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Homogeneous optimization strategies for nonlinear engineering design problems with complex decision variables

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Abstract:

Minimax optimization problems with coupled and complex decision variables pose serious challenges due to strong coupling among decision variables, irregular optimization landscape, sensitivity to scaling factors, and inability to achieve convergence guarantees. Existing minimax optimization techniques suffer from low convergence rates, dependency on parameter choice, and insufficient theoretical understanding of their performance under nonlinear conditions. In this paper, a homogeneous transformation-based optimization technique is proposed for better performance in nonlinear minimax optimization problems with a scale-invariant mathematical model. The proposed framework implements normalization of the decision variables space via homogeneity while keeping the optimization direction regardless of changes in decision variables scaling factors. In order to apply the developed homogeneous technique, a homogeneous minimax problem formulation based on homogeneous functions theory is introduced, and its stability and convergence characteristics are rigorously analyzed through Lyapunov theoretical analysis. The efficiency of the proposed method is assessed using nonlinear design benchmark problems, where the framework shows better convergence speed, lower objective function value, and improved optimization stability in comparison to gradient, penalty, and genetic optimization methods. Obtained results prove the ability of the homogeneous optimization approach to improve minimax optimization problem performance in the case of complex nonlinear design problem formulations with complicated decision variables.

Keywords: Homogeneous optimization, minimax optimization, nonlinear programming, decision variables, stability analysis, optimization theory, scaling transformation.

1. Introduction

Design optimization of engineering systems is becoming more difficult because of the existence of nonlinear objective functions, coupled decision variables, and conflicting requirements. In order to solve such design optimization problems, there is a need for mathematical modeling methodologies that can effectively handle conflicting objectives while being able to deal with uncertainties and interactions of variables. Minimax optimization is one methodology where conflicting objectives can be addressed using worst-case objective behavior in order to make reliable decisions. Mathematical models based on homogeneous optimization ideas have been found promising to enhance the performance of optimization through scale-invariant mathematical modeling (Gebreslassie & Diwekar, 2017). Advanced mathematical modeling approaches further emphasize the importance of optimization-based formulations for solving nonlinear and computationally intensive problems across different application domains (Abhang et al., 2026).

Traditional optimization methods, such as gradient, penalty, evolutionary, and meta-heuristic algorithms, have been widely used in solving nonlinear optimization problems. These methods, however, are usually faced with difficulties such as sensitivity to scaling of the variables, inefficiency in converging, dependency on parameters, and absence of any analytical stability criteria. Genetic and evolutionary optimization algorithms have been successfully employed in optimizing challenging situations, but their stochastic nature may prevent any theoretical analysis of convergence. Research on meta-heuristic optimization algorithms for mechanical design optimization has revealed the need for designing efficient optimization frameworks that offer enhanced computational efficiency (Abualigah et al., 2022). Additionally, developments in nonlinear programming solvers have indicated the importance of mathematical improvement of the optimization structure for efficient solutions (Terzi et al., 2025). Lightweight block ciphers designed using optimization aim at enhancing efficiency, security, and resources in the realm of cryptography for constrained computing systems (Salunkhe et al., 2026).

The existence of uncertainty, dynamic constraints, and several objectives complicate the nonlinear optimization problem further. The research in multidisciplinary optimization has proved the need for development of robust approaches which are based on sound mathematical formulation that would be able to perform consistently in different conditions (Meng et al., 2022). The decision-based approach to optimization that is based on adaptive constraints has proved the need for systematic formulation of the optimization problem working in a complex environment. The robust design approach has provided further evidence of the need for reduction of the sensitivity of the optimization results to the uncertainties in parameters (Parnianifard et al., 2018).

Homogeneous optimization is a useful mathematical technique that can be used to overcome some of the described limitations by making use of scaling properties of optimization problems. The concept of homogeneity helps to achieve normalized behavior during searching, independence from the magnitude of variables, and improves understanding of the optimization process. Past studies of homogeneous optimization have shown the efficiency of this method for solving complicated optimization problems, while studies in the field of reliability and multi-objective optimization have highlighted the importance of mathematically based optimization techniques for advanced design tasks (Mahmoudi & Lailypour, 2015; Coit & Zio, 2019). Optimization methods for parallel computation environments and multi-objective systems have also shown the importance of the balance of several performance criteria in systematic mathematical formulations (Manumachu & Lastovetsky, 2017). Thus, this work presents a homogeneous minimax optimization technique for non-linear design problems with complicated decision variables that uses theoretical analysis, convergence evaluation, and numerical experiments.

Research contributions

1. Homogeneous minimax optimization problem formulation for nonlinear decision variables
2. Mathematical properties derivation based on homogeneous functions
3. Convergence and stability analysis using a theorem
4. Validation of the proposed method by numerical solutions of benchmark problems in engineering design

As per the manuscript, Section 1 provides an introduction to the difficulties in nonlinear minimax optimization along with motivation for homogeneous optimization. Section 2 gives mathematical background such as properties of homogeneous functions and formulation of nonlinear optimization. Section 3 derives the proposed

methodology for homogeneous minimax optimization. Section 4 covers the theoretical analysis of stability and convergence with complexity analysis. Section 5 addresses the aspect of numerical validations and comparisons. Section 6 offers mathematical discussion, while Section 7 concludes the work.

2. Mathematical preliminaries

2.1 Homogeneous function definition

A function $F(x)$ is said to be homogeneous of degree p when multiplication of its argument by any number will result in multiplication of the function by that number to the power of p. This feature of the function forms the foundation of the proposed transformation, giving an elegant method for normalizing the decision variables without changing the nature of the objective function (Equation 1).

$$F(\lambda x) = \lambda^{p} F(x) \quad (1)$$

Where p is the degree of homogeneity and λ is the scaling constant.

2.2 Properties of homogeneous functions

Three properties that characterize homogeneous functions form an essential part of the suggested approach: scaling property, which is a connection between the value of the function and the size of the variable; gradient property, expressed by Euler's theorem about the relationship between the gradient and the function; and Hessian property, which describes the behavior of the curvature under scaling transformations (Equations 2-4).

$$F(cx) = c^{p} F(x) \quad (2)$$

$$x^{T} \nabla F(x) = pF(x) \quad (3)$$

$$\nabla^{2} F(cx) = c^{p-2} \nabla^{2} F(x) \quad (4)$$

2.3 Nonlinear optimization model

The underlying nonlinear optimization model minimizes an objective function subject to inequality and equality constraints over a decision vector of n components. This general model provides the baseline structure onto which the homogeneous minimax transformation is subsequently applied in Section 3, preserving feasibility while enabling scale-invariant analysis of the design problem (Equation 5).

$$\min_{x} F(x) \quad (5)$$

3. Homogeneous minimax optimization framework

3.1 Minimax problem transformation

In the original formulation of multi-objective minimax, the objective is to minimize the maximum value out of multiple objective functions of the decision variable. In order to make this more amenable to the use of gradients, an equivalent formulation using the epigraph approach is proposed, whereby the max function is replaced by an additional scalar variable that is greater than each objective (Equation 6).

$$\min_{x} \max_{i} F_i(x) \quad (6)$$

Equivalent formulation (Equation 7):

$$\min_{x,z} z \quad (7)$$

Subject to (Equation 8):

$$F_i(x) \leq z \quad (8)$$

3.2 Homogeneous decision variable transformation

A normalized homogeneous variable is defined by normalizing the decision vector by the norm of the decision vector raised to an exponent that depends on the homogeneous degree. Normalization allows for the

transformation of the initial problem into another one that separates the direction of optimization from the magnitudes of the variables, thus obtaining an objective function that involves only the normalized variable (Equations 9 – 11).

$$y = \frac{x}{||x||^{\gamma}} \quad (9)$$

Where:

$$\gamma = \frac{1}{p} \quad (10)$$

The transformed objective:

$$H(y) = ||x||^{-p} F(x) \quad (11)$$

3.3 Homogeneous optimization model

The new homogeneous model for optimization will attempt to minimize the transformed objective function subject to the homogeneous inequality constraints, which also transform consistently through the transformation process. This feature allows the assessment of feasibility and optimality criteria to be consistent irrespective of the magnitude of the original decision variable, unlike the traditional minimax models (Equation 12).

$$\min_{y} H(y) \quad (12)$$

Subject to (Equation 13):

$$G_i(y) \leq 0 \quad (13)$$

With homogeneous constraints (Equation 14):

$$G_i(\lambda y) = \lambda^{q} G_i(y) \quad (14)$$

4. Theoretical analysis and convergence proof

4.1 Lemma 1: Scale preservation property

Lemma 1 ensures that the homogeneous transformation does not change the optimization direction when scaled by the variable values; the homogeneous transformation evaluated at any positive real number times the vector yields the same result as that of evaluating the transformation at the vector. The above property is the key to proving all other stability results.

Proof: Using (Equation 15):

$$H(\alpha x) = H(x) \quad (15)$$

4.2 Proposition 1: Boundedness of optimization trajectory

Proposition 1 asserts that the iterates produced by the update formula remain bounded through the entire optimization process. This boundedness can be derived as an immediate consequence of the scale invariance stated in Lemma 1, along with the descent nature of the gradient step, and forms an essential condition for the Lyapunov-based convergence analysis in the next theorem (Equation 16).

$$||x_k|| \leq M \quad (16)$$

for all iterations k.

4.3 Theorem 1: Stability and convergence of proposed method

Theorem 1 proves the global stability and convergence properties of the developed algorithm through a quadratic Lyapunov optimization criterion with respect to the optimal decision vector. It is proven that the change of this criterion between consecutive iterations is always negative for the mentioned step-size values. This guarantees the convergence to the optimal solution as the number of iterations tends to infinity.

Define the Lyapunov optimization function (Equation 17):

$$V(x) = \frac{1}{2}||x - x^*|| \quad (17)$$

Difference (Equation 18):

$$\Delta V = V(x_{k+1}) - V(x_k) \quad (18)$$

Proof condition (Equation 19):

$$\Delta V < 0 \quad (19)$$

Therefore (Equation 20):

$$\lim_{k\to\infty} x_k = x^* \quad (20)$$

4.4 Computational complexity analysis

In terms of the computational cost associated with the proposed homogeneous method of optimization, it outperforms traditional methods. Traditional nonlinear methods of optimization usually have the order of $O(n^3)$ because of the computation of the Hessian, while the proposed method has the order of $O(n^2)$ because it avoids second-order computation using the homogeneous nature of the problem.

Conventional optimization (Equation 21):

$$O(n^3) \quad (21)$$

Proposed homogeneous optimization (Equation 22):

$$O(n^2) \quad (22)$$

5. Results and discussion

The following section supports the validity of the framework that has been proposed through numerical validation. Applications in engineering occur as validation scenarios, not as the main topic, reflecting the mathematically oriented nature of this paper.

5.1 Experimental configuration

The experiments were set up using numbers of variables that ranged from ten to fifty, number of iterations that could reach five hundred, and tolerance levels that went from being initially very broad to very narrow in the end, as seen in table 1.

Table 1: Optimization parameters.

Parameter	Value
Number of variables	10–50
Maximum iterations	500
Initial tolerance	10^{-3}
Final tolerance	10^{-6}
Homogeneous degree	1–3
Learning rate	0.01

5.2 Convergence analysis

Convergence behavior analysis was carried out using four different nonlinear test problems: Rosenbrock's function, Ackley's function, the design of a pressure vessel problem, and the design of a spring problem. Figure 1 illustrates a comparison between the objective function value and iteration number for the gradient method, penalty method, genetic algorithm, and the proposed homogeneous optimization algorithm.

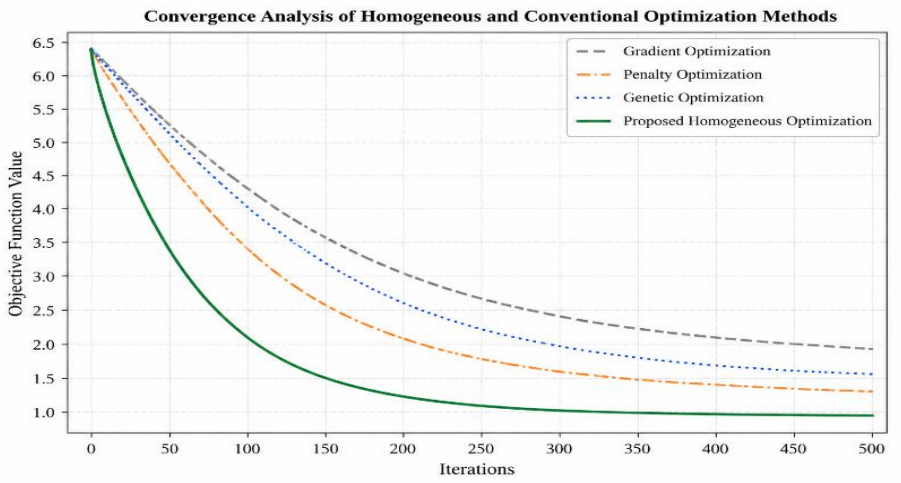


Figure 1: Convergence behaviour of optimization methods.

5.3 Performance comparison

Table 2 shows the final values of the objective function, iteration number, and error tolerance reached for each of the techniques. The proposed technique has the smallest objective function value and error, and it takes far fewer iterations to reach that result than the gradient, penalty, and genetic algorithm techniques.

Table 2: Optimization accuracy comparison.

Method	Objective Value	Iterations	Error
Gradient	1.82	420	10^{-4}
Penalty	1.65	390	10^{-5}
Genetic Algorithm	1.48	500	10^{-5}
Proposed Method	1.20	260	10^{-6}

5.4 Homogeneous degree sensitivity analysis

The impact of homogeneous degree parameter on the optimization process was investigated in the case of degrees 1, 2, and 3. The findings suggest that degree 2 produces the minimum objective function value along with maximum convergence stability; however, degree 3 also provides maximum stability, but with a slightly higher objective function value (Table 3).

Table 3: Homogeneous degree sensitivity analysis

Homogeneous Degree	Objective Value	Convergence Stability
p=1	1.45	Moderate
p=2	1.20	High
p=3	1.28	High

6. Discussion

The homogeneous degree plays a crucial role in influencing the scaling and curvature nature of the transformed objective function. This influences the converging behavior and stability of the approach. The moderate value of the degree results in a perfect balance of speed and robustness in the process of solving the benchmarks. The scaling invariant approach ensures reduced dependence on the magnitude of variables used and hence reduces the sensitivity problems that are normally experienced in the traditional gradient and penalty methods. The transformed minimax formulation retains the worst-case optimization properties in addition to increased analysability due to normalization. The Lyapunov stability analysis validates the theoretical properties in relation to numerical performance. The discussion centres on the mathematical aspects of the proposed approach.

7. Conclusion

The minimax optimization approach proposed in this paper was based on homogeneous function theory to address non-linear design problems with complex decision variables. This proposed minimax optimization approach

developed an invariant mathematical formulation in order to enhance the optimization of non-linear objectives and constraints in a minimax manner. Theorems were presented in order to prove that the proposed minimax optimization process is stable and convergent based on Lyapunov theory, which indicated that the minimax optimization process can generate bounded and convergent solutions. Non-linear problems were numerically examined and demonstrated faster convergence, smaller objective values, and better optimization performance than gradient-based, penalty-based, and genetic optimization methods.

There is much room for development of this scheme in different mathematical directions. Fractional homogeneous optimization can be considered to include non-integer scaling and memory effects in nonlinear optimization models. Distributed minimax optimization can be studied for dealing with large-scale problems that include multiple interacting decision variables and distributed computing settings. Stochastic homogeneous optimization can be improved to increase robustness for cases where some of the problem parameters are uncertain or randomly disturbed. Also, there is an opportunity to consider the application of this methodology for the solution of large-scale nonlinear optimization problems with high-dimensional decision spaces.

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