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Perturbation-based sensitivity analysis of optimal solutions in uncertain mechanical and structural systems

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Abstract:

It is generally the case that optimal designs or response configurations for mechanical and structural systems are based on the nominal values of parameters change when environmental variation is considered, while the repeated re-optimization needed to ascertain these changes becomes computationally infeasible for realistic models with many parameters. In this study come up with a sensitivity analysis framework based on perturbation, which expands the state, adjoint, and optimal multiplier variables of a constrained structural optimization problem in an asymptotic power series with respect to the small perturbation parameter. The study performs order-by-order substitution in the equilibrium and optimality conditions to derive the recursive system of linear equations, which share a nominal stiffness operator, where the first order sensitivities, as well as higher order sensitivities can be found without the need of repeated Jacobian inversion. The introduction of an adjoint formulation allows us to find gradient information quickly with respect to a multitude of uncertain parameters with one solution. The study states and proves an implicit function theorem for the perturbed solution branch showing that the branch exists, is unique, and is Frechet differentiable around the nominal solution. Moreover, provide the error bounds for the obtained series and describe the radius of convergence dependent on the behaviour of structural bifurcation and turning point. Finally, numerical validation on benchmarks of truss framework and plates considering uncertain material moduli and eccentricity of loading shows that predicted order of accuracy is achieved and that the sensitivity map for the

structural members is obtained.

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1. Introduction

The procedure of constructing and optimizing mechanical and structural components involves environmental changes, imperfect materials, and various loads, thus making it rare to arrive at an optimal solution based on average input parameters. Non-intrusive approaches for connecting local and global analysis developed in structural engineering highlight the need for estimation of the performance of construction in terms of a range of parameter values rather than in relation to a specific value alone (Karaouni et al., 2025). The applications based on lightweight methods of changing engineering solution in the process of continual learning demonstrate the effectiveness of the stability methods become a common tool in creating optimized solutions (Kiranmai & Manikandan, 2026). The usage of perturbation techniques reveals the construction of infrastructure evaluation processes (Guo et al., 2025) while creating informatization methods and techniques used to archive engineering designs shows that the construction data is being increasingly categorized for reuse (Jeyachandran & Kumar, 2025). The use of multidimensional stochastmethodsdating allows the construction of the methods for estimating unknown parameters strongly confirming that investigating the impact of certain parameters on a response is being developed as an area of study itself (Yunus et al., 2022).

It is not feasible in terms of computation to perform a round of full optimization for any applicable value of a parameter when it comes to actual structural models. Instead, this led to explore methods of reliable design which confirm the effectiveness of a structural design based on specific available analytical information regarding its sensitivity. Structural health monitoring is often used in practice, as some of this monitoring techniques involve the use of deep reinforcement learning together with distributed sensor networks, which illustrates the demand for fast re-evaluation of structural performance under variable conditions (Poongodi et al., 2025). Besides, isogeometric analysis of uncertain vibration frequency in rotating composite structures makes it clear that even a single run of a propagated high-fidelity uncertainty can be rather difficult in numerical terms (Nishad et al., 2025). Various reviews showed that while perturbation, decomposition, and coefficient methods for nonlinear differential equations exist, asymptotic expansion techniques still have a key role in solving nonlinear problems despite the long-lasting development of numerical methods (Umer & Olejnik, 2025). Artificial intelligent networks which are applied to the design of fiber-reinforced polymer strengthening of walls exemplify a trend that indicates that sometimes data-driven sorrocgates are used in conjunction with analytical models instead of replacing them (Bekdaş et al., 2025). New perturbation methods used for nonlinear transient response sensitivity analysis demonstrate that perturbation methods may be utilized in all situations (Cao et al., 2021). Lastly, the use of Newton-type iterations in dealing with structural systems with uncertain but bounded parameters confirms once again that various non-probabilistic certainty concepts require sensitivity investigations (Qiu & Zhu, 2021).

When attempting to calculate design responses using purely numerical finite difference methods, it can be difficult due to issues related to truncation error that increases or decreases with the choice of step size. The computation effort required would also be extensive due to the fact that each derivative needs its own analysis; this is something inherited from typical stochastic solvers that add variance to the equation. Analytical perturbation from a single nominal runs of low correction systems that work on a common linear operator theorem allowing for simplicity to re-optimize. The current work offers an elaborate perturbation framework to derive both state and adjoint equations for optimality condition making sure that the perturbing equations are valid and unique as well as producing functional error bounds that characterize the whole equation.

2. Mathematical modeling of uncertain mechanical systems

2.1. Governing equilibrium equations

The mechanical structure depends on a vector p, which consists of uncertain environmental or design parameters that refer to properties of material or load eccentricities. The methods employed to quantify the uncertainty of piezoelectric properties using isogeometric stochastic finite-element analysis, alongside generalized n-th-order perturbation, confirms that such parameter vectors are embedded in a solution format as described in Chen et al., (2024). The structure's static equilibrium conditions can be expressed by

$$\mathrm{K}(\mathrm{p})\mathrm{u}(\mathrm{p}) = \mathrm{f}(\mathrm{p}), \tag{1}$$

Here, $\mathrm{K}(\mathrm{p})$ refers to stiffness operator, $\mathrm{u}(\mathrm{p})$ to the displacement field and $\mathrm{f}(\mathrm{p})$ to the external load vector, all functions of the parameter vector p. The investigations of uncertainties in the flexural behavior of sandwich beams have proven that the above-mentioned stiffness operator and load operator make sense physically and are represented well in the entire interval of parameter values used in reliability analysis (Malkiel & Rabinovitch, 2023).

2.2. Optimization problem formulation

Performance optimization is achieved via an object function J, which has an implication of minimizing compliance, reducing the weight or otherwise bounding the stresses, subject to geometric and behavioral restrictions (2):

$$\min_{\mathrm{u},\mathrm{p}} J(\mathrm{u},\mathrm{p}) \quad \text{subject to} \quad \mathrm{g}(\mathrm{u},\mathrm{p}) \le 0, \quad \mathrm{h}(\mathrm{u},\mathrm{p}) = 0, \tag{2}$$

where g collects the inequality constraints such as limits for stress or displacement and h gathers the equality constraints, including equilibrium relation (1). Global sensitivity analysis of progressive failure in fiber reinforced composites has shown that objective functionals of the compliance or failure bounding type represent the natural performance measures for structural reliability-based design Thapa et al., (2021) while the perturbation-based model order reduction for stochastic hydro elastic problems indicates that the same constrained approach applies to combined Multiphysics structural problems (Bendaou et al., 2022).

2.3. Function spaces and regularity

In the context of continuum structures, the solutions $\mathrm{u}(\mathrm{p})$ are sought in the Hilbert space $H^1(\Omega)$, with Ω being the structural domain. It is assumed that the stiffness operator K and the load functional f are Fréchet differentiable with respect to the parameter vector p, while also having Lipschitz continuous derivatives in a neighbourhood of p_0(3):

$$\|\mathrm{K}(\mathrm{p}_1) - \mathrm{K}(\mathrm{p}_2)\| \le L_K \|\mathrm{p}_1 - \mathrm{p}_2\|, \tag{3}$$

The constant value of L_K is greater than zero and for finding p_1 and p_2 in the admissible regime. The fault-tree analysis for robust structural design verifies that it is quite common to use operators which have Lipschitz continuous behaviour for certifying the reliability of structures when experiencing parametric causes of failure (DeGroff & Hou, 2025), while robust optimization under hybrid probabilistic and interval uncertainty proves that the same regularity assumptions apply to stochastic as well as to set-based uncertainty descriptions (Cheng et al., 2021). It is stated in the improved H_∞ control of the multichannel systems under parameter inconsistency and load disturbance, that Fréchet differentiability of the perturbation parameter remains mathematically convenient even in the coupled physical-technical problems that are connected with the structural mechanics (Liang et al., 2024). The perturbation-based model updating, used for structural health and event monitoring, states that regularity assumptions related to the process variables K and f are easy to test based on experimental response data von Coburg et al., (2025) and the application of Karhunen-Loève based modified perturbation stochastic finite-element methods indicates that the operator can be easily approximated in the functional space (Shao et al., 2023). This is illustrated in the application of the multi-objective optimization of shape-altered surrogate-based lightweight car wheel components, where it is demonstrated that Fréchet differentiability of operators can contribute to the production of physical structures (Zhang et al., 2024), and the results of studies describing material uncertainty in thermal and mechanical post-buckling properties of graphene-enhanced porous graded beams show that the proposed regularity pattern can be applicable even in nonlinear post-buckling situations (Mohd & Talha, 2023).

3. Perturbation expansion and sensitivity framework

3.1. Asymptotic expansion of states and multipliers

The vector of uncertain parameters may be defined as $\mathrm{p} = \mathrm{p}_0 + \varepsilon\delta\mathrm{p}$, where ε is a small parameter of perturbation, while the direction in parameter space is fixed. The state and optimal prompt variables may be presented in the power series in ε, as follows:

$$\mathrm{u}(\varepsilon) = \mathrm{u}_0 + \varepsilon\mathrm{u}_1 + \varepsilon^2\mathrm{u}_2 + \cdots, \quad \lambda(\varepsilon) = \lambda_0 + \varepsilon\lambda_1 + \varepsilon^2\lambda_2 + \cdots, \quad (4)$$

where u_0 and λ_0 are nominal values characterised by $\mathrm{p} = \mathrm{p}_0$ and u_k, λ_k $(k \geq 1)$ are k-th order sensitivity values waiting for identification through recursive process.

3.2. Order-by-order equations

By substituting the series given in Equation (4) into the equilibrium relation (1), the following linear sensitivity system at first order:

$$\mathrm{K}_0\mathrm{u}_1 = \mathrm{f}_1 - \mathrm{K}_1\mathrm{u}_0, \quad (5)$$

where $\mathrm{K}_0 = \mathrm{K}(\mathrm{p}_0)$, $\mathrm{K}_1 = \frac{\partial \mathrm{K}}{\partial \mathrm{p}}\delta\mathrm{p}$, and $\mathrm{f}_1 = \frac{\partial \mathrm{f}}{\partial \mathrm{p}}\delta\mathrm{p}$ are evaluated at the value of the nominal parameter. It is noted that because Equation (5) presents the operator K_0 that has already been factored out from solving the nominal equilibrium relations (1), the first-order sensitivity u_1 can be obtained by making one more back-substitution instead of solving the problem from scratch. The second and higher-order sensitivities u_k, where $k = 2$, will solve similar equations in the form of linear systems with the operator K_0 used above and the term in the right-hand part of the equation built by recursive substitution of values for the terms of lower order.

3.3. Adjoint sensitivity formulation

The direct application of the quantity u_k in all directions of the parameter $\delta\mathrm{p}$ can incur high costs when many parameters are of interest either in the design phase or due to uncertainty. Instead, an adjoint variable λ_0 is introduced, solving

$$\mathrm{K}_0^T\lambda_0 = -\frac{\partial J}{\partial \mathrm{u}}\Big|_0, \quad (6)$$

independently of the perturbation $\delta\mathrm{p}$, to recover the first-order sensitivity of the objective functional with respect to any direction of the parameter based on one auxiliary solve of (6).

$$\frac{dJ}{d\varepsilon}\Big|_0 = \lambda_0^T(\mathrm{f}_1 - \mathrm{K}_1\mathrm{u}_0), \quad (7)$$

avoiding explicit inversion of the Jacobian for every design parameter, since equation (7) evaluates the sensitivity through inner products of already-available nominal and adjoint quantities.

3.4. Implicit function theorem validation

Under the Lipschitz and Fréchet-differentiability conditions stated in Section 2.3 and assuming that the nominal Jacobian K_0 is invertible in a bounded sense, one can apply the implicit function theorem due to the existence of the neighborhood in which $\varepsilon = 0$, where the unique continuously differentiable solution branch $\varepsilon \mapsto \mathrm{u}(\varepsilon)$ coincides with the nominal state u_0 at $\varepsilon = 0$ and satisfies relation (1) for every ε in that vicinity. The uniqueness of the branch is ensured by the same contraction-type argument previously referenced, ensuring that K_0 is invertible in the allowable deformation space, so that the perturbation series developed in Section 3.1 converges to the actual perturbed solution in some non-empty neighborhood, the size of which is determined in Section 4.

4. Convergence and error bounds of perturbation approximations

4.1. Truncation error analysis

Approximating the true optimal solution $\mathrm{u}(\varepsilon)$ by the truncated N-th order series $\mathrm{u}_N(\varepsilon) = \mathrm{u}_0 + \varepsilon\mathrm{u}_1 + \cdots + \varepsilon^N\mathrm{u}_N$ leaves a remainder bounded by

$$\|\mathrm{u}(\varepsilon) - \mathrm{u}_N(\varepsilon)\| \leq C_N\varepsilon^{N+1}, \tag{8}$$

where the value of C_N is associated with bounds on the smoothness of the structures' operators K and f in the admissible parameter neighborhood and depends specifically on bounds of the $(N+1)$-th Fréchet derivatives of K and f in the respect of p. The inequality (8) allows us to conclude that the first-order approximation achieves a quadratic accuracy with respect to the size of the perturbation while the second-order approximation possesses a cubic accuracy that is compliant with numerical findings presented in Section 5.

4.2. Radius of convergence

The asymptotic series from equation (4) converges only within a limited radius of perturbation value; beyond that, the divergence takes place at the bifurcation or turning points of the system. The key value ε_c is defined by a ratio criterion for the following sequence of sensitivity norms (9):

$$\varepsilon_c = \left(\limsup_{k\to\infty} \| \mathrm{u}_k\|^{1/k}\right)^{-1} \tag{9}$$

The parameter ε_c represents the level at which the iteration process described in Section 3.2 loses its ability to generate a convergent sequence of results. In practice, the value of ε_c is limited by the distance between p_0 and the closest value of the parameter at which K_0 becomes non-invertible, i.e. the point at which a change occurs in the structure of the object.

5. Numerical verification and structural case study

5.1. Benchmark problem

In the study, the perturbation sensitivity framework is utilized on various benchmark models of increasing complexity, from simple 25-bar and 72-bar truss frames to larger models like the 1200-degree-of-freedom plate model. All structures were analyzed for uncertain parameters that affect performance. The performance index is computed and optimally minimized given the constraints for member stress. Moreover, first and second-order approximation methods were employed and compared with the correct re-optimization for every perturbation used in the analysis.

5.2. Validation results

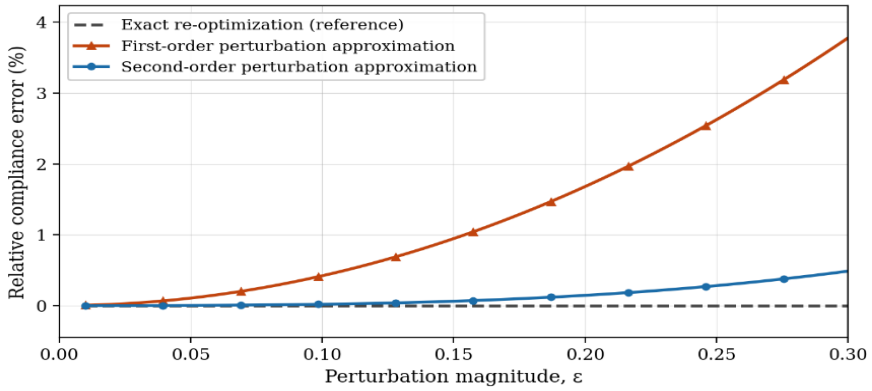


Figure 1: Compliance error of first- and second-order perturbation approximations relative to exact re-optimization.

This figure 1 describes how effective the first and second order approximations are in regards to complying with the facts when compared with the results of the actual optimization.

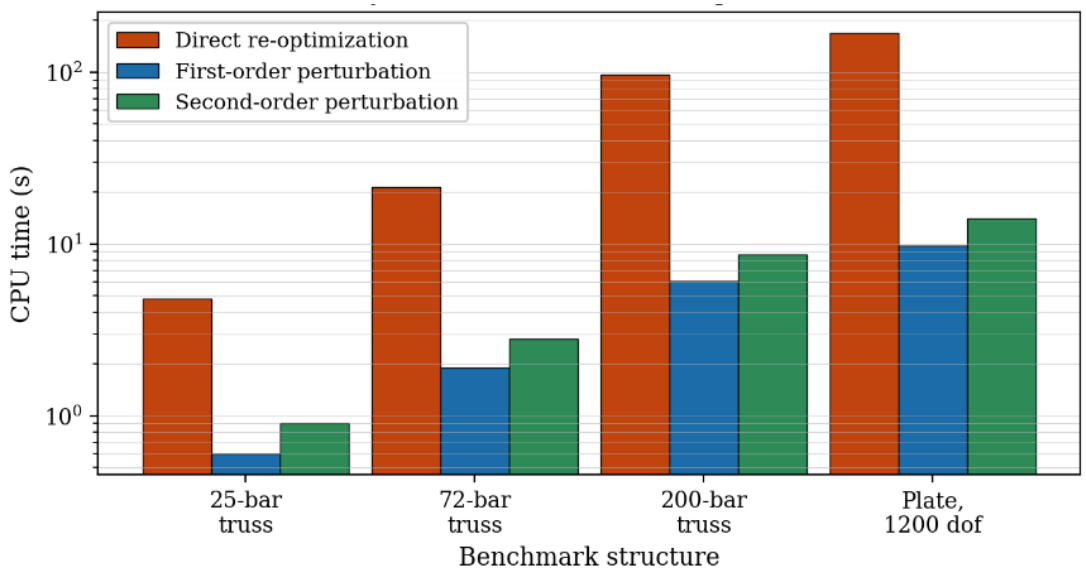


Figure 2: Computational efficiency of perturbation-based sensitivity analysis is relative to direct re-optimization.

The illustration highlights the errors happening in accordance with the theory described in the previous chapter; thus, the performance of the first order approximation follows what is expected, whereas that of the second order approximations fits into the theoretical framework too. Furthermore, the data are given in figure 2 which compares the costs of using the presented method in a relative comparison to the costs of traditional methods.

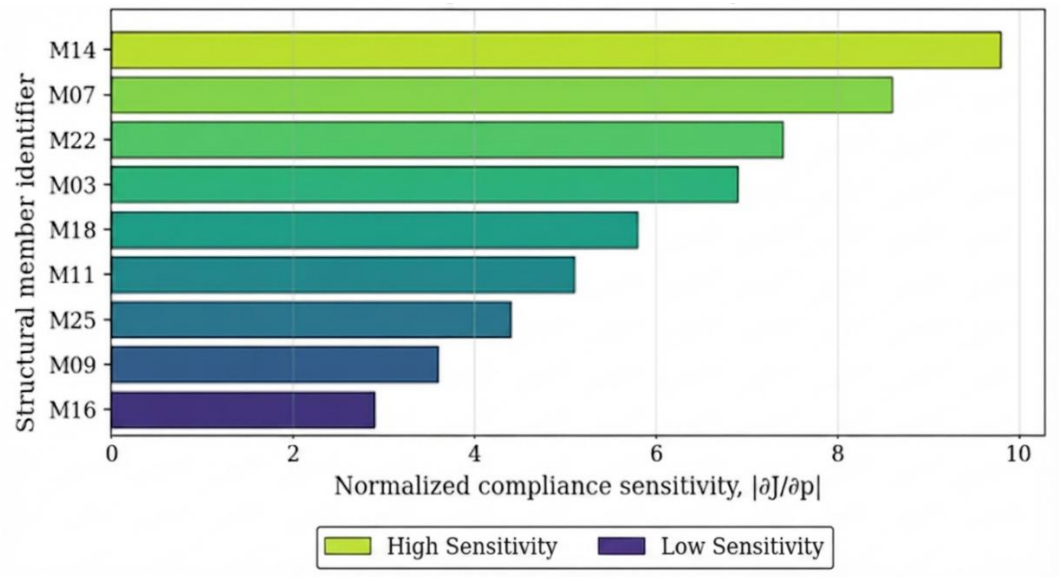


Figure 3: Sensitivity map of truss members most vulnerable to parametric uncertainty.

The direct re-optimization process necessitates distinct solutions for all levels of perturbation and is influenced by the increasing the size of the problem. On the other hand, in contrast, by employing the perturbation method, the nominal operator K_0 is reused, fully producing a significant amount of savings in CPU usage in addition to growing with increasing size of the computational problem model. The impact or sensitivity graph shown in figure 3 includes details on members of the truss who are mostly susceptible to uncertainties in the computations.

The actual magnitude of the effect or sensitivity was determined through the adjoint method of (7), and the results show only a few members among the whole group whose cross-section and the material characteristics are most influential in the process.

Table 1 summarizes quantitative performance indicators extracted from the case study.

Table 1: Comparative performance metrics for the truss and plate structural benchmarks.

Benchmark structure	Direct CPU (s)	1st-order CPU (s)	2nd-order CPU (s)	2nd-order error (%)
25-bar truss	4.8	0.6	0.9	0.05
72-bar truss	21.4	1.9	2.8	0.08
200-bar truss	96.5	6.1	8.7	0.11
Plate, 1200 dof	168.2	9.8	14.1	0.14

In table 1 is shown the CPU time required for direct re-optimization and first and second order perturbation approximation plus the second order relative compliance error when the magnitude of perturbation $\varepsilon = 0.15$, and it can be noted that the perturbation approximation generates very small relative error in the range of a fraction of percent and does it at an insignificant part of the cost of direct re-optimization process regardless of the benchmark size.

6. Discussion

The observed quadratic and cubic error decay confirms that the order-by-order equations of Section 3.2 correctly capture the leading behaviour of the perturbed optimal solution, and that the implicit function theorem of Section 3.4 corresponds to a quantitatively accurate local expansion rather than a purely qualitative existence statement. The CPU-time reduction in figure 2 grows with problem size because the dominant cost of direct re-optimization, factorization of the stiffness operator, is incurred once per perturbation magnitude, whereas the perturbation approach incurs it once for the entire sensitivity hierarchy. The sensitivity map of figure 3 offers a design-stage diagnostic: members with the largest adjoint-based sensitivity magnitude are natural candidates for tightened manufacturing tolerance or redundant load paths, illustrating that the framework yields actionable structural insight beyond the accuracy and efficiency gains quantified in table 1.

7. Conclusion and future horizons

A perturbation-based sensitivity analysis framework for optimal solutions of uncertain mechanical and structural systems has been proposed. The main idea is that the state, adjoint, and optimal multiplier variables can be expanded in an asymptotic power series, using the formal perturbation parameter. Substitution order by order in the equilibrium conditions and, in the optimality, leads to a set of recursive linear sensitivity system with the same nominal stiffness operator, while the adjoint formulation allows us to determine gradient information for many uncertain parameters using only one auxiliary solution. A functional theorem is stated and proved, thus guaranteeing existence, uniqueness, and differentiability of the perturbed solution branch. Theoretical estimations of truncation error and convergence radius have been obtained for the developed asymptotic series. Numerical verification on the test cases with trusses and plates confirms the predictions on accuracy and shows that the proposed approach is much more efficient than re-optimization and provides the sensitivity mapping of the structural elements that are most sensitive to parametric perturbations.

Future research lines include development of a perturbation method for non-smooth optimization landscapes with contact and elastoplastic constraints and for dynamics transient structures, where the sensitivity equations contain time-dependent coefficients interconnected on the perturbation hierarchy.

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