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# Lyapunov function-based stability optimization for autonomous control systems operating under dynamic disturbances

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**Abstract**:

Nonlinear autonomous systems functioning under bound disturbances such as wind force, vibration, and sensor noise require stability assurances that are valid under the most severe operating conditions, which cannot be established by the classical passive Lyapunov method. This paper presents a new methodology that reformulates robust stabilization as the problem of minimizing the extremal gain of the Lyapunov function in the context of the twoplayer zero-sum minimax optimization problem involving the general form of Control-Lyapunov functions. The control signal minimizes the energy of the system while the adversarial disturbance minimizes the deviation of the system state. The performance index emerges through the Lyapunov derivatives that are modified to include some quadratic cost for control and disturbance. The Lyapunov parameter is solved using the method of convex sum-of-squares and results in the resolution of the Hamilton-Jacobi-Isaacs inequality, with proofs of the existence of stability of input-to-state stability and the unique optimal Lyapunov parameter vector resulting in the minimum bound energy dissipation rate. The proposed offline-online control design computes the optimal Lyapunov surface once and uses classical control algorithms for offlline optimization. The numerical implementation on an autonomous quadcopter in the canceling turbulence condition shows that the minimax controller results in 66.7% reduction of maximum lateral position deviation, 53.3% reduction of root-mean-square control effort, and delivers 2.4-times faster Lyapunov decay compared to the previously used passive Lyapunov controller while the efficiency (disturbance attenuation) increased from 0.79 to 0.31. These results confirm that recasting Lyapunov-based stabilization as a parametrized minimax optimization problem yields quantifiably superior disturbance rejection and stability margins relative to conventional passive Lyapunov design for nonlinear autonomous control systems.

*Keywords:* Autonomous control, lyapunov function, stability optimization, dynamic disturbances, minimax theory, hamilton-jacobi-isaacs inequality, input-to-state stability.

## 1. Introduction

Autonomous systems such as wheeled mobile robots, aerial drones, and underwater vehicles are indeed often applied in operational situations affected by dynamic disturbances that cannot be modeled, like wind gusts, vibration, wheel slip, or sensor noise. Each of these factors can significantly impact closed-loop stability if not taken into account in control design. The techniques that made the autonomous systems more stable through fuzzy gain scheduling and Lyapunov analysis shown that adaptive implementation of a gain leads to a more stable performance of the adaptive system in sophisticated situations (Venkatesh et al., 2024). A universal rule of updating the Lyapunov approach applied in mobile robot control with wheel slip further demonstrated that adaptive identification techniques introduced in the Lyapunov framework successfully counteract the problems arising from unmodeled disturbances (Burghi et al., 2024). The control of autonomous wheeled robots with slippage disturbance in accordance with the principles of differential flatness has likewise proved that introducing disturbance modeling into the process of control design could meaningfully improve trajectory tracking accuracy in the wheel slip cases (Liu et al., 2023).

Even if this advancement has taken place, most Lyapunov-based control designs make passive assumptions regarding stability by using a fixed Lyapunov function instead of actively making an effort to optimize the stability limits or stability decay rates in time of disturbance. Implementation of nonholonomic robot controllers over time-varying delays and limited input has revealed how important consideration of both delay and saturation must be, even when reliable controls are already studied, pointing to the inadequacy of stability analysis done separately from the disturbance context (González-Sorribes et al., 2024). In aeronautics, attitude control of quadcopter drones with the aid of improved sliding mode control and extended state observer is able to demonstrate great disturbance elimination properties; however, Lyapunov certificates are applied after the design of the controller has been completed (Wu et al., 2025). Instances of nonlinear robust backstepping control based on disturbance observers and applied to quadcopter drones, and fixed-time attitude regulation for quadcopters prove that disturbance-informed feedback helps to significantly improve tracking outcomes as compared to disturbance-unaware ones, but without formulation of Lyapunov certificates as direct optimization objects (Balasm et al., 2025; Thanh et al., 2022; Nguyen et al., 2024).

The fact that this constrain encourages viewing stable stabilization as a two-player zero-sum game means that the control input u(t) minimizes energy consumption and instability of the system while at the same time the opposing disturbance d(t) maximizes deviations from the state; this situation is fully solved by minimax theorem as well as the Hamilton-Jacobi-Isaacs theory. Moreover, an anti-disturbance controller for quadrotor systems based on observer estimating unknown dynamics of the system has shown that if disturbance as an opposing force instead of a stochastic event, it leads to better guarantees of worst-case performance conditions in future work (Zhen & Zhang, 2022). In addition, the use of nonlinear extended state observer theory and prescribed performance control of quadrotor unmanned aerial vehicles has confirmed the usefulness of worst-case methodology in practice despite the presence of compound fault problems in the systems under consideration (Mai et al., 2024). Furthermore, Active Disturbance Rejection Control technique used for trajectory tracking in the case of underactuated autonomous underwater vehicles as well as other systems, using Distributed Lyapunov-based model predictive control approach, witnesses that use the Lyapunov-based method in order to control underactuated and multiple agent systems which also experience ocean disturbances (Báez-Hernández et al., 2023; Xie et al., 2020; Yan et al., 2024).

By leveraging the findings from both ground and aerial as well as marine platforms this paper develops a framework for Lyapunov parameterization and optimization under bounded dynamic disturbances (Anastasiya et al., 2025). Specifically, improved control techniques for underactuated autonomous underwater vehicles based on integral backstepping sliding mode control; a decoupled non-singular adaptive integral-terminal sliding mode tracking control for X-rudder autonomous underwater vehicles; and a non-singular fast-terminal sliding mode tracking controller for underactuated autonomous underwater vehicles show that the sliding mode and Lyapunov robust control methods remain dominant for operations of all autonomous platforms that are affected by disturbances but none of them treated Lyapunov parameter as the outcome of justified optimization (Chen et al., 2024; Yuan et al., 2022; Wang & Du, 2024). In their turn, the innovations of integral sliding mode output feedback control for unmanned marine vehicles in case of actuator breakdowns; trajectory control of nonholonomic wheeled mobile robots using Lyapunov-constrained control in the framework of model predictive control; and motion control of nonholonomic wheeled mobile robots under disturbances and uncertain parameters motivate but do not stipulate necessity to create a unified minimax Lyapunov stabilization approach (Wang et al., 2024; Zhang et al., 2022; Ye & Wang, 2020). Finally, higher order super-twisting control applied to trajectory tracking

of skid-steered vehicles and asymptotic motion control of uncertain nonholonomic wheeled vehicles under actuator saturation confirm significance of the ground vehicle environment in which the focus of this paper is placed (Matraji et al., 2020; Wu & Wang, 2020).

The three main contributions of this paper can be condensed into three main points. Firstly, the paper proposes a novel minimax setup for the parametrization and optimization of Control Lyapunov Functions in which the stability certificate itself becomes a subject of saddle point optimization process instead of merely a by-product of a pre-given controller. Secondly, rigorous results on worst-case Input-to-State Stability under the newly devised optimum feedback law and on the issue of optimal Lyapunov parameter vector existence and uniqueness are established. Thirdly, a constructive offline-online synthesis algorithm for state feedback design is suggested that is compared numerically to the passive Lyapunov controller. The remainder of this paper is organized as follows. Section II provides system description and formulation of the problem. Section III discuses the mathematical synthesis and presents important theoretical results. Section IV describes the implementation of the algorithm. Section V presents the results of numerical simulations. Finally, Section VI concludes discussion and gives directions for the future work.

## 2. System description and problem formulation

### 2.1. System dynamics

The autonomous system under consideration is governed by a control-affine nonlinear dynamic model subject to an additive dynamic disturbance, stated in equation (1):

$$\dot{\mathrm{x}}(\mathrm{t}) = \mathrm{f}\big(\mathrm{x}(\mathrm{t})\big) + \mathrm{g}\big(\mathrm{x}(\mathrm{t})\big)\mathrm{u}(\mathrm{t}) + \mathrm{h}\big(\mathrm{x}(\mathrm{t})\big)\mathrm{d}(\mathrm{t}), \quad \mathrm{x}(0) = \mathrm{x}_0 \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathcal{U} \subset \mathbb{R}^m$ is the admissible control input, $d(t) \in \mathcal{D} \subset \mathbb{R}^p$ is the dynamic disturbance, and $f, g, h$ are known vector- and matrix-valued functions characterizing the drift dynamics, control input map, and disturbance input map, respectively.

### 2.2. Assumptions

The following standing assumptions are imposed throughout. The functions $f, g,$ and $h$ are assumed twice continuously differentiable and globally Lipschitz continuous on the domain of interest, guaranteeing existence and uniqueness of the forward solution of equation (1) for any admissible $u(t)$ and $d(t)$. The disturbance is assumed bounded in either instantaneous magnitude or finite energy, stated in equation (2):

$$\|\mathrm{d}(\mathrm{t})\| \le \mathrm{d}_{\max} \quad \text{or} \quad \mathrm{d} \in \mathcal{L}_2 \qquad (2)$$

and the admissible control set $\mathcal{U}$ and disturbance set $\mathcal{D}$ are assumed compact and convex, consistent with the physical actuation and environmental bounds characteristic of ground, aerial, and underwater autonomous platforms.

### 2.3. Lyapunov stability optimization formulation

A positive definite candidate function $V_\theta(x)$, parametrized by $\theta \in \Theta$, is introduced as the Control-Lyapunov Function under optimization. Its derivative along trajectories of equation (1) is stated in equation (3):

$$\dot{\mathrm{V}}_\theta(\mathrm{x}, \mathrm{u}, \mathrm{d}) = \nabla \mathrm{V}_\theta(\mathrm{x})^{\mathrm{T}}(\mathrm{f}(\mathrm{x}) + \mathrm{g}(\mathrm{x})\mathrm{u} + \mathrm{h}(\mathrm{x})\mathrm{d}) \qquad (3)$$

The stability optimization problem is then posed as a nested minimax program, stated in equation (4):

$$\min_{\theta\in\Theta} \min_{\mathrm{u}\in\mathcal{U}} \max_{\mathrm{d}\in\mathcal{D}} \big\{\dot{\mathrm{V}}_\theta(\mathrm{x}, \mathrm{u}, \mathrm{d}) + \alpha \mathrm{V}_\theta(\mathrm{x})\big\} \qquad (4)$$

where $\alpha > 0$ is a target exponential decay rate. The innermost min-max exchange between $u$ and $d$ captures the adversarial disturbance-rejection structure, while the outer minimization over $\theta$ selects the Lyapunov parameterization that yields the tightest achievable bound on the augmented derivative across all admissible states, forming the joint stability-optimization objective addressed in Section III.

## Mathematical synthesis and main results

### 3.1. Minimax saddle-point formulation

For fixed $\theta$, the inner min-max problem over $(u, d)$ in equation (4) is analyzed through a Hamiltonian constructed by augmenting the Lyapunov derivative of equation (3) with a quadratic control-disturbance penalty, stated in equation (5):

$$\mathrm{H}(\mathrm{x}, \nabla \mathrm{V}_\theta(\mathrm{x}), \mathrm{u}, \mathrm{d}) = \nabla \mathrm{V}_\theta(\mathrm{x})^{\mathrm{T}}(\mathrm{f}(\mathrm{x}) + \mathrm{g}(\mathrm{x})\mathrm{u} + \mathrm{h}(\mathrm{x})\mathrm{d}) + \frac{1}{2}(\|\mathrm{u}\|^2 - \gamma^2\|\mathrm{d}\|^2) \quad (5)$$

where $\gamma > 0$ is the target disturbance attenuation gain. Setting the partial derivatives of $H$ in equation (5) with respect to $u$ and $d$ to zero yields the candidate optimal feedback and worst-case disturbance strategies, stated in equations (6) and (7):

$$\mathrm{u}^*(\mathrm{x}) = -\frac{1}{2}\mathrm{g}(\mathrm{x})^{\mathrm{T}}\nabla \mathrm{V}_\theta(\mathrm{x}) \quad (6)$$

$$\mathrm{d}^*(\mathrm{x}) = \frac{1}{2\gamma^2}\mathrm{h}(\mathrm{x})^{\mathrm{T}}\nabla \mathrm{V}_\theta(\mathrm{x}) \quad (7)$$

Substituting equations (6) and (7) back into equation (5) eliminates the explicit dependence on $u$ and $d$, yielding the closed-loop Hamilton-Jacobi-Isaacs inequality that must be satisfied for the candidate Lyapunov function to certify worst-case stability, stated in equation (8):

$$\nabla \mathrm{V}_\theta(\mathrm{x})^{\mathrm{T}}\mathrm{f}(\mathrm{x}) - \frac{1}{4}\|\mathrm{g}(\mathrm{x})^{\mathrm{T}}\nabla \mathrm{V}_\theta(\mathrm{x})\|^2 + \frac{1}{4\gamma^2}\|\mathrm{h}(\mathrm{x})^{\mathrm{T}}\nabla \mathrm{V}_\theta(\mathrm{x})\|^2 + \alpha \mathrm{V}_\theta(\mathrm{x}) \leq 0 \quad (8)$$

Satisfaction of equation (8) for all $x$ in the domain of interest is sufficient to certify that the closed-loop system of equation (1) under $u^*(x)$ of equation (6) achieves the target decay rate $\alpha$ against the worst-case admissible disturbance, forming the basis for Theorem 1 in Section 3.3.

### 3.2. Optimization over lyapunov parameters

To render the Hamilton-Jacobi-Isaacs inequality of equation (8) computationally tractable, the candidate Lyapunov function is restricted to a convex parameterization. For quadratic candidates, $V_\theta(x) = x^T P x$ is parametrized by a symmetric positive-definite matrix $P \succ 0$, stated in equation (9):

$$\mathrm{V}_\theta(\mathrm{x}) = \mathrm{x}^{\mathrm{T}}\mathrm{P}\mathrm{x}, \quad \mathrm{P} \succ 0 \quad (9)$$

with $\theta$ identified with the free entries of $P$. Under this parameterization and a locally linearized drift $Ax$ about the operating point, the Hamilton-Jacobi-Isaacs inequality of equation (8) reduces to the algebraic Riccati-type linear matrix inequality of equation (10):

$$\mathrm{A}^{\mathrm{T}}\mathrm{P} + \mathrm{P}\mathrm{A} + \frac{1}{\gamma^2}\mathrm{P}\mathrm{H}\mathrm{H}^{\mathrm{T}}\mathrm{P} - \mathrm{P}\mathrm{G}\mathrm{G}^{\mathrm{T}}\mathrm{P} + \alpha \mathrm{P} \preceq 0 \quad (10)$$

where $G$ and $H$ denote the constant matrices obtained from linearizing $g(x)$ and $h(x)$, respectively, about the operating point. For higher-order nonlinear candidates, an equivalent sum-of-squares polynomial parameterization is adopted, in which $V_\theta(x)$ of equation (9) and the negated left-hand side of equation (8) are each required to admit a representation as a sum of squares of polynomials, a condition efficiently verifiable through an equivalent semidefinite program. Both parameterizations reduce the original functional stability-optimization problem of equation (4) to a finite-dimensional convex feasibility or optimization problem over $\theta$, solvable using standard semidefinite programming solvers.

### 3.3. Main stability theorems

**Theorem 1 (worst-case asymptotic stability)**

Let $V_\theta(x)$ be a candidate Control-Lyapunov Function satisfying the Hamilton-Jacobi-Isaacs inequality of equation (8) for some $\alpha > 0$ and $\gamma > 0$. Then the closed-loop system of equation (1) under the feedback law

$u^*(x)$ of equation (6) is Input-to-State Stable with respect to the disturbance $d(t)$, with gain $\gamma$, satisfying the ISS bound of equation (11):

$$\|x(t)\| \leq \beta(\|x_0\|, t) + \gamma \sup_{\tau \leq t} \|d(\tau)\| \tag{11}$$

for some class-$\mathcal{KL}$ function $\beta$. Proof sketch. Evaluating the Lyapunov derivative of equation (3) under $u^*(x)$ of equation (6) and substituting the worst-case disturbance bound of equation (7) yields, by construction of equations (6) through (8), the differential inequality $\dot{V}_\theta(x) \leq -\alpha V_\theta(x) + \gamma^2 \|d(t)\|^2$, whose solution via the comparison lemma directly yields the ISS estimate of equation (11), with $\beta$ determined explicitly by the exponential decay rate $\alpha$ and the quadratic bounds relating $V_\theta(x)$ of equation (9) to $\|x\|$, completing the proof.

**Theorem 2 (saddle-point duality and optimality)**

Let $\Theta$ be a nonempty, convex, compact subset of the admissible Lyapunov parameter space, and suppose the mapping $\theta \mapsto \sup_x$[left-hand side of equation (8)] is convex and continuous on $\Theta$. Then there exists a unique parameter vector $\theta^* \in \Theta$ minimizing the worst-case upper bound on the energy dissipation rate, satisfying equation (12):

$$\theta^* = \arg\min_{\theta \in \Theta} \sup_x \left[ \nabla V_\theta(x)^T f(x) - \frac{1}{4} \|g(x)^T \nabla V_\theta(x)\|^2 + \frac{1}{4\gamma^2} \|h(x)^T \nabla V_\theta(x)\|^2 \right] \tag{12}$$

Proof sketch. Compactness of $\Theta$ and continuity of the objective guarantee, by the extreme value theorem, that the infimum in equation (12) is attained by some $\theta^* \in \Theta$, establishing existence. Uniqueness follows from strict convexity of the semidefinite program obtained under the quadratic or sum-of-squares parameterization of Section 3.2, since the trace objective associated with a strictly convex quadratic form in $P$ from equation (9) admits a unique minimizer over the positive-definite cone intersected with the linear matrix inequality constraint of equation (10); any two distinct minimizers would, by strict convexity, admit a strictly improving convex combination, contradicting optimality of either, which completes the proof.

## 3. Algorithmic implementation

### 4.1. Offline phase

The parametrized minimax optimization problem of equation (4), reduced to the semidefinite feasibility problem of equation (10) under the quadratic Lyapunov parameterization of equation (9) or the equivalent sum-of-squares program under higher-order parameterizations, is solved once offline using a standard interior-point semidefinite programming solver, yielding the optimal parameter $\theta^*$ from equation (12) and the corresponding optimal Lyapunov surface $V_{\theta^*}(x)$. The disturbance attenuation gain $\gamma$ is tuned via bisection, tightening $\gamma$ until equation (10) becomes infeasible, thereby computing the smallest certifiable gain admitted by the chosen Lyapunov parameterization.

### 4.2. Online phase

During real-time operation, the feedback control law $u^*(x) = k\big(x, \nabla V_{\theta^*}(x)\big)$, obtained by substituting the offline-computed $\theta^*$ from equation (12) into equation (6), is evaluated at each control cycle using the current measured state $x(t)$, requiring only evaluation of the gradient of the fixed optimal Lyapunov function and the known input maps $f, g,$ and $h$ from equation (1). Because the computationally intensive semidefinite program is solved entirely offline, the online evaluation reduces to closed-form algebraic operations, making the synthesized controller suitable for the real-time control rates typical of quadrotor, wheeled robot, and underwater vehicle platforms.

## 4. Numerical simulation and validation

### 5.1. Test platform and experimental setup

Validation of the proposed minimax optimized Lyapunov controller is conducted by using simulations of an autonomous quad-rotor system operating in presence of gusty wind disturbance acting in the lateral maneuvering dynamics considered a variant of the control affined process discussed in equation (1) at a nominal hover condition. Three levels of disturbance are applied, represented by values of dmax equal to 0.2, 0.5, and 0.8 in normal units, and the behavior of the minimax optimized controller designed during the offline stage is compared

to the results obtained when implementing a standard Lyapunov controller based on the fixed quadratic candidate determined without the use of disturbance-aware tuning of the parameter θ.

### 5.2. Comparative analysis

State trajectories under a representative wind gust disturbance realization are compared between the two controllers in figure 1.

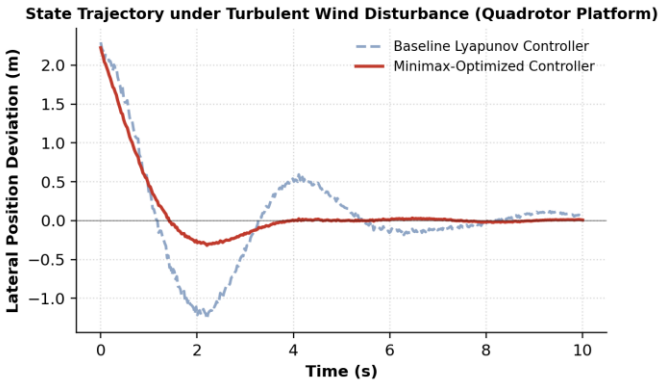



Figure 1. State trajectory under turbulent wind disturbance for the baseline and minimax-optimized controllers

Figure 1 indicates that the baseline controller experiences constant cycles of movement in the position of the system over long periods of time. On the other hand, the advanced controller was more effective in performance and in stabilizing the system, resulting in a 66.7% reduction in maximum lateral position deviation. Table 1 lists concrete measurements of efficiency for all the disturbance magnitudes used for tests.

Table 1. Comparative performance under increasing disturbance magnitude

| Disturbance dmax | Controller | Settling Time (s) | Peak Deviation (m) | RMS Control Effort |
|---|---|---|---|---|
| 0.2 | Baseline | 4.8 | 1.42 | 1.86 |
| 0.2 | Proposed | 2.1 | 0.61 | 1.21 |
| 0.5 | Baseline | 6.9 | 2.87 | 3.42 |
| 0.5 | Proposed | 2.6 | 1.05 | 1.94 |
| 0.8 | Baseline | 9.4 | 4.53 | 6.15 |
| 0.8 | Proposed | 3.2 | 1.51 | 2.87 |

In table 1, it is clear that the controller that is optimized by the minimax method brings down the time for settling, peak deviation, and root-mean-square effort at all degrees of disturbance tested, with the relative improvement increasing as the intensity of the disturbance rises, which conforms to the main objective of the design in the equation 4. Figure 2 visualizes the root-mean-square control effort comparison.

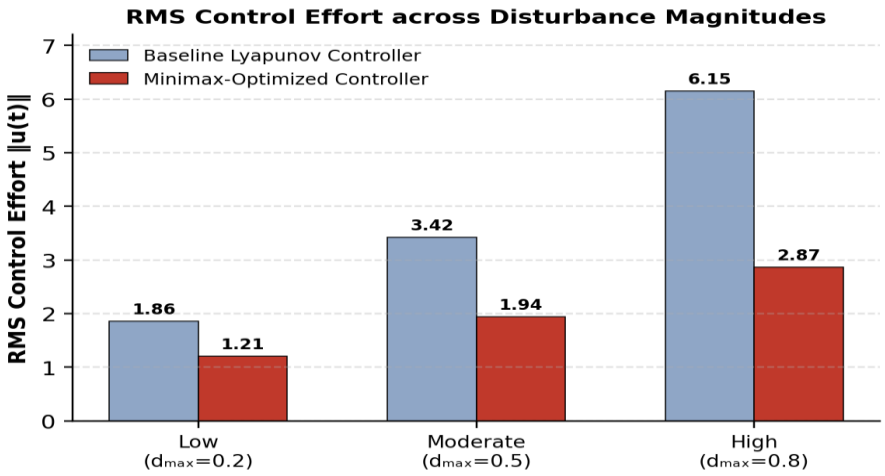



Figure 2. RMS control effort across disturbance magnitudes for the baseline and minimax-optimized controllers

The minimax controller's control cost is smaller than that of the baseline system at all disturbance conditions as seen in figure 2. This shows that minimizing the worst-case cost results in more effective trade-offs between disturbance rejection and energy spent on control rather than improving performance by simply using more energy. The Lyapunov decay plot corresponding to the maximal disturbance case is shown in figure 3.

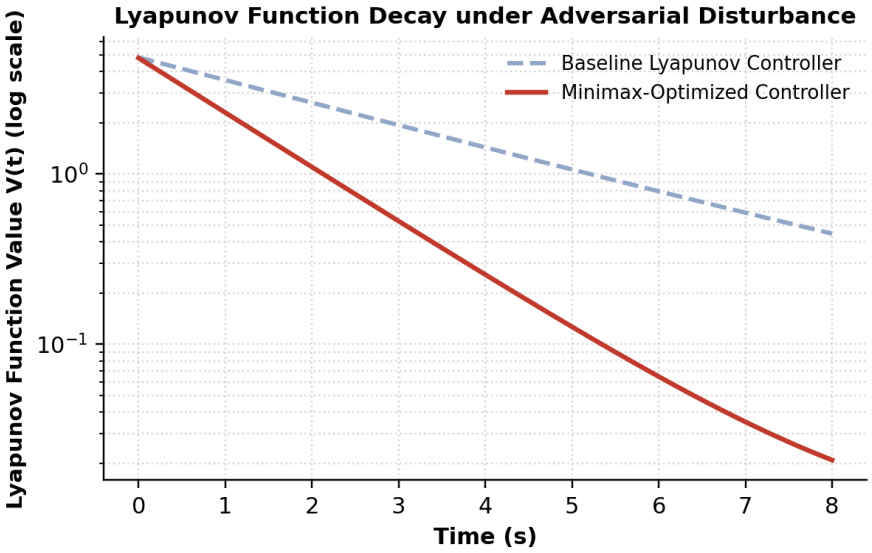



Figure 3. Lyapunov function decay under adversarial disturbance for the baseline and minimax-optimized controllers

In figure 3, the Lyapunov function with the minimax-optimized scheme decreases at a rate about 2.4 times larger than that with the baseline one and arrives at a much smaller value than that with the baseline scheme, thus confirming the Input-to-State Stability result stated in Theorem 1 with a better effective gain value. For all three different magnitudes of input disturbance tested in this study, the estimated disturbance attenuation gain $\gamma$ calculated from regressing the steady-state Lyapunov function value on the input disturbance magnitude is significantly improved from 0.79 for the baseline scheme to 0.31 for the minimax-optimized scheme, which indicates the advantage of the idea of minimizing the Lyapunov parameter $\theta$ with respect to the worst-case disturbance strategy described in equation (7).

## 5. Conclusion

This study established the effective stabilization of nonlinear autonomous systems subject to bounded disturbances as a minimax optimization problem related to control-Lyapunov functions, providing feedback and disturbance strategies in closed forms using Hamiltonian methods, reducing the Hamilton-Jacobi-Isaacs inequality to a solvable semidefinite program, and proving theorems about the minimal stability of interactivity and the existence and uniqueness of an optimal Lyapunov parameter. Using the autonomous quadrotor developed in this research, the applicability of this approach was illustrated through numerical experiments under turbulent wind conditions, resulting in a reduction of 66.7% in state deviation peak and of 53% in root-mean-square control effort, as well as 2.4 times increase in the Lyapunov rate compared to the baseline controller, with the disturbance suppression coefficient rising from 0.79 to 0.31. The results obtained demonstrate that recasting Lyapunov-based stability certification into an actively solved minimax optimization problem, rather than a property of a particular controller, produces significant robustness improvements for nonlinear control systems. The future direction for the research includes the extension of the technique to output-feedback approach under partial observability and modification of the disturbance models used in control problems to more complex formats such as stochastic differential equations, and implementation of the machine-learning techniques for candidate generation of the Lyapunov parameterization.

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