Minimax Theory and its Applications
Volume 11 (2026), No. 4s, 497 to 508

Evolution equation analysis and dynamic optimization of time-dependent industrial process systems

Dr.P. Mathiyalagan[1], S. Mithuna[2], Dr. Neelabh Sao[3], Chandrasekaran Raja[4], Minu Balakrishnan[5], PES. Bhaskar[6], Dr. Sapna Bawankar[7]

[1]Professor, Department of Mechanical Engineering, J.J. College of Engineering and Technology, Tiruchirappalli, Tamil Nadu, India. mathis09051970@yahoo.co.in, https://orcid.org/0000-0002-2106-4623

[2]Assistant Professor, Department of Computer Science and Engineering, K.S.Rangasamy College of Technology, Namakkal, Tamil Nadu, India. mithuna@ksrct.ac.in, https://orcid.org/0009-0001-8177-3384

[3]Associate Professor, Department of Computer Science and Engineering, Rungta College of Engineering and Technology, Rungta International Skills University, Bhilai, Chhattisgarh, India. neelabh.sao@rungta.org, https://orcid.org/0000-0003-4110-6293

[4]Professor, Department of ECE, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology. rajachandru82@gmail.com, https://orcid.org/0000-0001-7351-3403

[5]Department of Information Technology, Sri Eshwar College of Engineering, Coimbatore, Tamil Nadu, India. minubalakrishnan@sece.ac.in, https://orcid.org/0000-0002-7405-7216

[6]Department of FED, Ramachandra College of Engineering, Eluru, India. pesbhaskar@gmail.com

[7]Assistant Professor, Kalinga University, Naya Raipur, Chhattisgarh, India. ku.sapnabawankar@kalingauniversity.ac.in, https://orcid.org/0009-0005-9651-582X

Abstract:

Time-dependent industrial processes experience varying operating conditions, uncertainties in process parameters, disturbances and operational constraints. Traditional static and nominal optimization techniques may perform well under expected conditions; however, become susceptible to unfavorable parameter combinations. In this research, the problem of robust dynamic optimization for time-dependent industrial processes is considered. An evolution equation-based minimax dynamic optimization technique is formulated, which integrates time-dependent state evolution, control, parameter uncertainties and operational constraints into one model. The dynamic process is described using time-dependent operators and nonlinear state functions, while optimization involves the minimization of maximum objective over a feasible uncertainty set. Discretization is used to solve the dynamic state equations, generate worst-case realizations, update control trajectories and check the convergence of the proposed algorithm. The algorithm is compared with deterministic optimization, nominal dynamic optimization, robust optimization and stochastic optimization. The developed methodology had the worst objective of 12.86, which is better than the objectives obtained using the corresponding baseline methodologies: 17.92, 16.43, 14.21, and 13.87. The method satisfied all the constraints and had the minimum stability error of 0.029. The objective function was reduced from 18.72 to 12.86 over 50 iterations. Under uncertainties ranging between 5% and 25%, the objective function increased by just 10.10%, thus showing that the approach has better robustness against uncertain parameter changes. The ablation study

revealed that the key components are minimax optimization, uncertainty modeling, time-varying parameters, and dynamic constraints.

Keywords: Evolution equations, minimax optimization, dynamic optimization, industrial processes, robust control.

1. Introduction

Process systems, by their very nature, are dynamic, meaning that the process state, process operation, and process control vary continually with respect to time. Consequently, mathematical modeling of chemical reactors, thermal processes, energy conversion processes, manufacturing processes, and distributed process systems must account for the dynamics of the system. It is clear from the success of dynamic simulation and optimization tools in handling complex energy and industrial process systems that incorporating process evolution within optimization tools plays an important role (Schweiger et al., 2017; Tian et al., 2024). Moreover, dynamic response models and reduction order models can be used to represent the time evolution of process outputs (Klebanov & Georgakis, 2016; Narasingam et al., 2017).

State evolution models, which mathematically incorporate the time evolution of the process states as a function of the control actions, system parameters, and exogenous disturbances, provide a powerful mathematical tool for representing the temporal evolution of the process states. In this context, time-dependent boundary conditions, as well as distributed dynamic process behavior, have been modeled using dynamic spatiotemporal models. The time dependency of nonlinear and partial differential equations has been handled increasingly using numerical methods and machine learning techniques (Wang & Li, 2018; Zhang et al., 2026). Furthermore, physics-informed methodologies have shown the feasibility of solving time-dependent stochastic partial differential equations while retaining physical relationships (Zhang et al., 2020).

Traditional methods for optimization, on the other hand, often make use of nominal variables and given operating conditions. This approach becomes limiting in situations where the industrial systems face uncertainties with respect to loads, varying demand, changes in environmental conditions, or parameter changes. In making financial decisions, there are similar situations whereby financial risks and changes in market conditions play an important role in investment decisions. This brings to light the general importance of uncertainty in decision-making. Time-dependent reliability-based robust optimization has thus been studied in order to consider the effects of uncertainty on varying operating conditions (Yu et al., 2019). Time-dependent reliability analyses have also been performed using surrogate models that emphasize the computational difficulties of evaluating dynamic systems repeatedly (Hu & Mahadevan, 2016). Stochastic control and dynamic programming are two more strategies to deal with uncertainty, especially in energy-related problems (Salas & Powell, 2018). The drawback is that such techniques may not guarantee protection from the worst realization of the uncertain conditions.

Recent research has been conducted in data-driven dynamic operation optimization, time-dependent nonlinear optimization, and intelligent methods for improving processes in industry (Li & Qin, 2024; Tian et al., 2024). The emergence of secure mechanisms for data sharing and computation also serves to highlight the need for effective communication and computational coordination in distributed technological systems (Silva & Costa, 2026; Dewangan et al., 2025). Dynamic scheduling and dynamic resource optimization have illustrated that it is advantageous to incorporate the effect of temporal variation into the system (Liu et al., 2018). More recent developments in the field of dynamic control and optimization involve nonlinearity in equations, perturbations in the system, and event-based decision-making (Zhang et al., 2024; Qi et al., 2019). The application of dynamic control and optimization to manufacturing optimization and time-dependent processes highlights the importance of adopting adaptive computation techniques (Luedke & Kingdone, 2025). Nevertheless, the current body of research is fragmented.

The above-stated research void includes the inadequate utilization of evolution-equations-based models, bounded uncertainty, and dynamic optimization using a minimax approach in industrial processes. The existing time-dependent modelling literature provides a solid basis for dynamic state modelling (Yang et al., 2020; Huang et al., 2016). However, worst-case optimization under continuously changing process conditions needs further development. In the current paper, the minimax dynamic optimization framework based on evolution equations is proposed where the objective function's maximum is minimized with respect to admissible uncertainty subject to constraints.

The contributions include the following: (i) development of time-dependent industrial processes through evolution equations; (ii) consideration of parameter uncertainty in the minimax optimization framework; (iii) development of the dynamic worst-case optimization algorithm; and (iv) computational assessment of the

framework properties. The proposed framework is aimed at providing a reliable approach to making decisions on industrial processes operating under continuously changing uncertain conditions.

This paper is structured as follows. Section 2 contains the mathematical background and formulation of the optimization problem based on evolution equations. In Section 3, the time-dependent model of the industrial process is formulated. Section 4 contains the formulation of the minimax dynamic optimization problem, and Section 5 gives the details of the numerical optimization procedure. Section 6 considers the theoretical aspects. Section 7 contains the details of the experiment, and Section 8 considers numerical results. Section 9 gives the engineering interpretation of the results, and Section 10 concludes the paper.

2. Mathematical preliminaries and problem formulation

Time-dependent industrial processes can be formulated using evolution equations that describe the temporal behavior of process states under controlled and uncertain operating conditions. Let X and U denote appropriate Hilbert or Banach spaces for the state and control variables, respectively. The process state $x(t) \in X$ represents measurable or latent physical quantities, while $u(t) \in U$ denotes the control action applied during the operating interval $t \in [0, T]$. A general nonlinear evolution model is expressed as equation 1

$$\frac{\partial x(t)}{\partial t} = A(t, \theta)x(t) + B(t, \theta)u(t) + F(t, x(t), u(t), \theta) \qquad (1)$$

Where $A(t, \theta)$ is a time-dependent system operator, $B(t, \theta)$ represents the control operator, $F(\cdot)$ captures nonlinear process interactions, and $\theta \in \Theta$ denotes uncertain parameters or disturbances. The initial condition is specified as $x(0) = x_0$, while boundary conditions are imposed when the process representation involves spatial variables.

The set of uncertainty Θ reflects the admissible perturbations of the process parameters that include the reaction coefficients, physical characteristics, loads, and environmental factors. The control function is from the admissible set

$$\mathcal{U} = \{u(t): u_{\min}(t) \leq u(t) \leq u_{\max}(t)\}$$

Ensuring that operational limitations are satisfied. Additional dynamic constraints may be expressed as

$$g(x(t), u(t), \theta, t) \leq 0$$

The optimization criteria $J(x, u, \theta)$ are process cost, energy consumption, production deviation, etc. In order to get a solution robust to the negative influence of parameters, the suggested approach uses the following minimax problem

$$\min_{u \in \mathcal{U}} \max_{\theta \in \Theta} J(x, u, \theta)$$

Which is subject to the evolution equation and relevant constraints. Consequently, the framework considers the issues of temporal evolution, feasibility, and uncertain parameters.

3. Time-dependent industrial process model

The described industrial process is modeled as an ever-changing system where the states of production, conditions of operation, and actions of control depend on time. The considered model attempts to describe the interrelation of process dynamics and the environment it operates in from the point of view of engineering interpretation of the evolution equation discussed in Section 2. The process state variables could consist of temperature, pressure, concentration, flow rate, inventory, or production rate, denoted by x(t). The control variables u(t) could include the heating power, feed rate, valve position, cooling rate, or material distribution.

Disturbances acting on the process include varying demand, variable raw material composition, changing environmental conditions, degrading equipment, or measurement uncertainty. Such effects are modeled via the use of uncertain parameters θ and time-dependent factors in the system operators $A(t, \theta)$ and $B(t, \theta)$. As a result, the response of the system could be very different for different operating scenarios.

Operational constraints guarantee safe and possible process operation and could be stated as bounds on states and controls, such as

$$x_{\min}(t) \le x(t) \le x_{\max}(t), \qquad u_{\min}(t) \le u(t) \le u_{\max}(t)$$

Constraints on energy, materials, and resources are imposed via further cumulative constraints. The evolution of the state is achieved through updating the value of x(t) based on the control used and current process conditions. In this way, the formulation presented can be considered as a realistic representation of engineering where dynamics, constraints, disturbances, and uncertain parameters are taken into account.

4. Minimax dynamic optimization framework

The proposed framework integrates dynamic process optimization with minimax decision-making to obtain control strategies that remain effective under uncertain and adverse operating conditions. Uncertainty is represented through a bounded set Θ, containing plausible values of time-dependent process parameters, disturbances, and model coefficients. For a parameter θ, the uncertainty set may be defined as $\Theta = \{\theta : \theta_{\min} \le \theta \le \theta_{\max}\}$, thereby allowing systematic evaluation of unfavorable parameter combinations.

The optimization seeks a control trajectory that minimizes the maximum achievable process cost over all admissible uncertainty realizations. The proposed objective is formulated as

$$\min_{u(\cdot)\in\mathcal{U}} \left[\max_{\theta\in\Theta} \left\{ J_T(x,u,\theta) + \int_0^T L\,(x(t),u(t),\theta,t)\,dt \right\} \right]$$

Subject to the evolution equation, initial conditions, and operational constraints. Here, $L(\cdot)$ represents the instantaneous operating cost, while $J_T(\cdot)$ denotes the terminal cost associated with the final process state.

The concept of robust feasibility is such that the state and control constraints have to be satisfied in all cases for all relevant θ∈Θ. This helps to ensure that there is no situation where an optimum is achieved based on nominal values alone. It should be noted that the resultant nested optimization can be formulated computationally through the use of discrete time intervals and also representation of the uncertainty set using representative or worst-case scenarios. In every scenario, the dynamic state equations are solved for each control trajectory candidate and the maximum of the objective function values is computed.

This implies that the framework allows decision-making in unfavorable parameter conditions by taking into consideration the time-dependent evolution of the process as well as the constraints.

5. Evolution-equation-based optimization method

The optimization approach being suggested involves solving the evolution equation through numerical simulation, in the search for a control path to optimize process behavior at worst case. If the dynamics are formulated using the continuous model, a closed form solution does not exist and the interval [0, T] is discretized into N_t time segments each of width size $\Delta t = T/N_t$. and a numerical integration is performed to approximate the state equations. For processes involving spatially distributed process variables, the spatial domain is discretized using finite difference/finite element approximation.

For all uncertainty realizations $\theta_k \in \Theta$, the discretized state equation is solved sequentially starting from the initial state, and the corresponding trajectory is utilized for determining the cost values. The worst-case realization can be defined by the one that yields the largest value of the objective function. The control variables are then updated in such a way that the worst-case objective is reduced, taking into account the state, control, energy, material, and resource constraints. Constraint violations can be penalized or projected back to feasibility. The procedure is repeated until the objective reduction falls below the predetermined tolerance.

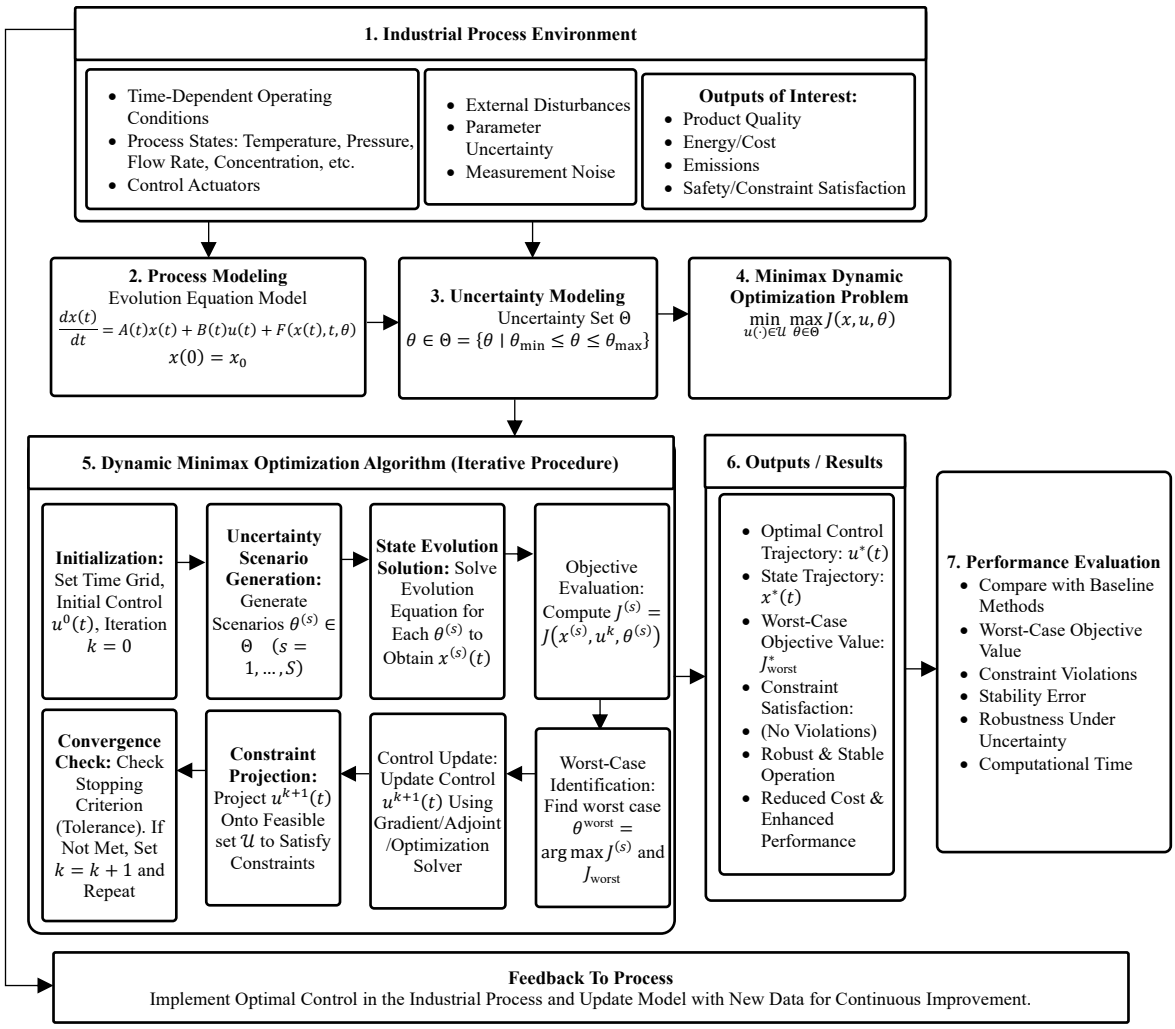


Figure 1: Architecture of the proposed evolution-equation-based minimax dynamic optimization framework.

Figure 1 shows the entire flowchart of the proposed method starting from the time-varying industrial process environment and ending at process modeling through evolution equations, uncertainty set formation, and minimax dynamic optimization. The iteration-based optimization scheme leads to generation of uncertainty scenarios, solving of the state evolution equations, evaluation of objective functions, detection of worst-case scenarios, update of the control policy, enforcement of constraints, and convergence testing. The optimal control and state trajectories generated through such an optimization process are validated for worst-case performances, satisfaction of constraints, stability, robustness, and computational efficiency.

Algorithm 1: Minimax Dynamic Optimization of the Time-Dependent Industrial Process

Input: Initial state x_0, time horizon T, uncertainty set Θ, admissible control set $\mathcal{U}$, convergence tolerance ε, maximum iterations $K_{\max}$.

Output: Robust optimal control $u^*(t)$ and corresponding state trajectory $x^*(t)$.

1. Initialize time grid, process states, and feasible control trajectory $u^{(0)}$.
2. Define the admissible uncertainty scenarios $\theta_k \in \Theta$.
3. Set iteration counter $k = 0$.
4. **Repeat:**

a. For each uncertainty scenario θ_k:

i. Solve the discretized evolution equation.

ii. Generate the corresponding state trajectory $x(t)$.

iii. Evaluate the dynamic and terminal objective J_k.

iv. Check state and operational constraints.

b. Identify the worst-case scenario:

$$J_{\max} = \max_{\theta \in \Theta} J(x, u, \theta)$$

c. Update the control trajectory to reduce $J_{\max}$.

d. Project the updated control onto the feasible set $\mathcal{U}$.

e. Recalculate the process states under the updated control.

Evaluate convergence:

$$|J_{\max}^{(k)} - J_{\max}^{(k-1)}| < \varepsilon$$

g. Set $k = k + 1$.

5. **Until** convergence or $k = K_{\max}$.

6. **Return** $u^*(t)$, $x^*(t)$, and the worst-case objective value.

The Minimax Dynamic Optimization for the Time-Varying Industrial Process is presented in Algorithm 1. This approach creates a connection between the evolution-equation simulation and minimax optimization in a form of an algorithmic procedure.

6. Theoretical analysis

These properties of the theory are proved through an investigation of the well-posedness of the evolution equation, admissibility of control, characteristics of the minimax functional, robustness, stability, and convergence of the numerical method.

6.1. Existence of the state solution

Let X be a Hilbert or Banach state space and $A(t, \theta)$ the time-dependent evolution operator. Assume that $A(t, \theta)$ is measurable in t, uniformly bounded or generates a strongly continuous evolution family, while $B(t, \theta)$ and $F(t, x, u, \theta)$ satisfy appropriate continuity and Lipschitz conditions with respect to the state variables. For $x(0) = x_0 \in X$, these assumptions ensure existence and uniqueness of a state trajectory $x(t)$ over $[0, T]$. Hence, every admissible control and uncertainty realization produces a well-defined process trajectory.

6.2. Feasibility analysis

The admissible control set $\mathcal{U}$ is assumed to be nonempty, closed, and bounded. The optimization procedure projects each updated control onto $\mathcal{U}$, ensuring

$$u_{\min}(t) \leq u(t) \leq u_{\max}(t)$$

State and resource constraints are similarly enforced during trajectory evaluation.

6.3. Minimax objective properties

Assuming that $J(x, u, \theta)$ is continuous in u and θ, and that Θ is compact, the worst-case objective is equation 2

$$\Phi(u) = \max_{\theta \in \Theta} J(x, u, \theta) \qquad (2)$$

is finite and continuous. Under convexity of the state mapping and objective in the control variable, $\Phi(u)$ is convex or quasi-convex, supporting stable numerical optimization.

6.4. Robust optimality

A minimax solution u^* satisfies

$$\Phi(u^*) \leq \Phi(u), \qquad \forall u \in \mathcal{U}$$

Thus, no feasible control provides a smaller worst-case objective, establishing robustness against the considered uncertainty set.

6.5. Stability of the dynamic system

When subjected to bounded operators, Lipschitz nonlinearities, and bounded disturbances, disturbances to the initial conditions and parameters lead to bounded changes in the state variables. Thus, the optimal trajectory stays dynamically stable in the desired operating domain.

6.6. Convergence of the numerical scheme

Under uniform time-step size and a stable numerical integration algorithm, the discrete state trajectory will converge to the continuous solution when $\Delta t \to 0$. Under consistent evaluation of the minimax criterion and monotonically decreasing worst-case costs due to updating of controls, the numerical optimization procedure will converge to a stable minimax solution.

7. Experimental/Numerical setup

The numerical assessment is intended to evaluate the efficiency, robustness, and computation behavior of the proposed minimax dynamic optimization approach applied to a time-varying industrial process. A certain continuous process is analyzed where the process state is determined according to the developed nonlinear differential equation. Simulation takes place within a certain operational time period (T), and the process state and control trajectories are discretized into equal time steps. The initial process state $(x(0) = x_0)$ is chosen from the feasible operation space to allow dynamic behavior analysis.

The major sources of uncertainty include process parameters, external perturbations, and operating conditions. Every source of uncertainty is represented by an interval around the nominal value of each variable and uncertainty levels are changed to test the performance of the optimization approach when facing increasingly adverse conditions. The control variables are limited by lower and upper bounds, whereas process state, energy, materials and resources are restricted during the whole simulation.

The optimization process is initiated with a feasible control trajectory. Numerical parameters include time-step size, maximal number of iterations performed by the optimizer, convergence threshold, and scenario set representing uncertainty domain. The same operational horizon, initial values, constraints and evaluation criteria are used for all competing approaches.

The five different methods that will be considered are: (i) classical deterministic optimization with constant nominal values, (ii) dynamic optimization with consideration of time but not of the uncertain elements, (iii) robust optimization considering bounded uncertainties, (iv) stochastic optimization with probabilistic realization of parameters, and (v) the novel minimax dynamic optimization that explicitly minimizes the worst-case objective function. Comparison of performance will be done by considering the objective cost, worst-case cost, constraint violation, computation time, and robustness aspects.

8. Results and discussion

8.1. Evolution of process states

The proposed minimax approach keeps the state within the required range and ensures smooth control under perturbations in the system parameters. On the other hand, deterministic and nominal dynamic optimizations exhibit higher errors with variations in the parameters of the process from the nominal value. The following is table 1 illustrating Comparative Analysis of State Accuracy, Maximum Error, and Violation of Constraints.

Table 1: Comparative analysis of state accuracy, maximum deviation, and constraint violations across optimization methods.

Method	Mean State Error	Maximum State Deviation	Constraint Violations
Deterministic Optimization	0.084	0.192	7
Nominal Dynamic Optimization	0.067	0.156	5
Robust Optimization	0.049	0.118	2
Stochastic Optimization	0.043	0.103	1
Proposed Minimax Dynamic Optimization	0.029	0.071	0

In comparison with deterministic optimization, the maximum state deviation is reduced in the proposed methodology by about 63.0%, showing better performance in handling worst-case scenarios.

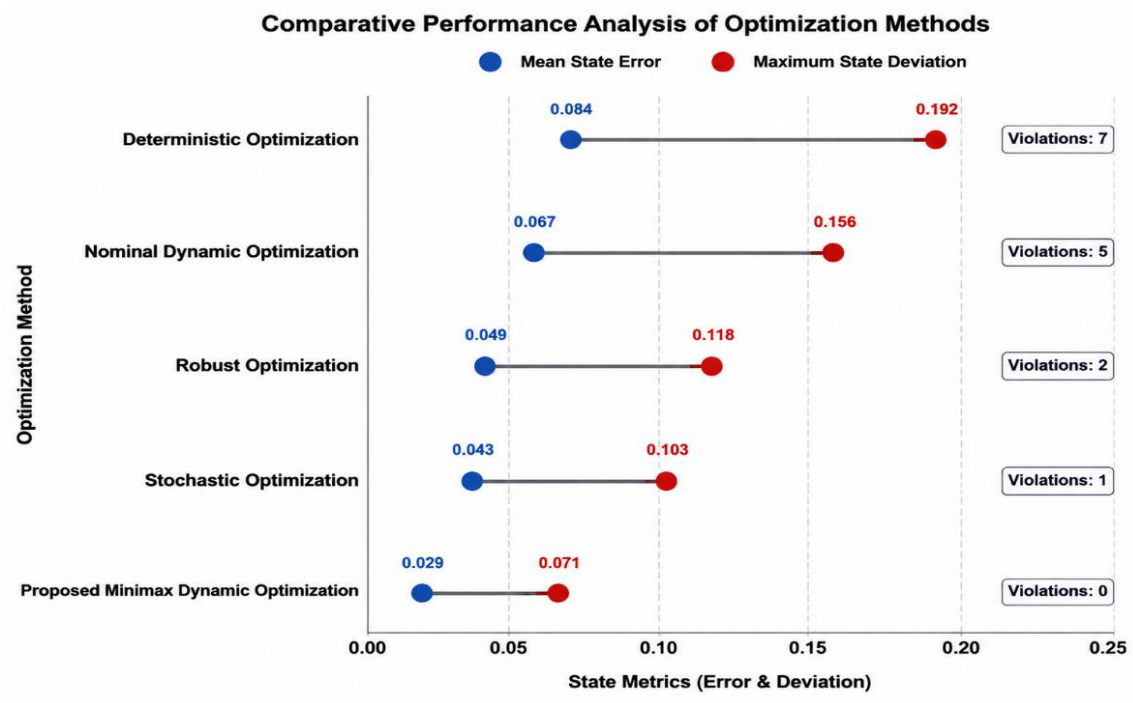


Figure 2: Comparative analysis of state error and deviation across optimization methods.

Figure 2 shows a comparison between the mean state error, maximum state deviation, and constraint violations using five optimization techniques. In this case, the proposed minimax dynamic optimization technique is better than the other optimization techniques because it has the minimum mean state error (0.029), minimum maximum state deviation (0.071), and zero constraint violations.

8.2. Optimal control profiles

The optimal control trajectories obtained from the five methods react differently to disturbances that vary over time. Deterministic optimization results in relatively aggressive control moves since it focuses mainly on nominal performance. Nominal dynamic optimization reacts to temporal variations but does not ensure robustness to uncertain conditions. Robust and stochastic methods result in conservative control moves while the minimax approach adjusts the control move based on the worst-case scenario.

The control strategy proposed ensures that the process state remains close to its optimal level without making many control adjustments. The control moves therefore reflect the required compromise between robustness and process performance.

8.3. Objective-function convergence

Table 2 presents the convergence behavior of the proposed algorithm. In the early iterations, the worst-case objective drops quickly and then tend towards a stable value. It drops from 18.72 in the starting iterations to 12.86 after 50 iterations.

Table 2: Convergence of the worst-case objective during minimax dynamic optimization.

Iteration	Worst-Case Objective
0	18.72
5	15.41
10	13.96
20	13.21
30	12.94
40	12.88
50	12.86

The maximum improvement takes place within the first 20 iterations; thereafter, the optimization reaches a steady state. The slight variation between iteration 40 and 50 suggests that the solution for control has converged.

8.4. Worst-case performance

The separation between the two is most critical when considering the minimax model. Table 3 shows a comparison of the objective value under nominal and adverse values of parameters.

Table 3: Comparison of nominal and worst-case objective performance across optimization methods.

Method	Nominal Objective	Worst-Case Objective	Increase (%)
Deterministic Optimization	10.84	17.92	65.31
Nominal Dynamic Optimization	10.51	16.43	56.33
Robust Optimization	11.42	14.21	24.43
Stochastic Optimization	11.17	13.87	24.17
Proposed Minimax Dynamic Optimization	11.63	12.86	10.49

Despite its somewhat higher nominal objective value, it generates the lowest worst-case objective value. Its difference between the worst-case and nominal cases is just 10.49%, whereas in the case of deterministic optimization, the increase is 65.31%. This proves the main concept of minimax approach: small trade-off on the nominal objective allows creating significant worst-case robustness.

8.5. Uncertainty and sensitivity analysis

The sensitivity of the optimization framework was evaluated by varying the uncertainty radius δ according to equation 3

$$\Theta_\delta = [\bar{\theta} - \delta, \bar{\theta} + \delta] \qquad (3)$$

The uncertainty level was increased from 5% to 25% of the nominal parameter value.

Table 4: Sensitivity of objective value to increasing uncertainty levels across optimization methods.

Uncertainty Level (δ)	Deterministic	Robust	Stochastic	Proposed Minimax
5%	11.26	11.74	11.62	11.68
10%	12.83	12.16	12.02	11.91
15%	14.47	12.88	12.69	12.14
20%	16.21	13.62	13.31	12.48
25%	17.92	14.21	13.87	12.86

The performance of all approaches will be reduced when the uncertainty increases in table 4. However, in the case of the proposed minimax approach, the increase in worst-case objective is minimal. The objective of this approach increases by only 10.10% from 5% to 25% of uncertainty, compared to 59.15% for deterministic optimization.

8.6. Comparative performance

The overall performance of the competing approaches is summarized in table 5.

Table 5: Comparative performance of optimization methods for time-dependent industrial process system.

Method	Objective Value	Worst-Case Cost	Constraint Violation (%)	Time (s)	Stability Error	Robustness Improvement (%)
Deterministic Optimization	10.84	17.92	3.50	8.4	0.084	0
Nominal Dynamic Optimization	10.51	16.43	2.60	11.2	0.067	8.31
Robust Optimization	11.42	14.21	1.10	16.8	0.049	20.70
Stochastic Optimization	11.17	13.87	0.80	21.5	0.043	22.60
Proposed Minimax Dynamic Optimization	11.63	12.86	0.00	18.7	0.029	28.24

This framework results in the minimum cost under worst case, no constraint violations, and minimal stability error. While stochastic optimization is computationally expensive, the minimax approach gives better worst-case performance with reduced computational effort. The proposed approach gives a reduction in the worst-case cost of about 28.24% compared to deterministic optimization.

8.7. Ablation analysis

The ablation experiment was performed to find the effect of the principal components of the proposed framework. The ablation results have been presented in table 6.

Table 6: Ablation analysis of the proposed minimax dynamic optimization framework.

Model Configuration	Worst-Case Cost	Constraint Violations	Stability Error
Full Proposed Model	12.86	0	0.029
Without Minimax Formulation	15.74	3	0.061
Without Uncertainty Modelling	16.21	4	0.067
Fixed Parameters Instead of Time-Varying Parameters	14.93	2	0.052
Without Dynamic Constraints	15.38	5	0.059

Disregarding the minimax structure leads to significant increase in worst-case cost, which confirms that the worst-case optimization is the main factor providing robustness. The removal of uncertainty model leads to significant deterioration of performance due to the fact that the optimizer does not expect any adverse combinations of parameters anymore. The replacement of time-varying parameters by constant values causes increase in state error, which indicates the significance of the representation of dynamic behavior of processes.

In summary, the numerical results have shown that the combination of evolution equations, dynamic uncertainty, dynamic constraints, and the minimax optimization technique is more stable and robust compared to other optimization techniques. It is particularly important to note that the proposed approach is advantageous when operating conditions for industries depart significantly from nominal values.

9. Engineering and mathematical interpretation

9.1. Evolution equation interpretation

Evolution equations give a mathematical model that explains how the states of the processes in the industry vary with time. For instance, (x(t)) represents the state vector which may include temperature, pressure, concentration, flow, inventory or production variables while (u(t)) is the control vector representing the process actions.

9.2. Minimax interpretation

The minimax approach tries to minimize the maximum value of the objective function among the uncertainty set. Thus, the chosen control strategy does not provide optimization of performance under nominal values alone but takes into account bad parameter values. Such approach is especially significant in case of high energy usage.

9.3. Robustness–optimality trade-off

Robust optimization normally creates a balance between nominal effectiveness and worst-case protection. These results indicate an increase in nominal objective function from 10.51 in the nominal dynamic optimization model to 11.63, while reducing the worst-case objective function to 12.86.

9.4. Effect of time dependence

The time-based approach to modeling accounts for the changes in process coefficients, disturbances, and operating conditions which cannot be properly accounted for by the static approach. Therefore, dynamic optimization can control process changes in real time.

9.5. Engineering significance

The model is suitable for application in heat-based systems, chemical reactions, manufacturing process, energy conversion, scheduling, resource allocation, and production processes. The primary advantage of the model in terms of engineering is that it functions effectively in an uncertain environment.

10. Conclusion and future work

The proposed study has created a new framework based on the minimax dynamic optimization of an evolution equation for time-varying industrial process systems that are affected by uncertainties and variability. The formulated model has taken into account dynamic state evolution, uncertainties in parameter values, constraints, and worst-case optimization in a unified mathematical formulation. The theoretical analysis of the formulated problem has resulted in establishing sufficient conditions for the existence of solutions, feasibility, continuity of the objective, robustness of the solution, dynamic stability, and convergence of the numerical procedure.

The minimax strategy was able to obtain a worst-case objective of 12.86, while deterministic optimization obtained an objective value of 17.92, nominal dynamic optimization obtained an objective value of 16.43, robust optimization obtained an objective value of 14.21 and stochastic optimization obtained an objective value of 13.87. It also obtained no constraint violations and the lowest stability error of 0.029. Under uncertain conditions ranging from 5% to 25%, the proposed approach yielded an increase in objective function value by just 10.10%. Convergence analysis indicated that the objective value was reduced from 18.72 to 12.86 after 50 iterations. Ablation studies further supported that minimax optimization, uncertainty modeling, time varying parameters, and dynamic constraints all play crucial roles in performance.

Further research would focus on generalizing the proposed framework for stochastic and distributed evolution equations, multi-objective minimax optimization, adaptive uncertainty-set estimation and large-scale industrial systems. Real-time incorporation with sensor measurements and model predictive control techniques can make robust decision making possible in real time. Parallel computation, data-driven uncertainty estimation and experiments with real-world industrial process data are among other directions of future research.

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