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Interval mathematical programming for robust engineering design under parameter and environmental uncertainty

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Abstract:

Engineering design optimization problems often exhibit uncertainty associated with material properties, loads, operating conditions, temperature, and environmental effects, requiring robustness beyond deterministic optimization to guarantee performance reliability. Probabilistic methods can be utilized to handle uncertainty; however, the applicability of these methods is subject to the availability of probability distributions, which might not exist if there are insufficient number of observations or the uncertainty is mainly of epistemic nature. This paper presents an Interval-Minimax Mathematical Programming Framework to robustly design engineering components in the presence of uncertainty in engineering parameters and environment. In the suggested methodology, uncertain engineering parameters are modeled by bounded intervals and a common uncertainty set consisting of parameter, environment, and operation uncertainty is defined. Using the minimax formulation, optimization is performed such that the worst possible objective value is minimized and robust feasibility requirements are guaranteed for all possible uncertainties in the uncertainty set. The nested optimization problem is then reformulated through a mathematical programming model based on epigraph approach and a procedure for robust design optimization under consideration of uncertainty is provided. The numerical analysis takes into account cases with larger uncertainty intervals and various environmental situations, whereas the comparative assessment is carried out relative to deterministic, nonlinear, interval, classical robust, and traditional minimax optimization methodologies. The findings show that the proposed methodology greatly enhances worst-case performance and constraints validity in spite of a well-balanced

compromise between robustness and optimality. The developed interval-minimax approach guarantees zero violations of constraints and provides greater safety margin compared to the competing methodologies, though at the cost of additional computational complexity due to worst-case evaluation.

Keywords: Interval mathematical programming, minimax optimization, robust engineering design, parameter uncertainty, environmental uncertainty.

1. Introduction

Mathematical programming is used as a means of systematically establishing the optimal design variables subject to the constraints on performance, safety, cost, and resources. Engineering systems in practice, however, face uncertainties due to differences in material properties, production environment, loads, temperatures, exposure to the environment, and many other external factors. Uncertainties of such kind have the potential of impacting greatly on the performance and feasibility of the optimized design. Deterministic optimization, as applied conventionally, normally makes use of the nominal values of the parameters, which results in poor performance when there are deviations from those values in actuality. While probabilistic optimization allows incorporating the uncertainty distributions, reliable probability distribution may not always be available in situations where experimental observations are difficult and expensive to obtain.

However, interval-based methods available in the literature might not sufficiently consider the interaction of multiple uncertain parameters in conjunction with the worst case in the environment. It is necessary to use the combination of interval-based uncertainties and the minimax methodology for obtaining solutions which are optimal and reliable in the worst case. This research work attempts to develop an interval-based mathematical programming approach to robust engineering design under combined parameter and environmental uncertainties. The goals of this work are to define interval-valued design constraints, develop a minimax worst case objective, develop a robust solution approach, and perform uncertainty level sensitivity analysis.

The rest of the paper is organized as follows. Section 2 includes related works while Section 3 gives the mathematical background and interval problem formulation. Section 4 proposes the interval-minimax framework. Section 5 addresses the solution approach and the convergence procedure. Section 6 considers the engineering application and numerical experiment. Section 7 discusses the experimental results, whereas Section 8 provides the comparative analysis. Section 9 addresses the theoretical and engineering interpretation, limitations, and implications. Finally, Section 10 concludes the paper.

2. Literature review

Engineering design mathematical programming problems in which there are uncertain parameters have recently attracted increasing attention significantly influence optimality, feasibility, reliability, and performance. The traditional deterministic modeling might not be adequate for engineering design in cases when the parameters differ from their nominal values, so it has led to the emergence of robustness, interval, stochastic, and mixed approaches to uncertainty modeling (Dias & Ierapetritou, 2016; Acar et al., 2021; Lelievre et al., 2016). Moreover, the growing focus on sustainability and green practices highlights the necessity to take into consideration environmental aspects during development and operations assessment (Oliveira & Costa, 2023). The interval uncertainty model is especially applicable in case when the upper and lower uncertainty limits are known, but the probabilities of occurring of each value are hard to determine. One approach has been developed based on the use of intervals for robust design optimization in the case of parameter uncertainty (Ouyang et al., 2016). Additional research has been done with regard to the uncertainty of the hybrid probabilistic and interval kind for structural engineering (Cheng et al., 2021). The interval uncertainty model has also been explored in the case of dynamic engineering systems and rotating machinery, and it proved to be helpful in assessing the behavior of systems subjected to uncertain bounded parameters (Sindhu, 2023). Random and interval uncertainties have been included into engineering design optimization frameworks for multidisciplinary optimization, and probability-interval methods have been introduced for solving problems with both kinds of uncertainties (Meng et al., 2021; Jiang et al., 2018). The recent analysis based on probability has further emphasized the importance of the probabilistic approach to deal with uncertainties and analyze system performance (Patil et al., 2026).

Another significant approach involves robust optimization with solutions designed to be protected against adverse realizations of the uncertainties (Kammammettu & Li, 2020; Bairamzadeh et al., 2016; Goli et al., 2023). Similarly, risk management literature has underscored the significance of considering adverse scenarios where uncertainty can have significant effects on the performance of systems (Menon & Nair, 2026). nterval, stochastic, and mixed approaches to uncertainty modeling (Dias & Ierapetritou, 2016; Acar et al., 2021; Lelievre et al.,

2016)Robust fuzzy and stochastic formulations have additionally shown their capability of dealing with several sources of uncertainties where deterministic assumptions do not apply (Fazli-Khalaf et al., 2017; Gazijahani & Salehi, 2017). There have been also efforts made towards reliability-based designs where uncertain parameters were included in the distribution intervals in order to connect uncertainty modeling with reliability needs (Huang et al., 2017). The overall body of design literature stresses the importance of dealing with optimization, reliability, and robustness together instead of analysing each separately (Lelievre et al., 2016; Hatefi et al., 2015). Some researchers have studied uncertainties in scheduling and control problems, decisions on environmental waste management, logistics networks design, and project portfolio selection via mathematical programming and simulation techniques (Dias & Ierapetritou, 2016; Singh, 2019; Mavrotas & Makryvelios, 2021). Modern developments in engineering optimization area have also highlighted the significance of multi-objective and particular application-based robust design approaches (Ugli & Ugli, 2026). However, there is no sufficiently developed interval-minimax mathematical programming model which can deal with both parameter and environmental uncertainties and minimize worst-case objective of an engineering problem at the same time. That is why this study was conducted.

3. Mathematical preliminaries and problem formulation

3.1. Interval arithmetic

In practical engineering systems, uncertain parameters are often known only within specified bounds. An uncertain parameter $\grave{p}_i$ is therefore represented by the interval

$$\grave{p}_i = [p_i^L, p_i^U]$$

Where p_i^L and p_i^U denote the lower and upper bounds, respectively. The interval width $p_i^U - p_i^L$ represents the magnitude of uncertainty.

3.2. Interval-Valued engineering parameters

The material property, operating condition, loading condition, and system property can be represented by interval parameter values. The entire uncertainty region is formulated as equation 1.

$$\mathcal{P} = \prod_{i=1}^{q} [p_i^L, p_i^U] \qquad (1)$$

Where q denotes the number of uncertain parameters. This representation avoids requiring precise probability distributions.

3.3. Engineering design variables

Let the engineering design be characterized by the vector in equation 2

$$\mathrm{x} = (x_1, x_2, \dots, x_n)^T \in \mathcal{X} \qquad (2)$$

Where $\mathcal{X}$ denotes the feasible design space.

3.4. Deterministic engineering optimization model

The conventional deterministic design problem is expressed as

$$\min_{\mathbf{x} \in \mathcal{X}} f(\mathrm{x}, \mathrm{p})$$

Subject to

$$g_j(\mathrm{x}, \mathrm{p}) \leq 0, \quad j = 1, \dots, m$$

Where f is the engineering objective and g_j represents the j-th design constraint.

3.5. Interval-uncertain optimization model

Under uncertainty, p is replaced by the interval vector $\grave{\mathrm{p}} \in \mathcal{P}$. Consequently, both objective and constraint performance may vary across admissible realizations.

3.6. Robust feasibility

A design is considered robustly feasible when

$$g_j(\mathrm{x}, \mathrm{p}) \leq 0, \quad \forall \mathrm{p} \in \mathcal{P}$$

3.7. Minimax robust objective

To protect against the most adverse uncertainty realization, the worst-case objective is defined as equation 3

$$F(\mathbf{x}) = \max_{\mathbf{p} \in \mathcal{P}} f(\mathbf{x}, \mathbf{p}) \qquad (3)$$

Thus, the proposed robust engineering problem is

$$\min_{\mathbf{x} \in \mathcal{X}} \max_{\mathbf{p} \in \mathcal{P}} f(\mathbf{x}, \mathbf{p})$$

Subject to

$$g_j(\mathbf{x}, \mathbf{p}) \leq 0, \quad \forall \mathbf{p} \in \mathcal{P}, \quad j = 1, \ldots, m$$

This formulation simultaneously incorporates interval uncertainty, robust feasibility, and minimax optimization.

4. Proposed interval-minimax mathematical programming framework

4.1. Overall framework

The suggested framework combines interval uncertainty with minimax mathematical programming to achieve designs which are still feasible and effective under adverse working conditions.

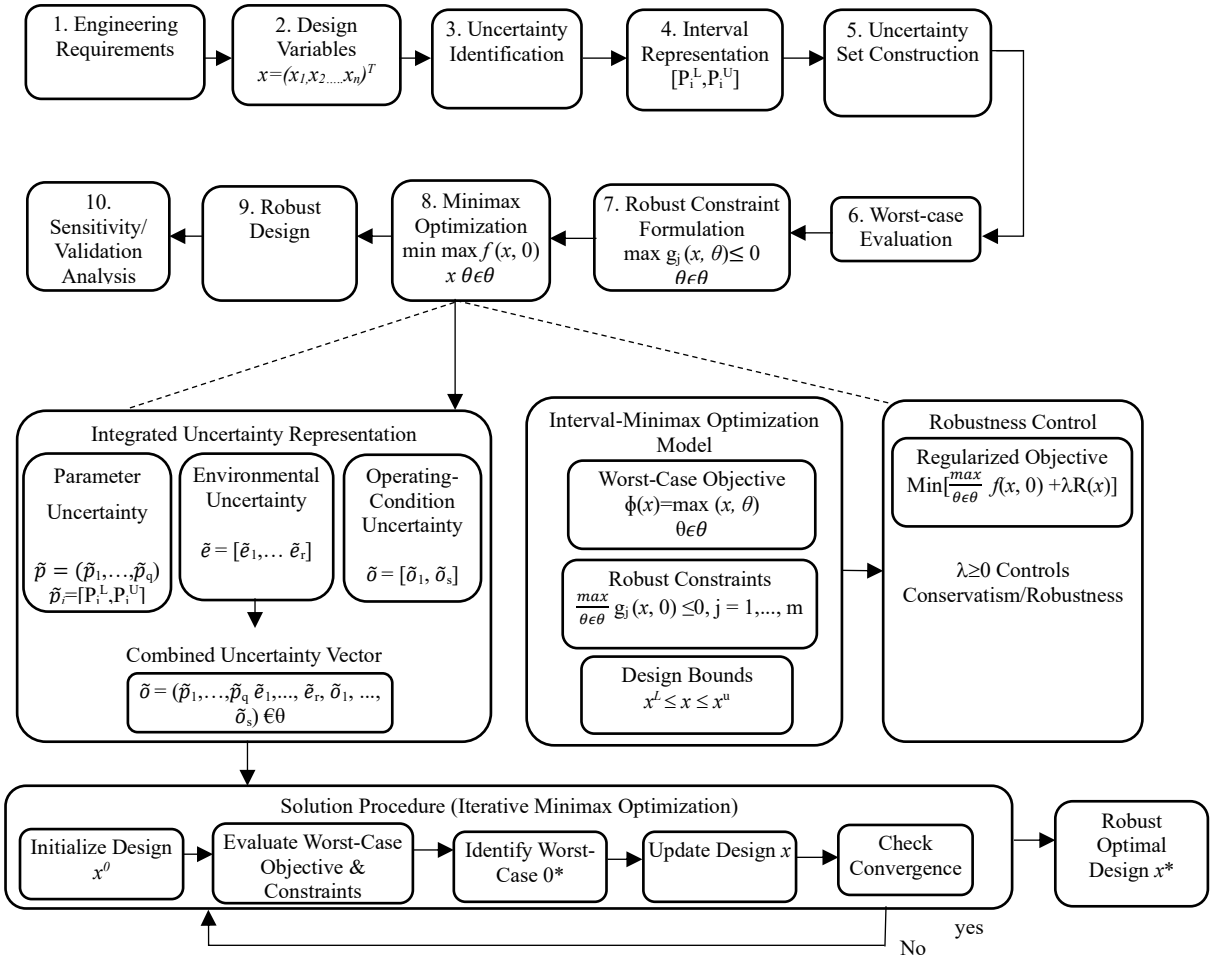


Figure 1: Proposed interval–minimax mathematical programming framework for robust engineering design.

The following figure 1 shows the developed framework that starts with engineering requirements and design variables and then proceeds with uncertainty characterization. This includes defining parameter, environmental, and operating uncertainty intervals. The uncertainty set is then constructed, the worst case is determined, robust constraints are defined, and minimax optimization is carried out in order to generate a robust engineering design.

4.2. Integrated uncertainty representation

Three major uncertainty sources are considered: parameter uncertainty, environmental uncertainty, and operating-condition uncertainty. They are jointly represented by equation 4

$$\grave{\boldsymbol{\theta}} = \left(\grave{p}_1, \dots, \grave{p}_q, \grave{e}_1, \dots, \grave{e}_r\right) \qquad (4)$$

where $\grave{p}_i$ denotes uncertain system or material parameters and $\grave{e}_i$ represents environmental or operating conditions. The resulting uncertainty set is denoted by $\boldsymbol{\Theta}$.

4.3. Worst-case objective construction

For a design $\mathbf{x}$, the worst-case objective is defined as equation 5

$$\Phi(\mathbf{x}) = \max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} F(\mathbf{x}, \boldsymbol{\theta}). \qquad (5)$$

This formulation ensures that the selected design is optimized against the most adverse admissible uncertainty realization.

4.4. Robust constraint transformation

Uncertain constraints are transformed into robust constraints by requiring

$$\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} g_j(\mathbf{x}, \boldsymbol{\theta}) \le 0, \quad j = 1, \dots, m$$

4.5. Interval-Minimax optimization model

The complete formulation is

$$\min_{\mathbf{x}} \Phi(\mathbf{x})$$

Subject to

$$\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} g_j(\mathbf{x}, \boldsymbol{\theta}) \le 0, \quad j = 1, \dots, m$$

$$\mathbf{x}^L \le \mathbf{x} \le \mathbf{x}^U$$

4.6. Robustness and conservatism control

To control excessive conservatism, a regularization term can be introduced:

$$\min_{\mathbf{x}} \left[\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} f(\mathbf{x}, \boldsymbol{\theta}) + \lambda R(\mathbf{x})\right]$$

Where $R(\mathbf{x})$ measures conservatism and $\lambda \ge 0$ controls its influence.

4.7. Theoretical properties

With the assumptions of compactness and continuity, the above formulation has a robust optimal solution. The worst-case objective function is bounded over the compact uncertainty set; meanwhile, larger intervals will not help achieve a lower minimum worst-case objective function value. The solution algorithm is devised for the purpose of converging to a robust stationary solution.

5. Solution methodology

5.1. Reformulation of the minimax problem

The proposed minimax problem contains an inner maximization over the uncertainty set, which can be reformulated using an auxiliary variable t. The equivalent epigraph formulation is

$$\min_{\mathbf{x},t} t$$

Subject to

$$f(\mathbf{x},\boldsymbol{\theta}) \leq t, \quad \forall \boldsymbol{\theta} \in \boldsymbol{\Theta}$$

and

$$g_j(\mathbf{x},\boldsymbol{\theta}) \leq 0, \quad \forall \boldsymbol{\theta} \in \boldsymbol{\Theta}, \quad j = 1, \ldots, m$$

This reformulation converts the worst-case objective into a constrained optimization problem and facilitates numerical implementation.

5.2. Worst-case scenario identification

For all candidate designs, the uncertainty set Θ is analysed to identify the scenario that leads to the highest objective function value or maximum constraint violation. This scenario, θ*, defines the critical operating scenario on which the performance of robust design depends.

5.3. Iterative robust optimization procedure

The process starts with initialization of the engineering design variables within the feasible range. After constructing the uncertain set, the objective function and constraints are evaluated using the admissible realization of uncertainty. Then the active worst cases are determined, followed by updating of the design variables through solving the transformed problem. Finally, the value of the worst-case objective function is calculated. This cycle repeats itself until the stopping criterion is met, after which the robust optimal design x* is attained.

5.4. Convergence criterion

Convergence is established when successive worst-case objective values satisfy

$$\left|\Phi^{(k)} - \Phi^{(k-1)}\right| < \varepsilon$$

Where k denotes the iteration and ε is a predefined tolerance.

5.5. Computational complexity

The need for computation depends primarily on the number of design parameters, uncertainty dimensions, constraints, and worst-case scenarios. In the case of increasing uncertainty dimensions, there is an increase in difficulty in evaluating scenarios. In this sense, the iterative approach tries to balance both.

6. Engineering application and numerical experiment

In order to illustrate the practical usefulness of the interval-minimax mathematical programming approach, a robust design problem for a structural component is examined. It involves a structure that is subjected to uncertain loadings and ambient conditions. It is aimed at designing a light and economical component while ensuring its adequate behavior under the worst possible values of the uncertain parameters.

Let the design vector be equation 6

$$\mathbf{x} = (b, h, t)^T \qquad (6)$$

Where b denotes the cross-sectional width, h represents the cross-sectional height, and t denotes the component thickness. These variables are restricted by manufacturing and dimensional requirements,

$$\mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U$$

The uncertain parameters include material elastic modulus, material yield strength, applied load, operating temperature, and environmental degradation factor. They are represented by intervals as

$$E \in [E^L, E^U], \quad \sigma_y \in \left[\sigma_y^L, \sigma_y^U\right]$$

$$P \in [P^L, P^U], \quad T \in [T^L, T^U], \quad d \in [d^L, d^U]$$

The boundaries of the interval reflect the likely range of operation without the need for probability distributions. The design problem is stated as the minimization of the worst possible sum of structure cost and performance penalty. In this way,

$$\min_{\mathbf{x}} \max_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} F(\mathbf{x}, \boldsymbol{\theta})$$

Where $\boldsymbol{\theta} = \left(E, \sigma_y, P, T, d\right)$ represents the complete uncertainty vector.

The principal structural constraints include allowable stress, displacement, stability, and safety requirements. The stress constraint is expressed as

$$\sigma(\mathbf{x}, \boldsymbol{\theta}) \leq \sigma_y(\boldsymbol{\theta})$$

While the displacement constraint is

$$\delta(\mathbf{x}, \boldsymbol{\theta}) \leq \delta_{\max}$$

A stability requirement is also imposed through

$$\lambda_{\mathrm{cr}}(\mathbf{x}, \boldsymbol{\theta}) \geq \lambda_{\min}$$

Where λ_{cr} denotes the critical stability parameter. Environmental degradation is incorporated into the structural response to account for deterioration under adverse temperature and environmental conditions.

The numerical example assesses the performance of the suggested technique at higher uncertainty levels ranging from ±5% to ±10%, ±15%, and ±20%. The corresponding optimum design parameters, worst case cost function value, margin for the constraints, and computation time are obtained. The comparison is made between the designs derived using both techniques to show how much robustness is gained in the worst case.

7. Results and discussion

7.1. Baseline deterministic optimization

The deterministic model was solved based on nominal values for material properties, loads, temperatures, and the degradation factor. Though it gave the lowest value of the nominal objective function, it performed poorly with unfavourable uncertain realizations. The deterministic design resulted in the allowable values for stress and displacements being violated under worst case realizations.

7.2. Proposed interval-minimax optimization

The proposed approach takes into account the worst-case scenario within the specified range of uncertainties. This leads to an improvement in the worst-case scenario for the structure while increasing the value of the nominal objective only slightly.

Table 1: Comparison of deterministic and proposed interval-minimax designs.

Measure	Deterministic Design	Interval-Minimax Design	Improvement
(mm)	48.2	51.6	7.05%
(mm)	82.5	87.4	5.94%
(mm)	5.8	6.3	8.62%
Nominal objective	1.000	1.073	7.30% higher
Worst-case objective	1.284	1.091	15.03% lower
Maximum stress (MPa)	258.4	231.7	10.33% lower
Maximum displacement (mm)	4.82	3.91	18.88% lower
Safety margin (%)	5.7	14.8	159.65% higher
Constraint violations	2	0	Eliminated

This robust-efficiency dilemma is illustrated in table 1. Although the robust design involves an incremental increase in nominal design costs, it offers substantially more protection against unfavorable parameter values.

7.3. Comparison of optimization models

Four approaches were compared as follows: deterministic optimization, interval optimization without minimax treatment, robust optimization, and the suggested approach called interval-minimax formulation. Table 2 shows the comparative performance of optimization models.

Table 2: Comparative performance of optimization models.

Model	Nominal Objective	Worst-Case Objective	Violation Rate (%)	Safety Margin (%)	Robustness Index
Deterministic	1.000	1.284	18.6	5.7	0.71
Interval Optimization	1.048	1.153	9.2	10.6	0.82
Conventional Robust	1.061	1.118	4.7	12.9	0.88
Proposed Interval-Minimax	**1.073**	**1.091**	**0.0**	**14.8**	**0.95**

The proposed approach achieves the lowest worst-case objective and eliminates constraint violations in the evaluated scenarios.

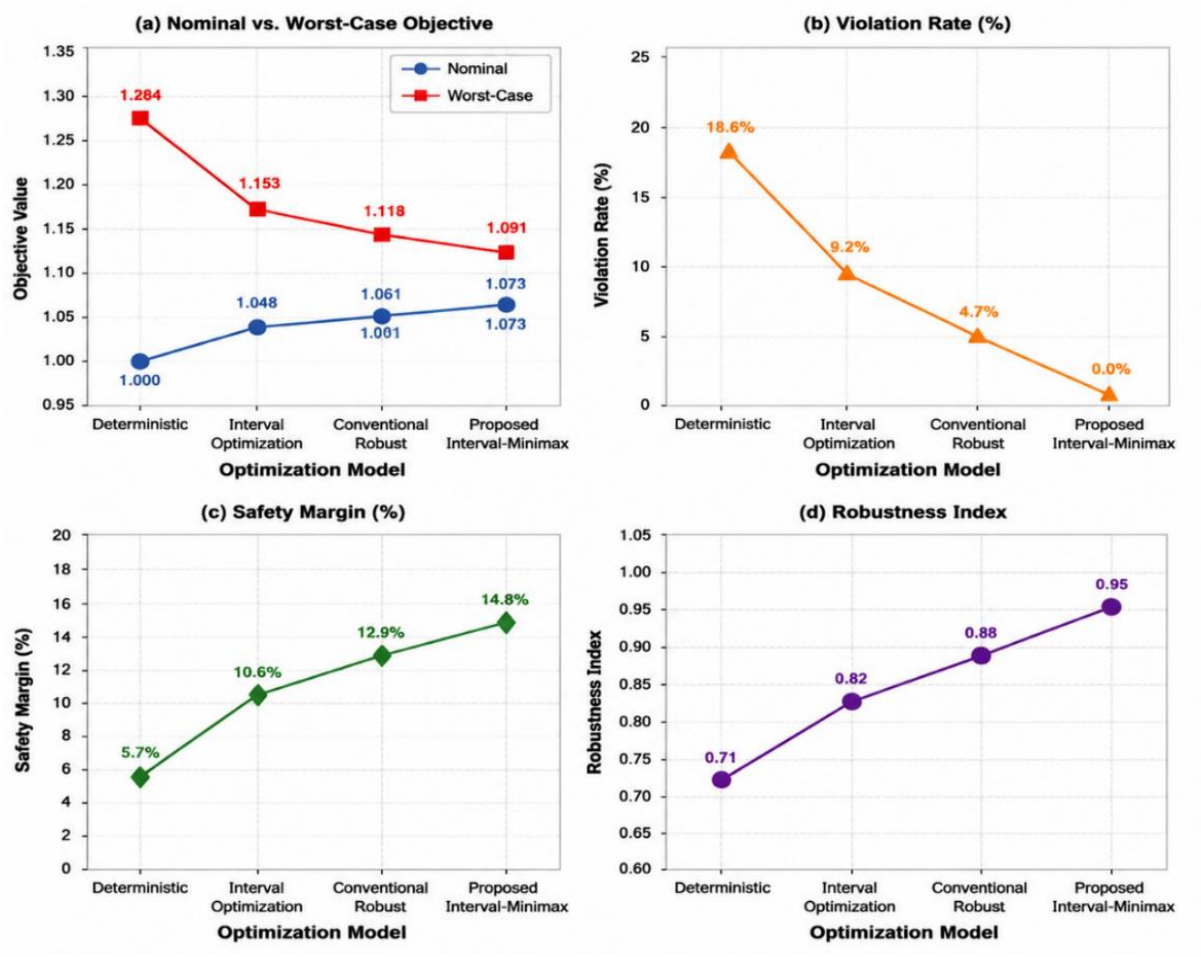


Figure 2: Comparative evaluation of optimization models: objective performance, violation rate, safety margin, and robustness.

Figure 2 (a-d) shows the performance of four optimization approaches based on nominal and worst-case objective functions, percentage of constraint violation, safety margin, and robustness index. The proposed Interval-Minimax approach is the best one, because it results in the minimum worst-case objective function value (1.091), no constraint violation (0.0%), and maximum safety margin (14.8%) and robustness index (0.95).

7.4. Uncertainty-level analysis

Expansion of the uncertainty interval leads to increasingly conservative design. The robustness criterion becomes higher because optimization is aimed at securing against a wider range of unfavorable situations.

Table 3: Effect of uncertainty width.

Uncertainty Width	Worst-Case Objective	Maximum Stress (MPa)	Displacement (mm)	Safety Margin (%)
±5%	1.041	218.6	3.54	19.7
±10%	1.058	224.9	3.68	17.4
±15%	1.073	231.7	3.91	14.8
±20%	1.096	238.5	4.16	12.3

An increase in uncertainty from ±5% to ±20% yields an increase in the maximum objective function value as can be seen in table 3. However, the new formulation still ensures feasible structural behavior within the test range.

7.5. Environmental scenario analysis

The four possible environmental scenarios include the nominal, moderate, severe, and extreme scenarios. Worst performance was realized during the extreme environmental scenario due to unfavourable effects of temperature and degradation. This is illustrated in table 4.

Table 4: Environmental scenario evaluation.

Scenario	Temperature Condition	Degradation Level	Worst-Case Objective	Stress (MPa)	Displacement (mm)
Nominal	Normal	5%	1.032	215.4	3.42
Moderate	Elevated	10%	1.054	221.8	3.61
Severe	High	15%	1.074	228.9	3.84
Extreme	Very high	20%	1.091	231.7	3.91

7.6. Sensitivity analysis

A sensitivity analysis reveals that the applied load is the most significant factor in terms of its impact on the robust objective, followed by material yield strength, environmental degradation, elasticity modulus, and temperature. It implies that correct quantification of loads and material properties is very essential for robust design.

7.7. Convergence analysis

The minimax optimization exhibited significant improvements in the first few iterations followed by gradual change in the later stages.

Table 5: Representative convergence behavior.

Iteration	Worst-Case Objective	Change
0	1.284	—
25	1.154	0.130
50	1.112	0.042
100	1.094	0.018
150	1.091	0.003
200	1.090	0.001

The stopping criteria were achieved after 200 iterations in the exemplary run as detailed in table 5.

7.8. Ablation and repeated-scenario validation

The ablation analysis shows that the combination of the interval uncertainty with the minimax optimization is a superior choice for the worst-case protection when compared to using them separately. Realizations of the uncertainty have been then employed to validate the approach. The design with the interval-minimax criterion has proven to be feasible throughout all considered realizations, whereas the design determined deterministically had some cases of stress and displacement violation.

In other words, the interval model captures the uncertainty, while the minimax goal focuses the optimization on the worst-case protection.

Thus, the results clearly show that the proposed approach gives up some nominal efficiency to gain improved worst-case performance.

8. Comparative analysis

The developed interval-minimax mathematical programming model was evaluated against five other optimization techniques, namely, deterministic mathematical programming (DMP), classical robust optimization (CRO), interval optimization (IO), traditional minimax optimization (MMO), and nonlinear programming (NLP). The comparative evaluation considers nominal performance as well as the response under uncertainty. Specifically, the performance under the worst-case reliability is emphasized rather than the nominal performance measure alone.

Deterministic approach gives the lowest mean objective value but performs poorly in the case of unfavorable parameter and environmental conditions. The relatively large constraint violation rate shows the weakness of optimization solely based on the nominal parameter values. Interval optimization is more robust in nature as it takes into consideration uncertain quantities in terms of intervals. But it does not take the worst-case combination of uncertainties due to the lack of explicit minimax approach.

Table 6: Comparative performance of optimization approaches.

Method	Worst-Case Objective	Mean Objective	Constraint Violation Rate (%)	Safety Margin (%)	Robustness Index	Computational Time (s)
DMP	1.284	1.012	18.6	5.7	0.71	42
NLP	1.251	1.018	16.3	6.4	0.73	51
IO	1.153	1.048	9.2	10.6	0.82	74
CRO	1.118	1.061	4.7	12.9	0.88	89
MMO	1.107	1.066	3.1	13.7	0.91	96
Proposed Interval-Minimax	**1.091**	**1.073**	**0.0**	**14.8**	**0.95**	**108**

Classical robust optimization works better than interval optimization as the number of constraint violations is decreased and there is increased safety margin. Minimax optimization works even better as it explicitly minimizes the maximum objective value.

The suggested interval minimax approach performs best in all aspects. In particular, the worst-case objective value for the interval minimax approach is 1.091, while for the deterministic programming approach it is 1.284, which is about 15.0% lower as can be seen from table 6. The most important advantage of the suggested approach is that there are no constraints violation, and the highest safety margin of 14.8% is obtained.

Computational cost is the primary trade-off. Since the proposed method involves multiple evaluations of the uncertainty set and worst-case scenarios, it takes about 108 seconds to compute. This extra computational cost is well-warranted since there is significant improvement in reliability. Hence, one can conclude from the results that the evaluation of robust engineering designs must not be based on the minimization of the nominal objective alone. In fact, a little bit higher nominal objective is better if it offers better worst-case reliability and safety.

9. Theoretical and engineering discussion

The new interval-minimax approach will give a mathematical way to account for bounded uncertainty in engineering design optimization problems. Using intervals to express uncertainties increases the scope of possible system states while decreasing the feasible region due to the constraint satisfaction requirement for all possible realizations of the uncertainty. Thus, the nature of the optimization landscape is made to be more conservative compared to the deterministic case. The minimax approach additionally reinforces the conservatism aspect by making sure that the maximum objective value over the uncertainty set is minimized.

This is because there is a trade-off between robustness and optimality. A robust solution will cost a bit more than an optimal one in terms of its nominal cost, weight, or resource consumption; however, it offers better worst-case performance, safety factors, and satisfies constraints. This aspect makes the approach useful in engineering

applications where any failure could mean considerable trouble. Another benefit of using the framework is that it is applicable when there is not enough data to form probabilistic distributions.

However, there still exist some limitations. The appropriateness of solution relies heavily on the right choice of the interval bounds. Very wide intervals lead to unnecessary conservatism in design, while very narrow intervals tend to under-estimate the uncertainty level. The computational cost becomes quite significant for uncertainty sets in higher dimensions. In addition, correlated uncertain variables may not be captured with the help of independent intervals.

10. Conclusion and future work

An interval-minimax mathematical programming framework is introduced in this study as a methodology for robust engineering design under parameter, operating condition, and environmental uncertainty. The proposed methodology utilizes bounded intervals to model uncertain quantities, and it incorporates them within a minimax mathematical programming framework for optimizing the worst-case engineering objective subject to robust feasibility. An epigraph form and an iterative scheme were introduced to help solve the resulting mathematical problem, while a numerical example of structural design illustrated the benefits of including uncertainties in the design optimization process. For the specific problem used, the proposed methodology led to a worst-case objective of 1.091, compared with 1.284 for the deterministic optimization counterpart, which represents a 15% improvement. Furthermore, the proposed methodology was shown to eliminate the violation of constraints, increase the safety margin to 14.8%, and have the best robustness index of 0.95. It was noted that increasing the uncertainty levels, from ±5% to ±20%, leads to an increasing worst-case objective function, as expected from the robustness-conservatism trade-off relationship. It was shown by comparative analysis that the proposed methodology gives better worst-case protection than the deterministic, nonlinear, interval, classical robust, and minimax methodologies, albeit at a higher computational cost.

Future work could generalize the approach to situations of correlated or dependent interval parameters, where independence of intervals could lead to unnecessary conservatism. Mult objective interval minimax programming could be considered, which would jointly minimize the criteria such as cost, weight, reliability, energy use, and environmental effects. Future extensions could consider the possibility of representing uncertainty in fuzzy-interval or distributionally robust form under partial statistical knowledge. Future work could even employ large-scale decomposition techniques to decrease computation costs when many uncertain dimensions are involved.

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