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Projection-free numerical algorithms for large-scale constrained optimization in networked engineering applications

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Abstract:

In the paper, a prospect of large-scale optimization problems faced during the operation of networking engineering systems such as smart grids, multi-agent systems based on sensor arrays, or systems for distribution of resources are discussed since it deals with high-dimensional coupled constraints making metric projections difficult. The use of conventional numerical projection-based methods in practice involves the repeated solution of quadratic problems in each iteration, leading to costs increasing super-linearly with the increase in network size, thus limiting applicability in real-time engineering practice. In this paper, projection-free numerical methods based on linear minimization oracles are proposed; in particular, the responses of conditional gradient methods (Frank-Wolfe algorithm) are applied to decentralized networks. The study presents an optimization problem defined over a communication graph, as well as present assumption considering smoothness, convexity, and boundedness in respect to both feasible set and the function goal of optimization. The study proves sublinear convergence of the primal and its dual formulations in general convex case; in addition, for strong convex/Polyak-Łojasiewicz case, it is proved that linear convergence is possible. The communication complexity bounds that evaluate the amount of data exchange between nodes to achieve epsilon-optimality is offered along with the results concerning convergence proofs. The conducted numerical simulation in respect to a distributed resource allocation case study illustrates the established rates of the algorithm efficiency, demonstrates the advantages of the proposed method

over projection-based problems, and discusses the scalability of the proposed methods as concerning implementation of up to five hundred nodes in a network.

Keywords: Projection-free optimization, Frank-Wolfe algorithm, conditional gradient method, distributed optimization, networked engineering systems, convergence analysis, linear minimization oracle.

1. Introduction

In present-day engineering, infrastructures nowadays are manifesting into huge networks that are interconnecting which means that decision variables are present across various operators under different operational limits. Multi-agent sensor arrays, networks of distributed energy systems, and control systems with communication constraints lead to optimization problems where number of variables and constraint complexity are proportional to number of network nodes (Chen et al., 2023). There is similar scaling issues encountered in area of materials and manufacturing where search for viable design configuration given numerous physical and cost constraints increases rapidly with increase in number of design variables (Sharma & Maurya, 2025; Gaur & Agrawal, 2025). Large measurement and inference networks that are used to monitor infrastructures exhibit the same trend when it comes to constraint dimension which places even a stronger accent on numerical methods that are not becoming more expensive with increase in network size (Fallah et al., 2025).

The main numerical procedure in constrained optimization known as projective gradient descent and some of its other improved versions require a metric projection on the feasible set at each step of the algorithm. When the feasible set is determined by a set of linear or conical inequalities represented throughout the whole network, then this metric projection becomes more of a quadratic or semidefinite programming problem whose cost depends on the dimensionality of the problem (Jiang et al., 2024). Moreover, in decentralized systems this problem becomes more complex since it does not have a full description of the problem but has to solve all the feasible sets as local-global nested problems (Wang & Yi, 2023; Hou et al., 2024). Hence, while hundreds of nodes can be engaged in solving the original problem, the total cost of metric projections makes even traditional implementations insufficient for effective decision-making purposes. Enhanced ant colony optimization was introduced to improve the search capability and convergence performance of traditional ant colony optimization for solving the traveling salesperson problem (Agrawal et al., 2012a).

An alternate family of numerical techniques evades the projection step by substituting it with a linear minimization oracle that solves a linear problem in the same feasible domain. This principle is at the core of the conditional gradient or Frank-Wolfe family of algorithms whose per-iteration cost is typically ten times lower than the cost of the projection (Zhang et al., 2021). The adaptation of this concept to decentralized federated and consensus scenarios maintains the privacy of the involved parties and minimizes the necessary communication while preserving the good computational performance of the linear oracle (Zhang et al., 2025). The non-smooth and functionally constrained versions add even more to the scope of applicability of the projection-free idea to situations when the constraint set is defined implicitly through inequality functionals rather than explicitly through some geometric representation (Asgari & Neely, 2024) and examples of using alternative interior-point algorithms with time-varying constraints show the benefits of using projection-free methods (Lupien et al., 2024). An enhanced bee colony algorithm was developed to optimize the solution process for the traveling salesperson problem by improving the balance between exploration and exploitation in swarm intelligence-based optimization (Agrawal et al., 2012b).

In this article, a decentralized projection-free numerical framework for addressing large-scale constrained optimization problems in networked engineering infrastructures. Firstly, a general formulation of a constrained optimization problem over the framework of a communication graph, accompanied by a limited set of assumptions on smoothness, boundedness, and convexity that are applicable to engineering constraints created. Secondly, a novel approach based on the combination of a local linear minimization oracle and gradient tracking in distributional conditional-gradient updating introduced. Thirdly, the existence of sublinear convergence of the Frank-Wolfe gap for a general convex problem and linear convergence for a special variant with a generalized Polyak-Łojasiewicz condition additionally providing a bound on the communication volume needed to achieve a specific accuracy level proved. Finally, this new method using the case studies of resource allocation in distribution networks to calculate the savings in computation time and communication expenses achieved as compared to projection-based methods was illustrated.

2. Problem formulation and preliminaries

2.1 General mathematical setup

Consider the constrained optimization problem

$$\min f(\mathbf{x}) \quad \text{subject to} \quad \mathbf{x} \in \mathcal{X},$$

where $f: \mathbb{R}^n \to \mathbb{R}$ is an even function of the gathering engineering goal, such as cost of production, delay in the network, or loss of resources, while $\mathcal{X}$ is a compact, convex set that incorporates the operational and network restrictions of the infrastructure in question. That analyzes the set $\mathcal{X}$ when dealing with the games and aggregative problems where coupling is also in place due to the common resources used by players involved in the process of distributed aggregative games (Li et al., 2025; Xu et al., 2022).

2.2 Network graph model

In this context, the networked engineering infrastructure can be described through the undirected communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. In this graph, the set $\mathcal{V} = \{1, \dots, N\}$ represents the total number of agents involved in the system, including buses, sensors, and controllers, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents where agents exchange local information from each other. Each agent $i \in \mathcal{V}$ has a local variable $\mathbf{x}_i \in \mathbb{R}^n$, and all copies of each agent's variable are reconciled using the consensus process, as explained in Section III. As a result, the variable $\mathbf{x} = (\mathbf{x_1}, \dots, \mathbf{x}_N)$ is feasible with respect to $\mathcal{X}$. It's assumed that the graph is linked and the mixing matrix $\mathbf{W} = [w_{ij}]$ is doubly stochastic concerning the topology of the graph (Deng et al., 2024; Mo et al., 2025).

2.3 Standing assumptions

(A1) L-smoothness: The function f has continuous derivatives in an open set that contains the locations of the points of the set $\mathcal{X}$, and its gradient is characterized by Lipschitz continuity, so that the following relation holds for any two points x and y.

(A2) Convexity and compactness: The set $\mathcal{X}$ is convex and compact with a diameter $D = \sup\{\|\mathrm{x} - \mathrm{y}\|: \mathrm{x} \text{ and } \mathrm{y} \text{ are in } \mathcal{X}\}$, as the function f is assumed to be convex in $\mathcal{X}$. The property of compactness provides a guarantee of existence of the global minimizer of the function, as well as limits the diameter term in estimating the convergence discussed in Section IV, as it is consistent with the conditions of boundedness considered in multi-scale distributed convex-optimization works (Ni & Wang, 2022).

(A3) Decentralized-information structure: Each agent of the system receives information only from a local component of function f, and, therefore, communicates with its neighbors only. Analytic gradients of the function are inaccessible, which provides a ground for the utilization of local subgradient information or zeroth order-gradient information, the agent not having any access to analytic gradients of the function (Nguyen & Balasubramanian, 2023). This form of information reception has led to the statement of the linear-minimization-oracle framework proposed in Section III to be more suitable than the solution of a network-wide projection problem, which is a projection-free approach applied whenever it is applicable (Bomze et al., 2021).

Assumptions (A1) -(A3) could be conformed to the context of projection-free literature, since the conditions looked at coincide with reasonable feasible sets used in engineering practices such as box constraints on generation and activation limits together with balance constraints on linear flows used for traffic–assignment and power flow analysis (Suranto & Siregar, 2025).

3. Algorithmic design: projection-free framework

3.1 Linear minimization oracle

The projection operator is substituted for the linear minimization oracle (LMO), which takes in a certain point $\mathbf{x}^k$ and the gradient vector $\nabla f(\mathbf{x}^k)$ and produces some vertex of $\mathcal{X}$ aligning against the negative direction of the gradient.

$$\mathbf{s}^k = \arg\min_{\mathbf{s}\in\mathcal{X}} \langle \nabla f(\mathbf{x}^k), \mathbf{s} \rangle$$

For engineering applications here, $\mathcal{X}$ is either some polytope or just a simple compact convex set; hence running the LMO boils down to solving a linear programming problem or calculating the vertex in a closed form rather than engaging in a quadratic programming or semidefinite programming associated with the metric projection (Yurtsever et al., 2021).

3.2 Distributed update rule

The agent i keeps a variable named $\mathbf{y}_{i,k}$ for the local gradient-tracking, which estimates the global gradient average through communication with neighbours as given:

$$\mathbf{y}_{i,k} = \sum_j w_{ij}\, \mathbf{y}_{j,k-1} + \nabla f_i\big(\mathbf{x}_{i,k}\big) - \nabla f_i\big(\mathbf{x}_{i,k-1}\big),$$

with $\mathbf{y}_{i,0} = \nabla f_i\big(\mathbf{x}_{i,0}\big)$. The local linear minimization step then uses $\mathbf{y}_{i,k}$ in place of the true gradient:

$$\mathbf{s}_{i,k} = \arg\min_{\mathbf{s}\in\mathcal{X}_i} \big\langle \mathbf{y}_{i,k}, \mathbf{s} \big\rangle, \quad \mathbf{x}_{i,k+1} = \mathbf{x}_{i,k} + \gamma_k\big(\mathbf{s}_{i,k} - \mathbf{x}_{i,k}\big),$$

Where $\mathcal{X}_i$ is the local part of the coupled feasible set that can be reached by i agent and γ_k can be defined as a step size such that γ_k is in range $(0,1]$ and is either following diminishing step regime or constant step regime.

3.3 Algorithmic procedure

The procedure executed by agent i at iteration k proceeds through the following steps:

Step 1: Communicate the local iterate and gradient-tracking variable to neighboring agents in $\mathcal{G}$.

Step 2: Update the gradient-tracking variable $\mathbf{y}_{i,k}$ using the consensus mixing rule of Section III.2.

Step 3: Solve the local linear minimization oracle to obtain the vertex direction $\mathbf{s}_{i,k}$.

Step 4: Update the local iterate $\mathbf{x}_{i,k+1}$ by taking a step of size γ_k toward $\mathbf{s}_{i,k}$.

Step 5: Increment k and repeat until the duality gap falls below a prescribed tolerance ε.

This approach extends the concepts of the randomized block-coordinate and event-triggered communication techniques that were created for distributed Frank-Wolfe methods, where only a portion of the coordinates or communication cycles are triggered at every time step (Gao et al., 2025; Zhu et al., 2023).

4. Convergence analysis and theoretical guarantees

4.1 Duality gap

For $\mathbf{x} \in \mathcal{X}$, the Frank-Wolfe duality gap is defined as

$$g(\mathbf{x}) = \max_{\mathbf{s}\in\mathcal{X}} \langle \nabla f(\mathbf{x}), \mathbf{x} - \mathbf{s} \rangle$$

Since f is convex, $g(\mathbf{x}) \geq f(\mathbf{x}) - f(\mathbf{x}^*)$ for any minimizer $\mathbf{x}^*$ of f over $\mathcal{X}$, so that $g(\mathbf{x})$ furnishes a computable upper bound on the optimality gap without requiring knowledge of $f(\mathbf{x}^*)$.

4.2 Sublinear convergence for general convexity

Theorem 1. Under Assumptions (A1) -(A2), with step size $\gamma_k = \frac{2}{k+2}$, the iterates generated by the centralized linear-minimization update satisfy

$$f(\mathbf{x}^k) - f(\mathbf{x}^*) \leq \frac{2LD^2}{k+2}$$

for every $k \geq 1$, where D is the diameter of $\mathcal{X}$.

Proof sketch. The descent inequality comes from the convexity and L-smoothness conditions, specifically $f(\mathbf{x}^{k+1}) \leq f(\mathbf{x}^k) - \gamma_k g(\mathbf{x}^k) + \cdots$ where γ_k is substituted by $\frac{2}{k+2}$. The induction on the recurrence gives the final result as well as the descent argument of Frank-Wolfe.

Theorem 1 states that the convergence rate for the objective is of order one in terms of k and it is exactly the same as the rate of convergence for centralized method in the more general setting of convex optimization problems. It also applies to the decentralized network with the only difference that the convergence is not exact due to the mixing property of the network.

4.3 Linear convergence under strong convexity or a polyak-łojasiewicz condition

If f satisfies a Polyak-Łojasiewicz condition on $\mathcal{X}$, or is μ-strongly convex, then the away-step algorithm, which allows the algorithm to stray away from the current point, avoids the erratic behavior that restricts the regular procedure to a sublinear pace.

Theorem 2. If the conditions in Assumptions (A1) -(A2) hold and the requirement of Polyak-Łojasiewicz with $\mu > 0$ holds as well, the method of away steps with suitable upper bound on step size will satisfy

$$f(\mathbf{x}^k) - f(\mathbf{x}^*) \leq \big(f(\mathbf{x}^0) - f(\mathbf{x}^*)\big) \cdot \rho^k$$

for some $\rho \in (0,1)$ depending on L, μ, and the pyramidal width of $\mathcal{X}$.

The proof sketch. The away-step analysis shows that the duality gap decreases geometrically, as long as changes to the set of selected vertices are prevented, where have only finitely many changes to this set before can apply the idea of geometrical decrease from the away-step method.

4.4 Communication complexity

Proposition 1 states that for achieving an ε-optimal solution of the form $f(\mathbf{x}^k) - f(\mathbf{x}^*) \leq \varepsilon$, the decentralized gradient-tracking algorithm is expected to incur $\mathcal{O}\left(\frac{1}{\varepsilon} \cdot \log\left(\frac{1}{1-\lambda_2(\mathbf{W})}\right)\right)$ rounds of communication, where $\lambda_2(\mathbf{W})$ is the magnitude of the second largest eigenvalue of the mixing matrix $\mathbf{W}$, with each communication round wasting $\mathcal{O}(n)$ real numbers on every edge of $\mathcal{G}$.

In this way, the above configuration quantifies the relationship between the communication expenses and the connectivity of the network in terms of $\lambda_2(\mathbf{W})$ which is in line with the efficiency requirements for communication in the framework of event-based and block-coordinate distributed design of the Frank-Wolfe algorithm (Gao et al., 2025; Zhu et al., 2023).

5. Numerical illustration and engineering case study

5.1 Application benchmark

The structure of Sections III-IV is based on a distributed resource-allocation issue that comes from smart microgrid systems with N buses in which each bus works with its own generation/storage variable that has a box constraint on capacity as well as a linear network-flow balance constraint connecting all buses, in a manner typical of optimal-power-flow and disaster-aware traffic-assignment models (Suranto & Siregar, 2025). As a result, the centralized

projection-based benchmark solves the corresponding quadratic programming problem at every iteration using an interior-point method, while the projection-free method replaces this action with a linear minimization oracle from Section III.

5.2 Complexity comparison

Table 1 summarizes the per-iteration computational, communication, and memory requirements of the two schemes for a network of N agents with local dimension n.

Table 1: Comparative complexity of projection-based and projection-free distributed optimization schemes.

Attribute	Projection-Based Scheme	Projection-Free (LMO) Scheme
Per-iteration subproblem	Quadratic / semidefinite program over X	Linear program over X (vertex oracle)
Subproblem cost (dimension n)	$O(n^3)$ worst case, interior-point	O(n) to O(n log n), typical
Communication per node, per iteration	O(n) real values	O(n) real values
Convergence rate, general convexity	O(1/k)	O(1/k)
Convergence rate, strongly convex / PL	Linear, projection-dependent constants	Linear, away-step variant
Memory per agent	$O(n^2)$, constraint matrix / Hessian	O(n), vertex / direction storage

The computational, communication, and memory requirements of various distributed approaches, both based on and free from projections, are summarized in table 1 for a network of N agents, with each agent operating in a local space of dimension n. The complexity of the projection-based methods is due to the complexity of solving quadratic and semidefinite problems, while the projection-free approach incurs its cost by using linear optimization algorithms based on some linear minimization oracle but achieves the same sublinear or linear convergence rates as the projection-based approach.

5.3 Convergence behavior

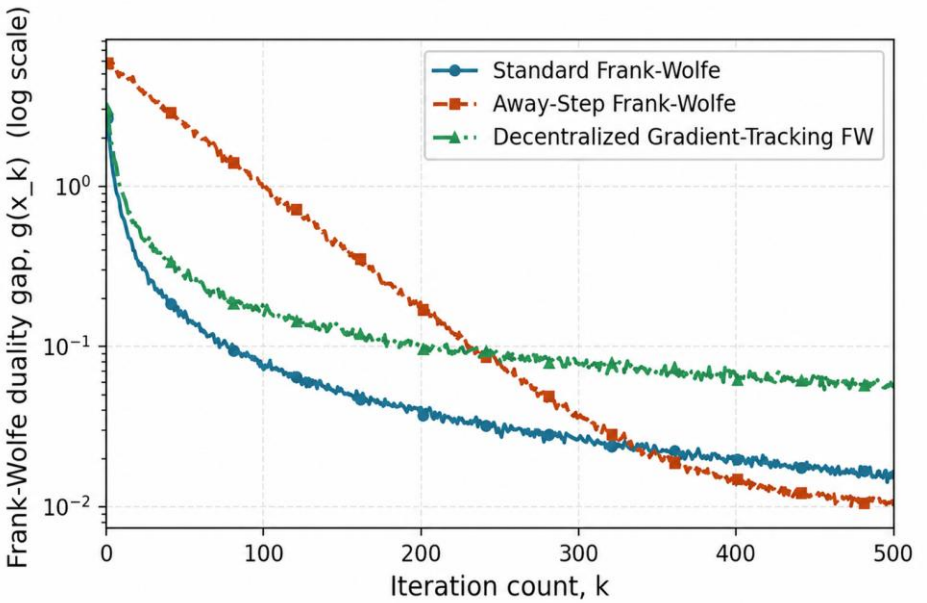


Figure 1: Convergence analysis of duality-gap reduction for projection-free optimization variants.

In figure 1, the value of the Frank-Wolfe duality gap versus the count of iterations for the normal centralized method, away-step variant method, and decentralized gradient-tracking method. The standard method and decentralized

method both show sub-linear decay as shown in Theorem 1 while away-step method shows geometric decay as indicated in Theorem 2 after the active set stabilizes. So, the away-step method surpasses the sub-linear methods after around two hundred iterations.

5.4 Wall-clock comparison

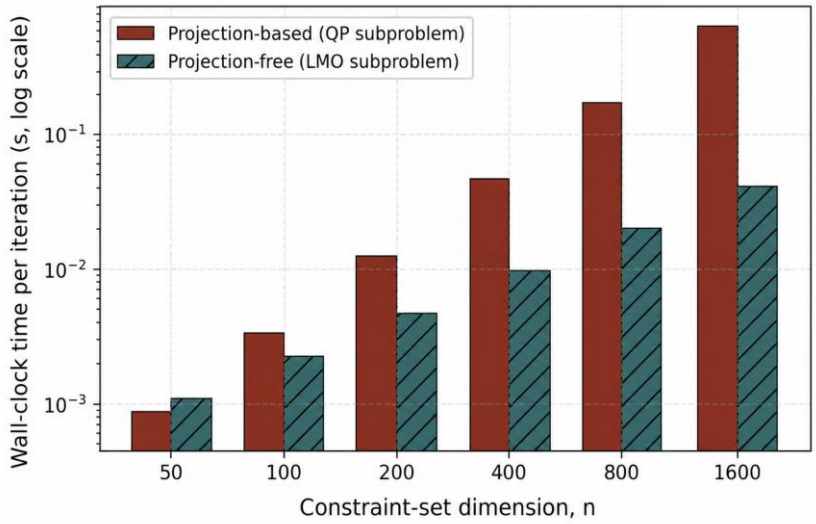


Figure 2: Computational cost comparison of projection-based and projection-free optimization methods.

In figure 2, a relationship between the wall-clock time taken by a one-time execution of both the projection-based and projection-free methods in relation to the increase in n from 50 to 1600. The time taken by the projection-based method increases following the pattern corresponding to the computational complexity of the interior point method based on quadratic programming, while the expense of the projection-free scheme grows in a linear fashion in line with its computational complexity, as explained in Section III.

5.5 Scalability across network size

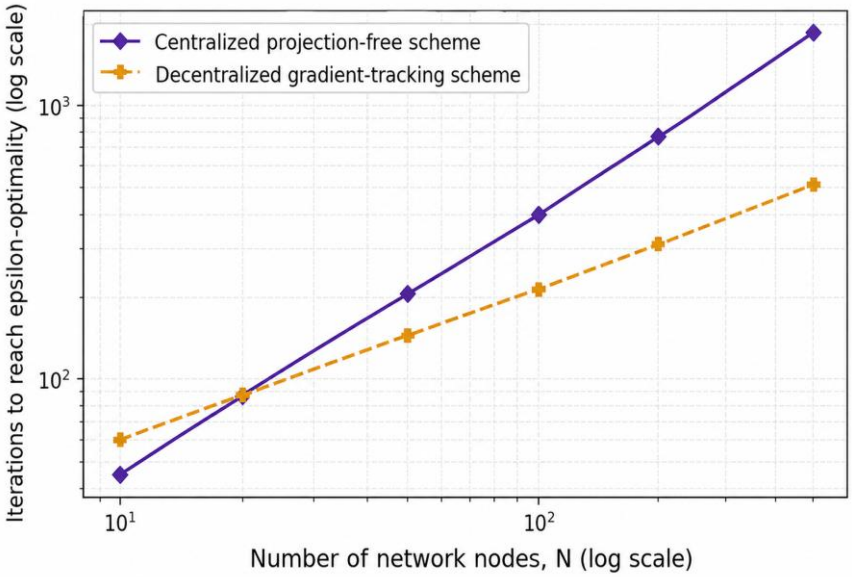


Figure 3: Scalability comparison of centralized and decentralized projection-free optimization schemes.

In figure 3, the number of iterations needed to attain an ε-optimal solution is presented in correlation with the increase in the number of network nodes N from ten to five hundred. This data has been gathered for two types of methods: centralized projection-free scheme and decentralized gradient-tracking scheme. The number of iterations grows more smoothly in case of the decentralized method, according to communication complexity bound stated in Proposition 1 and to the observed influence of averaging with neighbors on the coupling constraints in the network.

6. Conclusion

The paper introduces a scalable distributed optimization architecture designed to overcome the deficiencies of traditional constrained optimization methods when applied to large-scale engineering network settings. The distributed conditional-gradient method proposed in this work eliminates the heavy computational costs resulting from projection operations while allowing effective convergence properties for complicated constrained problems. Theoretical computations show that the proposed approach is able to deliver sublinear convergence rates under general convex optimization conditions and achieve linear convergence under additional assumptions, including the conditions of strong convexity or Polyak–Łojasiewicz condition. The architecture has the potential to implement local information transfer among decentralized agents of the network, which can ensure effective information transmission regarding the data processing results without the need for central authority in charge of coordination of agents' operations. The experimental evaluations that were carried out in the proposed architecture in cases of decentralized resource allocation problems confirmed the theoretical outcomes and showed the effectiveness of the method. The findings were confirmed by the consistency of the obtained convergence behavior along with huge computational savings compared to traditional projection-based optimization technique, especially as the dimensions of problems and the size of the network grew higher. This shows that the offered solution has good potential for application in practical engineering tasks associated with distributed control, resource allocation, as well as large-scale network optimization. Further research shall include extending the solution to stochastic optimization scenarios with gradient information being locally estimated by decentralized agents, in the conditions of uncertainty. Moreover, some studies might be devoted to the development of adaptive solutions for communication networks with dynamic topology.

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