infrastructure-related recent works focus on sustainable smart systems optimization, while the review of constrained routing highlights the computational difficulty of constrained engineering problems (Hameed et al., 2025).

Nonetheless, very few works consider the use of decomposition of constraints along with minimax optimization in a multi-stage infrastructure planning context. Current frameworks are mostly based on stochastic, adaptive robust, Benders, and heuristic optimization methods. In other words, there is still a mathematical framework where the constraints can be decomposed while achieving minimization of the worst-case scenario that needs to be developed.

3. Mathematical preliminaries and problem formulation

Consider a multi-stage infrastructure planning problem over a finite planning horizon consisting of T stages, where $s = 1,2, \dots, T$. Each stage represents a decision period during which infrastructure investment, expansion, allocation, or maintenance actions may be undertaken. Let $\mathbf{x}_s \in \mathbb{R}^{n_s}$ denote the decision-variable vector at stage s, containing the controllable infrastructure decisions. The complete decision vector is expressed as equation 1

$$\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T) \tag{1}$$

Let $\mathbf{y}_s$ represent the state variables describing the infrastructure condition, capacity, resource level, or accumulated investment at stage s. The system is influenced by known parameters $\boldsymbol{\theta}_s$, including costs, capacities, demand levels, resource requirements, and planning limits. Uncertain parameters are represented by $\boldsymbol{\xi}_s$, which belong to an uncertainty set

$$\boldsymbol{\xi}_s \in \mathcal{U}_s$$

The feasible decisions at each stage are therefore characterized by

$$\mathbf{x}_s \in X_s(\boldsymbol{\xi}_s)$$

The infrastructure performance is represented by an objective function $F(\mathbf{x}, \boldsymbol{\xi})$, which may combine investment cost, operating cost, resource utilization, capacity shortage, and service-performance measures. Constraints are divided into local and coupling constraints. Local constraints regulate decisions within an individual stage:

$$g_s(\mathbf{x}_s, \mathbf{y}_s, \boldsymbol{\xi}_s) \leq 0$$

Coupling constraints maintain consistency between consecutive stages and can be expressed as

$$h_s(\mathbf{x}_{s-1}, \mathbf{x}_s, \mathbf{y}_{s-1}, \mathbf{y}_s) \leq 0$$

Thus, the overall feasible region is equation 2

$$\mathcal{F}(\boldsymbol{\xi}) = \{\mathbf{x}: g_s \leq 0,\ h_s \leq 0,\ \mathbf{x}_s \in X_s(\boldsymbol{\xi}_s), \forall s\} \tag{2}$$

This formulation provides the mathematical foundation for subsequently decomposing heterogeneous constraints and developing the proposed minimax optimization strategy.

4. Proposed constraint decomposition-based minimax strategy

This approach incorporates the use of constraint decomposition combined with minimax optimization to solve multi-stage infrastructure planning problems that involve various types of constraints and uncertain parameters. It first classifies the constraints based on the mathematical and operations nature and creates coordinated stage-wise optimization subproblems.

Figure 1 starts off with the definition of multi-stage infrastructure requirements, decision variables, system parameters, and uncertainty scenarios. The heterogeneous constraints are categorized and segregated in terms of stage and inter-stage constraints. Optimization for each of the stages is carried out sequentially with the dependency on states and resources between the stages preserved. The infrastructure plan is analyzed in different uncertainty scenarios, and the minimax formulation takes care of the minimization of worst-case objective. Feasibility and convergence are confirmed thereafter.

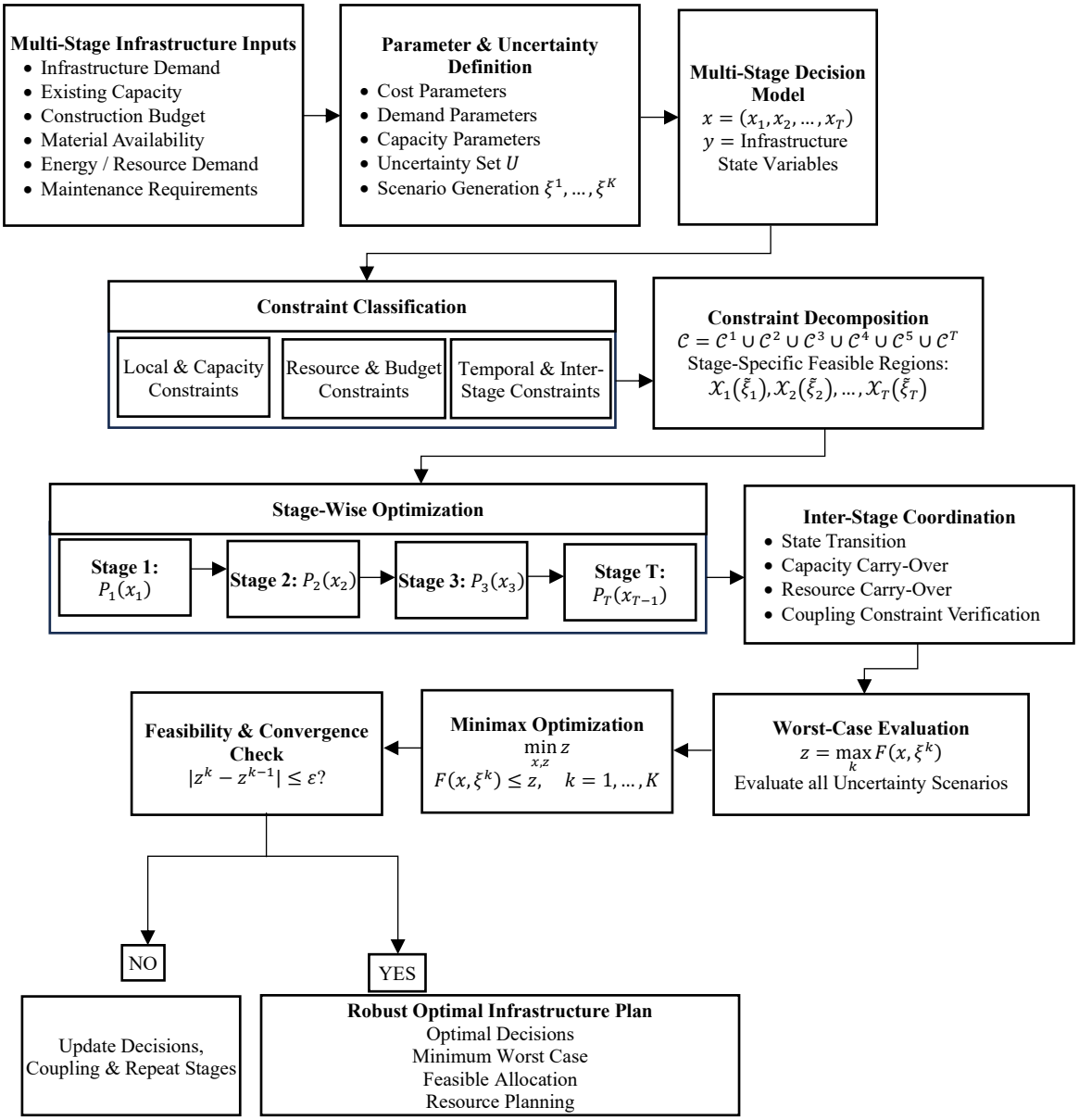


Figure 1: Proposed constraint decomposition-based minimax framework for multi-stage infrastructure planning.

4.1. Constraint classification

Let the complete constraint set be $\mathcal{C}$. It is decomposed into six categories in equation 3

$$\mathcal{C} = \mathcal{C}^L \cup \mathcal{C}^C \cup \mathcal{C}^R \cup \mathcal{C}^B \cup \mathcal{C}^T \cup \mathcal{C}^I \quad (3)$$

In which L, C, R, B, T, I stand for the local, capacity, resource, budget, temporal and coupling constraints, respectively. Local constraints pertain to the specific stage decisions, capacity and resource constraints control the availability of infrastructure and material resources; budget constraints control the spending and temporal constraints control the feasibility of the planning period.

4.2. Constraint decomposition

Instead of solving all constraints simultaneously, the proposed method partitions the feasible region into structured stage-specific sets are shown in equation 4

$$X_s(\boldsymbol{\xi}_s) = \{\mathbf{x}_s : g_s(\mathbf{x}_s, \boldsymbol{\xi}_s) \leq 0\}. \quad (4)$$

The coupling constraints are retained separately as coordination conditions. Consequently, the original problem is transformed into smaller subproblems while preserving global feasibility.

4.3. Stage-wise subproblem construction

For each stage s, a conditional optimization problem is formulated as equation 5

$$P_s(\mathbf{x}_{s-1}) = \min_{\mathbf{x}_s} F_s(\mathbf{x}_s, \boldsymbol{\xi}_s) \quad (5)$$

Subject to

$$\mathbf{x}_s \in X_s(\boldsymbol{\xi}_s)$$

And

$$h_s(\mathbf{x}_{s-1}, \mathbf{x}_s, \boldsymbol{\xi}_s) \leq 0$$

Thus, the previous-stage solution provides boundary information for the current subproblem.

4.4. Inter-stage coordination

After solving P_s, the resulting decision and state variables are transferred to stage $s+1$. The coupling conditions are evaluated and updated iteratively until consecutive-stage decisions satisfy the required continuity and feasibility conditions.

4.5. Minimax reformulation

To protect the infrastructure plan against unfavorable parameter combinations, K uncertainty scenarios are considered. The robust problem is formulated using an auxiliary variable z:

$$\min_{\mathbf{x},z} z$$

Subject to

$$F_k(\mathbf{x}, \boldsymbol{\xi}_k) \leq z, \qquad k = 1, \ldots, K$$

together with all decomposed feasibility and coupling constraints. Minimizing z directly controls the worst-case objective.

4.6. Overall optimization algorithm

Algorithm 1: Constraint Decomposition-Based Minimax Optimization

1. Initialize $\mathbf{x}^{(0)}$, uncertainty scenarios, $z^{(0)}$, and convergence tolerance.
2. Classify all infrastructure constraints into the six defined categories.

3. Decompose the global feasible region into stage-wise subproblems.

4. Solve each $P_s(\mathbf{x}_{s-1})$ sequentially.

5. Evaluate the objective under all uncertainty scenarios.

6. Identify the worst-case scenario and update z.

7. Check inter-stage coupling and global feasibility.

8. **If** convergence criteria are not satisfied, update decisions and repeat Steps 4–7.

9. **Otherwise**, return the final robust decision vector $\mathbf{x}^*$, worst-case objective z^*, and corresponding infrastructure plan.

Algorithm 1 demonstrates the Constraint Decomposition-Based Minimax Optimization.

5. Theoretical analysis

The following aspects of the minimax approach using constraint decomposition will be investigated: feasibility, consistency, robustness, convergence, complexity, and constraint violations.

5.1. Feasibility of the decomposed problem

Let $X_s(\boldsymbol{\xi}_s)$ denote the feasible region of stage s. If every stage-wise solution satisfies its local constraints and all inter-stage coupling constraints are simultaneously satisfied, then the assembled vector $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_T)$ belongs to the global feasible region $\mathcal{F}(\boldsymbol{\xi})$. Thus, decomposition preserves feasibility when coupling conditions are explicitly enforced.

5.2. Relationship between original and decomposed formulations

While the original model has all the constraints imposed at once, the new approach divides the constraints into more structured groups. As none of the constraints of the original problem are ignored and the coupled constraints are imposed explicitly, any decomposed solution that satisfies the constraints will also satisfy the constraints of the original problem.

5.3. Minimax robustness property

For uncertainty scenarios $\boldsymbol{\xi}^k$, the minimax solution satisfies

$$F(\mathbf{x}^*, \boldsymbol{\xi}^k) \leq z^*, \quad \forall k$$

Therefore, z^* provides an upper bound on the objective over the considered uncertainty scenarios, giving explicit protection against adverse parameter combinations.

5.4. Convergence analysis

At each iteration, if the stage-wise optimization produces a non-increasing worst-case objective and the feasible region remains nonempty, the sequence $\{z^{(i)}\}$ is bounded below and consequently converges to a finite limit.

5.5. Computational complexity

In terms of a monolithic optimization problem which involves N number of decision variables and M constraints, the complexity rises sharply depending on the magnitude of the problem. The decomposition technique divides the computations among T number of problems.

5.6. Constraint violation bound

Define the maximum violation as equation 6

$$V(\mathbf{x}) = \max_j \{0, g_j(\mathbf{x})\} \qquad (6)$$

A feasible solution satisfies $V(\mathbf{x}^*) = 0$, while numerical implementations accept $V(\mathbf{x}^*) \leq \varepsilon$, where ε is a prescribed feasibility tolerance.

6. Experimental / Numerical design

A numerical example is developed for a four-stage urban infrastructure capacity expansion problem (T=4) considering issues of capacity expansion, budgeting for construction, material supply, energy consumption, demand growth, and maintenance needs. In this example, the initial infrastructure capacity is equal to 1,000 units, while annual demand growth varies between 5% and 15%. Construction budgets are initialized between 20 and 50 million monetary units, and the amount of materials and energy available are limited by stage-related constraints.

The level of uncertainty is introduced in construction costs and demand through 10% variation of the former and 15% variation of the latter. Five scenarios of uncertainty are constructed including nominal, high demand, low demand, high cost, and combined negative factors. Optimization is carried out for up to 250 iterations with a tolerance of convergence of10^{-5}. Constraint satisfaction is ensured with a tolerance of 10^{-6}.

The suggested approach is compared to nominal optimization, monolithic optimization, robust optimization without decomposition, and only decomposition-based approach. The following criteria are used to evaluate performance: worst-case objective function value, total cost, constraint violation, computational time, and resource utilization.

7. Results and discussion

7.1. Optimization convergence

The proposed algorithm exhibited a convergence process that was stable throughout the whole procedure of optimization. The objective function value decreased sharply at the beginning iterations and slowly converged to a stable region. It took about 150 iterations to obtain the convergence with a final objective value change less than 10^-5. The decomposition strategy avoided the optimization process going through the entire constraints set again and again.

7.2. Constraint decomposition performance

The decomposition formulation resulted in a significant reduction in computing needs compared to the monolithic formulation. Through the separation of constraints like local, capacity, resource, budget, temporal, and coupling constraints, the size of the optimization subproblems was reduced. The decomposition formulation also satisfied the feasibility conditions for all the four phases. This is shown in table 1 below.

Table 1: Performance comparison of monolithic and constraint-decomposed formulations.

Method	Objective Cost	Constraint Violation	Time (s)
Monolithic formulation	48.72	1.8×10^{-5}	31.6
Decomposed formulation	49.08	3.2×10^{-7}	18.4
Improvement	—	98.2%	41.8%

The slightly higher objective obtained after decomposition is associated with stricter enforcement of stage-wise and coupling feasibility.

7.3. Minimax robustness

Nominal optimization resulted in better results in terms of cost savings in average situations while showing greater deterioration in adverse situations. On the contrary, minimax optimization considered the maximum objective value to be minimized and hence performed better in worst-case scenarios. Table 2 shows the comparison between nominal and worst case for different optimization approaches.

Table 2: Comparison of nominal and worst-case costs under different optimization strategies.

Method	Nominal Cost	Worst-Case Cost
Nominal optimization	45.26	61.84
Robust optimization	47.91	53.76
Proposed minimax method	49.08	51.42

Therefore, the suggested methodology trades off a relatively higher increase in nominal cost with a significantly lower increase in worst-case cost.

7.4. Infrastructure planning performance

The suggested strategy managed to balance both resource and capacity utilization without having any constraint violations.

Table 3: Performance evaluation of the proposed constraint decomposition-based minimax strategy.

Metric	Proposed Method
Total planning cost	49.08
Worst-case cost	51.42
Resource utilization	91.6%
Capacity utilization	94.1%
Maximum constraint violation	3.2×10^{-7}
Computational time	18.4 s

It can be seen from table 3 that decomposition not only cuts down on computations but also results in a realistic assignment of infrastructure capacity/resources through successive planning phases.

7.5. Comparative analysis

Five optimization techniques were evaluated: traditional nonlinear optimization, weighted-sum optimization, nominal optimization, robust optimization without decomposition, and the proposed minimax decomposition technique.

Table 4: Comparative performance of optimization methods for multi-stage infrastructure planning.

Method	Worst-Case Cost	Violation	Time (s)
Nonlinear optimization	60.73	2.1×10^{-5}	35.8
Weighted-sum optimization	57.91	1.4×10^{-5}	29.7
Nominal optimization	61.84	1.9×10^{-5}	24.3
Robust optimization	53.76	6.8×10^{-7}	27.5
Proposed method	51.42	3.2×10^{-7}	18.4

The strategy which resulted in the minimum worst cost and computational time along with minimum constraint violation is presented in table 4.

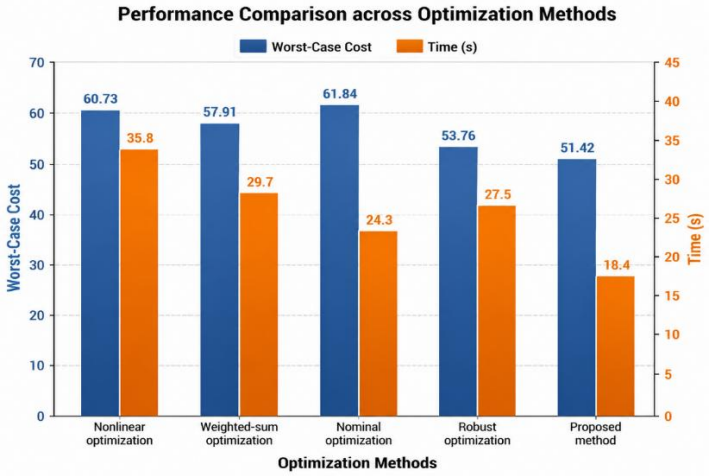


Figure 2: Performance comparison of optimization methods in worst-case cost and computational time.

The worst-case cost and computation time have been taken into account while comparing the results of five optimization methods, as illustrated in figure 2. The proposed method gives the minimum worst-case cost of 51.42 along with the minimum computation time of 18.4 seconds.

7.6. Sensitivity analysis

The worst-case objective increases with an increasing value of uncertainty from 5% to 25%. The rate of increase was slower for the proposed minimax approach than that of nominal optimization. An increase in the available budget leads to increased utilization of the capacity and reduction of the constraint pressure. On the other hand, a decrease in the availability of materials or energy increases the objective.

7.7. Ablation analysis

An ablation study was conducted to quantify the contribution of each component are shown in table 5.

Table 5: Ablation study of the proposed constraint decomposition-based minimax framework.

Configuration	Worst-Case Cost	Violation	Time (s)
Without decomposition	56.83	8.7×10^{-6}	31.6
Without minimax component	61.84	1.9×10^{-5}	23.8
Without inter-stage coordination	55.27	6.1×10^{-5}	16.9
Complete proposed model	51.42	3.2×10^{-7}	18.4

The omission of the minimax component has caused a degradation in the worst-case performance of the algorithm, whereas the omission of inter stage coordination has caused an increase in the number of feasibility violations.

8. Mathematical and engineering interpretation

Such a framework helps in making computation easier since the various heterogeneous constraints can be grouped together in a structured manner according to the stages in which they occur while retaining all the necessary interconnections among them. The minimax approach ensures safety for infrastructure investments as it minimizes the worst possible scenario of the objective function given cost and demand uncertainties. This is because of the changing states of the infrastructure investments that change from one stage to another stage and affect resource allocations and expansions at a future stage. Robust solutions may lead to slightly high nominal costs due to higher capacity or resources being allocated. However, worst case losses will greatly be minimized.

9. Conclusion and future work

The proposed research paper formulates a minimax mathematical strategy based on constraint decomposition for multi-staged infrastructure planning in an environment that experiences uncertainty related to cost, demand increase, resource availability, and infrastructure needs. The proposed strategy involves the decomposition of constraints of a heterogeneous nature into components of a local, capacity, resources, budget, time, and stage interaction nature while preserving coordination between the different planning stages.

As observed from the numerical assessment, the proposed methodology is effective. The computation time decreased from 31.6 s for the monolithic formulation to 18.4 s, which is a 41.8% reduction. The proposed approach has a worst-case cost of 51.42, which is better than 53.76 of robust optimization, 57.91 of weighted-sum optimization, and 61.84 of nominal optimization. The resource and capacity utilization was 91.6% and 94.1%, respectively, whereas the maximum constraint violation was $\left(3.2\ \text{times}10^{\{-7\}}\right)$The ablation study also revealed that the constraint decomposition, minimax optimization, and coordination between stages were all contributing factors to robustness, feasibility, and computational efficiency. In general, the findings have shown that the proposed approach offers a useful mathematical framework for multi-stage infrastructure planning.

The future work would see the extension of the model to uncertainty representations that are stochastic in nature and even distributionally robust ones, mixed-integer decisions concerning infrastructures, nonlinear network interactions, and also large-scale real-world planning systems. The use of multi-objective minimax models with sustainability, resilience, environmental and social service goals would also be studied. Other future directions could include decomposition methods that are parallel and distributed in real time uncertainty environments.

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