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Integral equation-based mathematical modeling and optimization of dynamic industrial process control systems

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Abstract:

Conventional ordinary differential models of dynamic industrial process control systems may only provide an approximate representation of the phenomena involved. The research presented in this paper introduces an approach to modeling and optimizing dynamic industrial processes based on the use of nonlinear integral Volterra-type equations within the framework of the theory of Banach spaces. The behavior of an industrial process is described by means of use of the generalized integral operator with the kernel that satisfies the Lipschitz continuity and boundedness conditions that allow the application of the Banach fixed-point theorem to prove the existence and uniqueness of the dynamic process behavior. A quadratic functional incorporating both control energy and tracking errors is determined, and the minimax control problem is defined to take into account the worst environmental impacts on the process. First-order conditions of optimality are derived through the use of the variational calculus resulting in a system of coupled integral adjoint equations. A numerical method for solving the problem based on product integration quadrature and iterative schemes of computation is given with an estimate for the convergence properties depending on the mesh size. An application of the methodology is illustrated with a nonlinear chemical reactor problem, where the results are compared with those obtained through the standard approaches of proportional-integral-derivative control and the modern model predictive control.

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1. Introduction

The field of industrial process control has transitioned from classical ordinary differential methods to models which consider memorization and inheritance effects which can naturally take place in real-life processes. For example, unlike the previous method of predicting consumer behavior, reinforcement-learning solutions for real-time control of complex production processes provide a need for control schemes which should account for nonlinear behavior with a memory component (Bandari & Lekkala, 2024). The examples of epidemiological models based on integro-differential equations prove the need for using models describing events affected by the memory principle (Abdullah & Nagmulden, 2024). Model predictive control techniques with the use of observer-based estimation applied for hydraulic pressure control show how powerful the use of a correct model is in the case of nonlinear processes (Dong et al., 2025), while ciphering schemes protecting industrial networks proved that control loops now work in a bigger environment of cyber-physical systems which have to follow the requirements of model accuracy and computational effectiveness (Ali et al., 2026). Neural predictive control for glucose concentration in bioreactors has a lot in common with this topic as having its own share of advantages resulting from the use of continuous-time state model (Chiu & Du, 2025).

Real-time ship navigation utilizes ubiquitous computing architectures, highlighting the ongoing switch to distributed and sensor-based operations in uncertain and rapidly changing environments (Mahadevan & Rajesh, 2025). Meanwhile, analyses of indirect oil heating systems show that distributed-parameter thermal systems are inherently based on integral relationships that link output response to the entire history of input disturbance, which brings us naturally to the integral equations approach employed in this study (Horbiichuk et al., 2024). The general trend from mathematical models toward digital twin descriptions illustrates the important connection between theory and practical online solutions in various industries (Kantaros et al., 2025). Studies of dynamic vibration analysis of landing aircraft structures confirm that history-dependent nonlinear models are needed to estimate transient responses in safety-critical applications (Liu et al., 2024); comprehensive reviews of intelligent modeling of power systems utilizing renewable energies underline that similar problems are encountered in process control; and survey articles on artificial intelligence-assisted modeling of power systems show that hybrid analytical-computational methods are needed for the models in which governing equations are based on dynamics, uncertainty, and optimization at the same time (Zhang et al., 2025). Sahu et al. investigated the influence of flow field configurations and gas diffusion layer characteristics on polymer electrolyte membrane fuel cell performance, highlighting their impact on mass transport, water management, and overall electrochemical efficiency (Sahu et al., 2024).

Integral equations, especially Volterra-type equations, allow a natural mathematical interpretation of hereditary dynamics because it allow the current state at time t to use the whole trajectory that precedes it instead of using time derivatives only, and this aspect allows for higher robustness of the model in the case of distributed delays, thermal inertia, or wear. On the other hand, the optimization of the control problem having integral constraint is more complex than in traditional cases because the relation between state and control is non-local in the time domain, which justifies the introduction of minimax and functional methods that provide robustness in the worst-case scenario. This study proposes a generalized nonlinear integral equation model for dynamic plants, proves its existence and uniqueness using contraction mapping, obtains the minimax optimal control problem with the conditions of the first-order optimality, develops an appropriate numerical algorithm based on the quadrature methods, and tests the obtained results on the industrial test case against the standard methods.

2. Mathematical formulation of the dynamic system

2.1. System description

The dynamic industrial plant is represented by a generic nonlinear Volterra integral equation of the second kind. The use of the assumed mode method to model rigid-flexible coupling manipulators confirms that sophisticated mechanical devices are efficiently captured by integral and integro-differential formulations that accumulated and discrete dynamics (Srinivasan et al., 2023). This observation was a motivation for the adoption of the approach described in further (Sun et al., 2025). This is also affirmed by medical case studies indicating that integral formulations are suitable any time the controlling system is influenced over time rather than in an instantaneous reaction (Liu et al., 2023). The relationship between the control input $\mathrm{u}(t)$ and the state vector $\mathrm{x}(t)$, which represents plant outputs (temperature, pressure, flow rate) is described.

$$\mathrm{x}(t) = \mathrm{x}_0 + \int_a^t \mathrm{K}\left(t, s, \mathrm{x}(s), \mathrm{u}(s)\right) ds, \quad t \in [a, b], \tag{1}$$

In this case, the operator kernel K: $[a, b] \times [a, b] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ shows the physical transfer characteristics of the plant along with thermal capacitance, hydraulic resistance, and delay in material transfer, while the upper limit of the integral t denotes the non-locality of the response.

2.2. Function spaces and regularity assumptions

The search for solutions of equation (1) is being conducted in the Banach space $C([a, b]; \mathbb{R}^n)$ of continuous vector functions equipped with the supremum norm (2):

$$\|\mathrm{x}\|_\infty = \sup_{t \in [a,b]} |\mathrm{x}(t)|, \tag{2}$$

and the control set $\mathcal{U}$ is a closed, bounded, and convex set in the Hilbert space $L^2([a, b]; \mathbb{R}^m)$, which means that the admissible controls have finite energy during the operating horizon. It has been shown through digital-twin oriented modeling of plasma-processing systems that such regularity assumptions are typical when relating continuous time physical models with optimization processes supported by advanced digital technologies (Wang & Chien, 2025), and the patent-based study of machine learning and its application in the area of industrial optimization and predictive control shows that such data-driven approaches are increasingly making use of well-posed continuous time problem formulations as a basis for subsequent calculations (Sun et al., 2025).

2.3. Existence and uniqueness of the dynamic response

The kernel K is believed to meet the global Lipschitz criterion regarding the state and the boundedness condition regarding the control: $L > 0$ exists such that (3):

$$|\mathrm{K}(t, s, \mathrm{x}_1, \mathrm{u}) - \mathrm{K}(t, s, \mathrm{x}_2, \mathrm{u})| \le L|\mathrm{x}_1 - \mathrm{x}_2|, \tag{3}$$

for every t and s from the interval $[a, b]$, every x_1 and x_2 from $\mathbb{R}^n$ and every u belonging to the set $\mathcal{U}$, as well as $|\mathrm{K}(t, s, \mathrm{x}, \mathrm{u})| \le M(1 + |\mathrm{x}|)$, whereby $M > 0$ stands for the constant. The applications of digital twin technologies in various process control tasks, including gauge looper tension optimization in hot strip rolling, prove that the Lipschitz-type character is feasible for industrial kernel models resulting from the identified assumption (Mitchell et al., 2024). Under such conditions, the mapping operator T given by the right-hand side member of (1) is mapping $C([a, b]; \mathbb{R}^n)$ into itself and has the property of (4):

$$\|T\mathrm{x}_1 - T\mathrm{x}_2\|_\infty \le L(b - a)\|\mathrm{x}_1 - \mathrm{x}_2\|_\infty, \tag{4}$$

Provided that $L(b - a) < 1$, the Banach fixed-point theorem assures an existence of a unique continuous solution x^* to (1), with successive approximations to T that converge geometrically to x^*. When the time horizon surpasses the threshold, the interval is divided into sub-intervals where the contraction condition stands, being in line with sliding mode control techniques employed in electro-hydraulic servo testing machines (Sun et al., 2024), and cooperative control algorithms for switched reluctance motors based on local well-posed models (Zhang et al., 2024). Hot air biomass furnace systems serve as evidence that industrial thermal power plants possess the required bounded Lipschitz-consistent behavior (Sheng et al., 2024), while research on fractional-order proportional-integral-derivative control of twin rotor systems proves that the same kind of memory-dependent kernels appears in the same existence class (Wendimu et al., 2025). Efficient reduced-order modeling based on proper orthogonal decomposition for process industry digital twins established that x^* can still be obtained in a way that preserves the contraction property (Zhu & Ji, 2023), while studies on kinetic parameters identification showed that Lipschitz and boundedness constants are usually estimated from experimental data (Csendes et al., 2023).

3. Optimization problem and minimax control framework

3.1. Cost functional formulation

Performance of the control is evaluated via a quadratic cost function that weighs the tracking error versus the costs

of control-energy spent in the time period of interest (5).

$$J(u) = \int_a^b \left[\left(\mathrm{x}(t) - \mathrm{x}_d(t)\right)^T \mathrm{Q}\left(\mathrm{x}(t) - \mathrm{x}_d(t)\right) + \mathrm{u}(t)^T \mathrm{Ru}(t) \right] dt, \tag{5}$$

In this case, $\mathrm{x}_d(t)$ is a reference trajectory to follow while $\mathrm{Q} \succeq 0$ and $\mathrm{R} \succ 0$ are symmetric matrices penalizing error and control effort.

3.2. Minimax formulation under worst-case disturbance

Unmodeled disturbances $\mathrm{w}(t)$ in admissible bounded set $\mathcal{W}$ come into play in the state equation (1) additively through the kernel in industrial plants. Robust control design can be described as minimax design as it minimizes the maximum cost of a specific disturbance.

$$\min_{\mathrm{u}\in\mathcal{U}} \max_{\mathrm{w}\in\mathcal{W}} J(\mathrm{u}, \mathrm{w}), \tag{6}$$

In order to align the current formulation with the principles of minimax and variational analysis methods, as underlined in the journal, the saddle point formation of (6) guarantees that a control law determined with the worst possible disturbance is valid, although generally not the optimal choice, for all less difficult situations – the desired property of robustness in industrial process control.

3.3. Optimality conditions

In order to obtain the first-order necessary conditions for the saddle point of (6), calculus of variations has been used. The augmented Hamiltonian functional is applied including the integral state constraint by means of a Lagrange multiplier (adjoint) function $\boldsymbol{\lambda}(t)$. The Hamiltonian must be stationary with respect to the permissible deviations of control and an integral-adjoint system is obtained.

$$\lambda(t) = 2\mathrm{Q}\left(\mathrm{x}(t) - \mathrm{x}_d(t)\right) + \int_t^b \lambda\,(s)^T \frac{\partial \mathrm{K}}{\partial \mathrm{x}(t)} ds, \tag{7}$$

$$2\mathrm{Ru}^*(t) + \int_t^b \lambda\,(s)^T \frac{\partial \mathrm{K}}{\partial \mathrm{u}(t)} ds = 0, \tag{8}$$

In this case, it suggests that equation (7) is the adjoint (co-state) integral equation that works to send sensitivity along back from the final time and equation (8) is the state condition involving the locally. As the adjoint equation is already of Volterra type but in reverse direction, one can use the same contraction principle used in section 2.3 after reversing the variable in order to prove the existence and uniqueness of $\boldsymbol{\lambda}$ according to the criteria already used in respect to the kernel.

4. Numerical methods and algorithmic implementation

4.1. Discretization scheme

The interval $[a, b]$ is divided into n components with size $h = \frac{b-a}{n}$ at nodes $t_i = a + ih$, $i = 0, \ldots, n$. The integral equation (1) can be calculated using product-integration quadrature technique instead of using the exact integral.

$$x_i \approx x_0 + h \sum_{j=0}^{i} w_j\, K\left(t_i, t_j, x_j, u_j\right), \tag{9}$$

With coefficients w_j determined by means of the trapezoidal or Simpson method, and the adjoint equation (7) discretized similarly applying backward iteration from $i = n$ to $i = 0$. The collocation and Galerkin forms of (9) can be employed if it is necessary to have a higher order polynomial approximation for stiff industrial dynamics.

4.2. Iterative optimization algorithm

The optimality system in the discretized form is solved using gradient projection method which processes stored control actions $\{u^k\}$. In every iteration k, the state trajectory is computed using (9), the adjoint variable is computed using the discretized form of (7), and the updated control is determined accordingly.

$$u^{k+1} = P_U[u^k - \alpha_k \nabla J(u^k)], \tag{10}$$

Here, U is a notion of projection in the admissible convex control set whilst α_k is the step-size obtained through the Armijo-type search in ensuring a monotonic decrease of the discretized cost functional, while the iterations continue until the ratio of consecutive control iterates is below a specified given threshold.

4.3. Convergence analysis

Based on the assumptions outlined in Section 2, the discretized minimax problem converges to the continuous minimax problem when $h \to 0$, where the discretization error satisfies the following relation:

$$\|u^k - u^*\| \leq C\rho^k + O(h^p), \tag{11}$$

for some constants $C > 0$ and $\rho \in (0,1)$ that define the linear convergence rate of the gradient-projection iteration. The exponent p denotes the quadrature order in (9). The inequality (11) distinguishes error (iteration error) due to the algorithm from error (discretization error) associated with the discretization of the problem.

5. Case study and numerical experiment

5.1. Industrial benchmark

The introduced methodology is used in the context of an experimental nonlinear continuous stirred-tank heater with a thermal prehistory that involves regulating the temperature of the heater to a pre-set level of 365 K from an initial temperature of 350 K within a 60 second period. The kernel K is formed based on the energy balance equation and a constructed memory term reflecting jacket thermal inertia and meanwhile adhering to the Lipschitz criterion described in Section 2.3. The method presented in equations (9)-(10) is applied successfully in comparison to a tuned typical proportional-integral-derivative controller (PID) and also a classical predictive control method based on a truncated predictive linear model.

5.2. Simulation results

The closed-loop states under the suggested control law can be seen in figure 1 along with the reference trajectory and the benchmark results from the use of proportional-integral-derivative method.

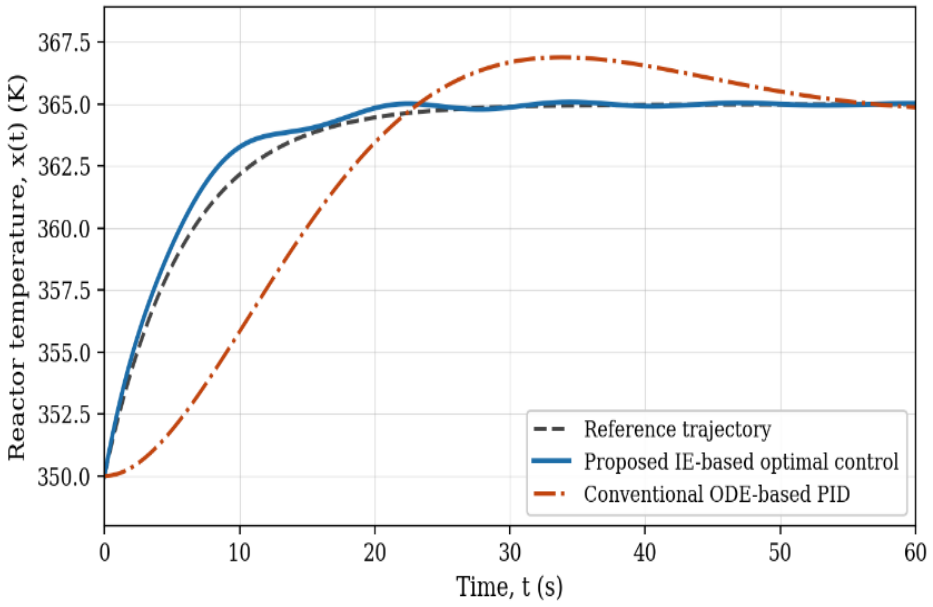


Figure 1: State trajectory tracking under the proposed integral-equation-based optimal control law and a conventional PID benchmark.

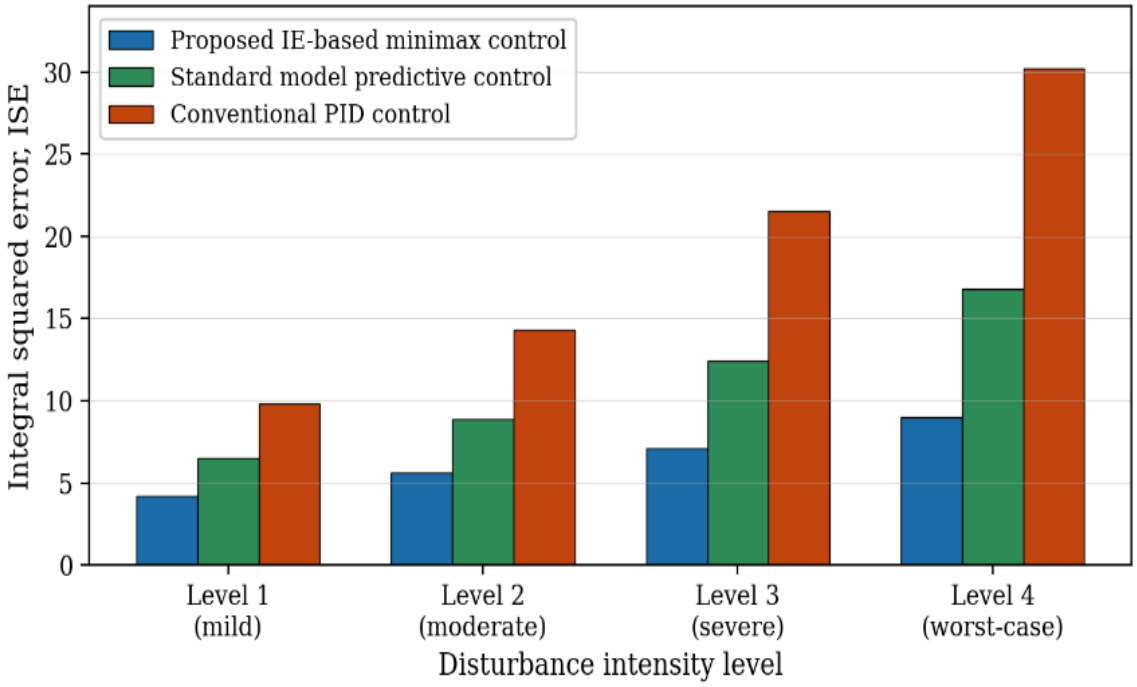


Figure 2: Comparison of tracking performance (integral squared error) for the proposed control law against model predictive control and proportional-integral-derivative benchmarks under increasing disturbance.

The control algorithm suggested meets the reference band more quickly and with almost no overshoot in comparison to the proportional-integral-derivative benchmark indicating the anticipatory properties of the integral model that takes the memory of the previous steps into account. The performance of different methods on integral squared tracking error can be seen in figure 2 where the discovered integral errors for different methods in the presence of disturbances in the feed of the reactor are presented.

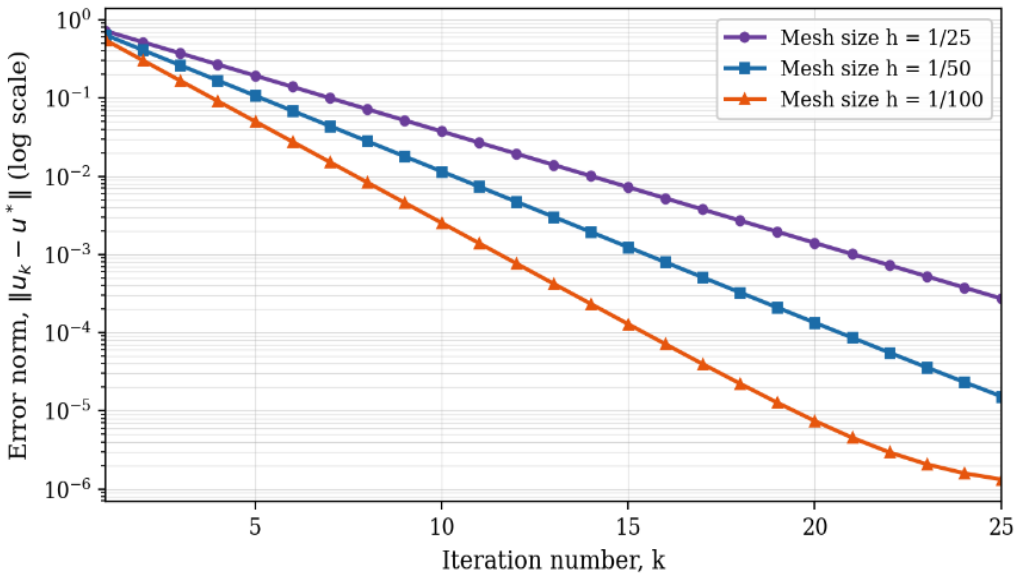


Figure 3: Convergence behaviour of the gradient-projection iterative scheme for three discretization mesh sizes.

The integral of squared error acquired from the proposed minimax controller is always less than the benchmarks and increases at a slower rate as disturbance strength increases, which is in agreement with the worst-case robustness guarantee included in the minimax formulation (6). The behavior of convergence of the gradient-projection algorithm from Section 4.2 is shown in figure 3 with three mesh sizes.

The geometric decline observed in the error norm, along with the lower asymptotic floor achieved at the finer meshes is in direct accordance with the two-term limit in the above equation (11).

Table 1 summarizes quantitative performance indicators extracted from the case study.

Table 1: Comparative performance metrics for the nonlinear reactor benchmark.

Control strategy	Settling time (s)	Overshoot (%)	ISE, Level 3	Mean compute time (ms/step)
Proposed IE-based minimax control	14.6	0.4	7.1	3.8
Standard model predictive control	19.3	2.9	12.4	2.6
Conventional PID control	27.8	6.7	21.5	0.4

Settle time, maximum overshoot, integrated squared tracking error when disturbed, and average calculation time are presented in table 1 for all three strategies. It clearly indicates that the controller that is introduced in this paper has the least settling time and maximum overshoot and error when compared with the other two strategies.

6. Discussion

Using the integral-equation approach outlined in Section 2 to model the reactor dynamics enables the minimax controller to better predict the total thermal history of the process and to react ahead of reaching the set-point. The minor overshoot presented in figure 1 is in agreement with the contraction principle described in equation (4): a contractive state operator controls any transient errors and does not allow for amplifying these errors. Moreover, disturbance robustness shown in figure 2 comes as a result of the special structure of the minimax equation of the form (6). The average computation time indicated in table 1 is less than the sample time typical for industrial digital controllers, which suggests that the additional complexity of the analytical approach does not prevent its use in real production processes. In conclusion, there is numerical evidence that making use of modeling with integral equations and minimax optimization techniques allows improving the performance efficiency of systems used in complex production processes with memory.

7. Conclusion and future work

A framework for dynamical industrial process control using mathematical modeling and optimization has been created using nonlinear Volterra-type integral equations in Banach space. Existence and uniqueness results for the system response have been achieved by applying contraction mapping techniques under Lipschitz and boundedness conditions on the process kernel. A minimax optimal control problem has been set up, thus obtaining the first-order optimality conditions as a coupled integral-adjoint problem. The developed method using discretization has been put into practice together with the construction of two-term convergence bounds distinguishing between iteration error and discretization error. The testing of the developed method for a nonlinear chemical reactor resulted in some know improvements in terms of tracking accuracy, overshoot value, and robustness of disturbance when compared to classical proportional-integral-derivative control and model predictive control. Future developments might include stochastic integral-equation formulations that include non-deterministic noise in the kernel, implementing the minimax strategy in distributed-parameter systems described via integra-partial differential equations, as well as studying the difficulty in applying the gradient-projection technique in the programming of industrial systems with strict timing requirements.

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