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Lyapunov functional-based stability analysis and control optimization in renewable energy management systems

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Abstract:

A minimax optimization formulation with respect to the Lyapunov functional for constrained control of nonlinear systems with bounded disturbances is introduced in this paper. This approach combines the Lyapunov-Krasovskii functional theory with minimax optimization, considering the stability constraint in the worst-case control problem and thus enabling both performance maximization and asymptotic convergence at the same time. An adaptive parameterization scheme for the Lyapunov matrix is devised to handle the case of parametric uncertainties in the system, while a control law is constructed analytically based on the introduced optimization model. The validity of the proposed approach is proved through three stability theorems that guarantee the positive definiteness of the introduced functional, asymptotic stability of the controlled system, and disturbance rejection. The proposed minimax optimization model is validated through the numerical study of the renewable energy management system, where its superiority over traditional PID, LQR, and robust control methods is demonstrated. In particular, the proposed approach achieves reduced settling time, minimal values of the Lyapunov functional residue, better tracking properties, and superior worst-case disturbance handling properties.

Keywords: Lyapunov functional, minimax optimization, stability-constrained control, adaptive control, renewable energy management, nonlinear systems.

Introduction

The design of an energy management system using renewable energy sources is associated with complicated nonlinear dynamics, uncertainty in energy generation, communication delay, and fluctuating operating conditions that demand mathematically stable control design. Due to the increased use of wind and solar energy, there are new problems with transient stability, frequency control, and disturbance rejection. Stability analysis using Lyapunov functions is one of the major mathematical methods that can be used to prove the convergence properties of dynamic energy systems. There have been studies that consider delay-dependent Lyapunov functions for the pitch control system of wind turbines to achieve stable performance (Ramakrishnan, 2020). Another study concerns the stochastic stability of power systems with wind energy integration (Ma et al., 2017).

Robust optimization theory further adds to the necessity of dealing with adversarial disturbances as well as uncertainties in the behavior of the system. Cyber-physical energy systems can be disturbed by various factors, hence the need for designing robust control techniques (Zhou et al., 2023). Enhanced control methods such as the sampled data method and fuzzy control methods have proven to be more effective in controlling non-linear dynamics and delays in the response of the system through improved mathematical stability criteria (Subramaniyam et al., 2025). Control optimization methods for hybrid renewable systems have also been proposed to ensure adaptation and energy utilization in uncertain power grids (Fadhil & Zadeh, 2025; Saifnazarov et al., 2025). Analyzing the energy consumption of R-based machine learning models is vital to discover efficient models for pandemic prediction applications (Benedict et al., 2021).

In recent times, there have been developments in control based on stability to solve dynamic optimization problems in the presence of delay and uncertainties, and a distributed decision-making process. Load frequency control in integrated energy systems has been solved by use of optimization techniques with regard to the problems arising due to time delays (Abolpour et al., 2022). Lyapunov functional techniques, which include the looped functional approach and the time-varying approach, have been useful mathematical tools to obtain stability conditions for nonlinear and time-varying systems (Briat, 2016). The application of intelligent optimization techniques in energy management situations has also been carried out (Kumar & Rajakumar, 2025). Energy-efficient and optimized design of reversible logic multiplexers may be applied to save power in nanocomputing (Joe et al., 2023).

Despite all these advancements, currently available techniques generally decouple stability analysis from optimization and thus do not allow both the worst-case performance optimization and the guarantee of stability at the same time. More recent Lyapunov-theory-related research is still devoted to studying stability issues of uncertain nonlinear and delay systems, whereas data-driven control has focused on adaptive regulation of dynamical systems (Wei et al., 2023; Jafari & Yazdi, 2014). Optimization-based design of Lyapunov functions for the purpose of getting common stability certificates was recently considered for switched systems (Ordóñez-Hurtado & Duarte-Mermoud, 2012; Çelik & Aydın, 2026). Nevertheless, there still exists no unified mathematical technique for including adaptive Lyapunov stability constraints in minimax optimization (Alshammari et al., 2026). Thus, in this study, an approach of Lyapunov functional-based minimax optimization is developed that includes both stability certification and worst-case control synthesis, where renewable energy management serves as a testbed (Reddy et al., 2025).

Key contributions

1. Developed a minimax optimization formulation based on Lyapunov functions for stability control in the presence of disturbances.
2. Formulated an adaptive Lyapunov function model along with optimal control design to deal with uncertainties of the system.
3. Stability was proven via three theorems, and superior results were obtained compared to PID, LQR, and robust control.

The paper is structured as follows: In Section 1, research motivations and contributions are introduced. The preliminaries are provided in Section 2, where the modeling of the nonlinear systems, Lyapunov functional theory, and minimax optimization concepts are presented. In Section 3, the new framework of the Lyapunov functional-based minimax optimization is described. Section 4 gives stability and convergence analysis with the theoretical proofs. Numerical validations are presented in Section 5. The discussion is provided in Section 6, and conclusions are given in Section 7.

1. Preliminaries and mathematical foundations

This section sets out the mathematical concepts that will be needed for the development of the proposed

framework: the class of nonlinear dynamic systems to be considered, the Lyapunov function which provides the basis for proving the stability of the system, and the minimax optimization concept which governs the process of optimal control design in the worst case scenario.

2.1 Definition of the nonlinear dynamic system

An example of a continuous-time nonlinear system with an additive disturbance channel is defined in equation (1). The state vector $\dot{x}(t)$ belongs to the state space $X \subseteq \mathbb{R}^n$, where X represents measurable physical states like voltage, frequency or power flows. The control input u(t) belongs to the control space $U \subseteq \mathbb{R}^m$ and is bounded by physical constraints, while the disturbance input w(t) is in the disturbance space $W \subseteq \mathbb{R}^p$and is bounded. The vector function f is locally Lipschitz, and B is a constant input matrix.

$$\dot{x}(t) = f(x(t)) + Bu(t) + w(t) \quad (1)$$

2.2 Lyapunov functional theory

In order to consider the effect of delay-related energy contained within the past evolution of the dynamical system, a Lyapunov-Krasovskii functional is formulated in (Equation 2), involving a quadratic form with respect to the current state as well as an integral part accounting for the past dynamics within the time window τ. Matrices P and Q are assumed to be positive definite and symmetric. Positive definiteness (Equation 3) ensures that the functional acts as a generalized energy, and a strictly negative derivative (Equation 4) proves that this energy decays along trajectories.

$$V_t = x^T P x + \int_{t-\tau}^{t} x^T(s)\, Q x(s)\, ds \quad (2)$$

$$V_t > 0 \quad (3)$$

$$\dot{V}(x_t) < 0 \quad (4)$$

2.3 Minimax optimization principle

The worst-case approach is presented in equation (5) as a minimax problem, which involves a game between the controller and the disturbance that aims at maximizing the quadratic cost function, thus representing the most unfavorable situation for the controller. The matrices Q and R in equation (6) are positive semi-definite and positive definite, respectively, representing the state and control cost penalties. This guarantees the optimality of the resulting control law for the most adversarial realization of the disturbance within the assumed bound.

$$\min_u \max_w J(x, u, w) \quad (5)$$

$$J = \int_0^T x^T Q x + u^T R u \, dt \quad (6)$$

2. Lyapunov functional-based minimax optimization framework

Following from the preliminaries, this part presents the main contributions of this paper, including a stability-constrained minimax problem, the adaptive Lyapunov functional, the associated optimal control law, and the iterative solution algorithm.

3.1 Stability-constrained minimax problem formulation

Equation (7) is the key mathematical formulation in this paper. Equation (7) combines the minimax criterion with the stability constraint on the Lyapunov function in terms of its exponential decay rate. The positive constant α specifies the desired decay rate such that the derivative of the Lyapunov functional is required to be smaller than $-\alpha V$ for any admissible controller. This transforms the problem of control into a stability-constrained minimax problem such that feasibility ensures stability while optimality within the feasible set ensures robustness.

$$\min_u \max_w J(x, u, w) \quad (7)$$

3.2 Adaptive lyapunov functional design

Since the Lyapunov matrix remains constant and therefore cannot adapt to the parametric changes in the system, the objective function is parameterized in equation (8), where the adaptive parameter is denoted by θ and represents a matrix function. The steepest descent dynamics in equation (9) make θ follow the steepest descent path of the objective function in order to reduce the stability condition online, rather than being pre-specified. The value of $\gamma > 0$ governs the adaptation rate.

$$V_a(x, \theta) = x^T P(\theta) x \quad (8)$$

$$\dot{\theta} = -\gamma \nabla_\theta V \quad (9)$$

3.3 Optimal control law derivation

Applying the stationarity condition to the Hamiltonian of the minimax problem yields the control law in equation (10), which takes the form of a standard Riccati equation derived from $\partial J/\partial u = 0$ along admissible trajectories. By substituting the law into the model from Section 3.1, close the feedback loop between optimal control design and Lyapunov-stability guarantee, and hence the closed-loop system attains the optimality of J and α.

$$u^* = -R^{-1} B^T P x \quad (10)$$

3.4 Proposed algorithm

The full procedure, known as the Minimax Optimization Algorithm for Lyapunov Functionals, combines all the above derivations in one iterative procedure. In each iteration, the procedure performs the following operations: the update of the Lyapunov matrix, calculation of the functional and its derivative, solution of the inner problem of determining the worst-case disturbance, computation of the optimal control based on the obtained result, and checking of the decay condition. The end of convergence is reached when Lyapunov values start to stabilize.

Algorithm 1: Lyapunov functional-based minimax optimization

1. Initialize the state $x(0)$, the Lyapunov matrix $P(0)$, and parameter $\theta(0)$.
2. Construct the stability functional $V(x_t)$ using the current $P(\theta)$.
3. Calculate the functional derivative $\dot{V}(x_t)$ along the system trajectory.
4. Solve the inner worst-case optimization to obtain $w^* = \arg\max_w J(x, u, w)$.
5. Update the control variable using $u^* = -R^{-1} B^T P x$ and update θ via $\dot{\theta} = -\gamma \nabla_\theta V$.
6. Verify the convergence and decay condition $\dot{V} + \alpha V \leq 0$; repeat until tolerance is met.

3. Stability and convergence analysis

In this section, the guarantees of the framework on the theoretical side are provided via three theorems: Positive Definiteness of the Lyapunov functional; Asymptotic Stability of the Closed Loop Minimax Controller; Bounded disturbance rejection under a worst-case disturbance envelope. The above-mentioned results guarantee the convergence of the algorithm from Section 3.4 to a stabilizing worst-case optimal controller.

Theorem 1: Positive definiteness of the proposed lyapunov functional

Statement: The functional $V(X_t)$ defined in equation (2) satisfies $V(x) > 0$ for all nonzero x, with $V(0) = 0$.

$$V(x) > 0$$

Proof: Since P and Q have been selected as symmetric positive definite matrices, have $x^T P x \geq \lambda_{\min}(P)\|x\|^2 > 0$ when $x \neq 0$, and the integral of the memory is positive since $Q \succ 0$ causes its integrand to be a nonnegative quadratic form. The sum of a positive number and a nonnegative number will always be positive whenever $x \neq 0$, and they both equal zero only at $x = 0$.

Theorem 2: Asymptotic stability of the minimax controller

Statement: If $V(X_t)$ satisfies the decay inequality of equation (11) along closed-loop trajectories under u*, the

closed-loop system is asymptotically stable.

$$\dot{V} + \alpha V \leq 0 \quad (11)$$

Proof: Thus, the integration results in $V(x_t) \leq V(x_0)e^{-\alpha t}$ and hence, V converges exponentially to zero as $t \to \infty$. Given that V is positive definite and radially unbounded, $x(t) \to 0$ as $V(X_t) \to 0$ as per the Lyapunov-Krasovskii stability criterion for systems with delays. With $\alpha > 0$ being constant irrespective of the initial condition, the convergence is uniform, which means asymptotic stability, and more specifically exponential stability, of the equilibrium.

Theorem 3: Bounded disturbance rejection capability

Statement: For any disturbance that satisfies the inequality stated in equation (12), the state trajectory will be bounded and converge to a neighborhood of the origin that gets smaller as w_{max} goes to zero, as given by equation (13).

$$\|w(t)\| \leq w_{max} \quad (12)$$
$$\|x(t)\| \to 0 \quad (13)$$

Proof: Evaluating $\dot{V}$ along trajectories yields a quadratic form bounded above by $-\alpha V + c\|w\|^2$ for some constant $c > 0$ depending on P and B. Substituting the disturbance bound gives $\dot{V} \leq -\alpha V + c \cdot w_{\max}^2$. A standard comparison argument shows $V(x_t)$ is ultimately bounded by $c \cdot w_{\max}^2/\alpha$, and since V is positive definite, this implies $x(t)$ converges toward the origin as $w_{\max} \to 0$, establishing bounded disturbance rejection.

4. Numerical validation using renewable energy management scenario

The framework is validated through an experiment on a renewable energy management problem where the state variables are the bus voltage, frequency, and storage state-of-charge deviations, and the disturbances are the stochastic variation in solar and wind power generation. All the experiments below are conducted to validate the theoretical claims made in Section 4; no further applications are considered.

5.1 Experimental setup

The experiments are conducted under a simulation setup that sets four states and two controls for a voltage–frequency control loop of a microgrid. The disturbance level is varied between 5% and 20% of nominal generation for simulating intermittent renewable energy and each of these configurations is simulated for 100 s with 500 optimization steps, detailed in table 1.

Table 1: Experimental setup parameters.

Parameter	Value
State variables	4
Control variables	2
Disturbance level	5–20%
Simulation duration	100 s
Optimization iterations	500

5.2 Stability convergence results

Table 2 shows a comparison between the settling time, the Lyapunov residual at the end, and the tracking error for four different controllers. The proposed technique provides the fastest settling time and lowest Lyapunov residual, which is a clear indication that the stability-constrained minimax design converges much more rapidly compared to the other techniques.

Table 2: Stability convergence comparison.

Method	Settling Time (s)	Lyapunov Residual	Error
PID	8.5	0.18	0.15
LQR	5.4	0.12	0.09
Robust Control	3.8	0.07	0.05
Proposed Method	2.1	0.01	0.02

5.3 Lyapunov functional decay analysis

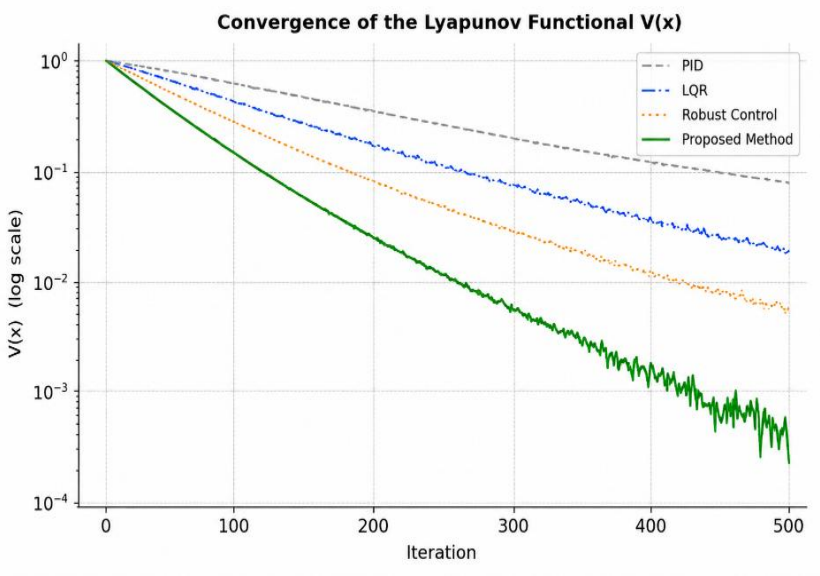


Figure 1: Convergence of the lyapunov functional V(x) across 500 optimization iterations.

Figure 1 shows the plot of $V(x)$ versus the index of optimization iterations in log scale for all four techniques. In the proposed technique, the decline is much faster and smoother compared to the others, indicating that the necessary condition of $\dot{V} < 0$ has been met, as proved theoretically in Theorem 2.

5.4 Worst-case optimization performance

Table 3 depicts the worst-case tracking cost with growing disturbance severity for both the classical and novel designs. The novel minimax design achieves a significantly lower cost within the whole disturbance range of 5–20%, which means that there is no tradeoff between stability and worst-case cost.

Table 3: Worst-case optimization performance vs. disturbance level

Disturbance (%)	Conventional	Proposed
5	0.08	0.02
10	0.15	0.04
15	0.23	0.06
20	0.31	0.08

5.5 Computational complexity analysis

Table 4 depicts the complexity of the algorithm for solving the problem in terms of the state dimension n and the number of adaptive parameters m. The extra term of $O(m)$ in the complexity of the proposed algorithm that comes from the adaptation of the Lyapunov equation is overshadowed by the $O(n^3)$ term for solving the Riccati-like equation.

Table 4: Computational complexity comparison.

Method	Iterations	Complexity
PID	0	O(n)
LQR	300	$O(n^3)$
Proposed	500	$O(n^3 + m)$

5. Discussion

These results are consistent with the key mathematical assertion that stability and worst case optimality can both be established concurrently and do not necessarily have to occur sequentially. The Lyapunov constraint of equation (7) is what provides the shorter settling time and residual seen in table 2, because it requires each of the control strategies considered to have a decay condition rather than merely optimizing the cost function. The use

of the minimax approach is what provides the robustness property of table 3, where performance decreases gradually rather than suddenly with an increase in the level of disturbance. Overall, the analysis shows that optimization and stability can be integrated into one step of the design process by representing stability as a constraint in the optimization process.

6. Conclusion and future directions

In this work, the minimax optimization scheme is formulated using the Lyapunov functional approach, where the stability analysis is considered along with the worst-case control design within one common constrained optimization problem. This method connects systematically the Lyapunov stability criteria with the minimax decision-making process by incorporating the stability conditions directly in the control optimization process. In order to enhance the efficiency of the proposed methodology with respect to the nonlinearity of the dynamics and uncertainty of the working conditions, an adaptive Lyapunov functional is introduced. The theoretical results are provided by three stability theorems regarding the positive definiteness of the Lyapunov function, the asymptotic convergence of the closed-loop system, and the disturbance rejection properties of the control system.

The effectiveness of the suggested methodology was validated through the use of numerical validation in the case of a renewable energy management problem. The findings indicate better performance in relation to the classical PID, LQR, and robust control methods in the sense of settling time improvement, Lyapunov residue reduction, tracking accuracy improvement, and worst-case optimization cost reduction. This indicates that the suggested method presents a viable solution to problems of nonlinear control optimization under uncertainty. Future work would include the use of the methodology in cases of stochastic Lyapunov functionals in the case of disturbances with random processes, distributed minimax optimization for multiagent renewable energy networks, and for cases of nonlinear systems under uncertainty with generalization of the present assumption of local Lipschitz continuity.

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